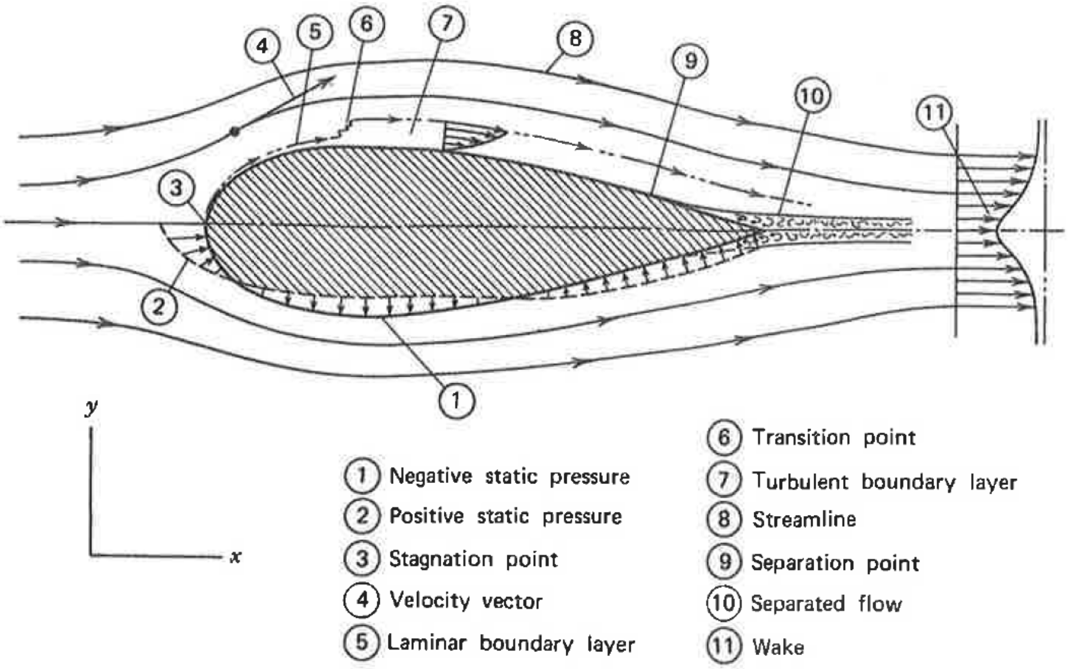
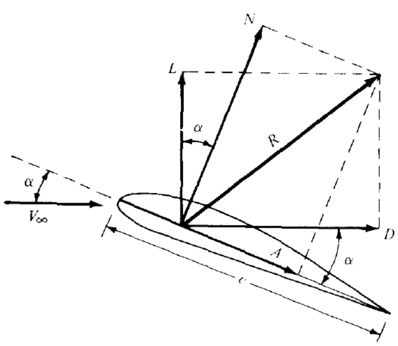
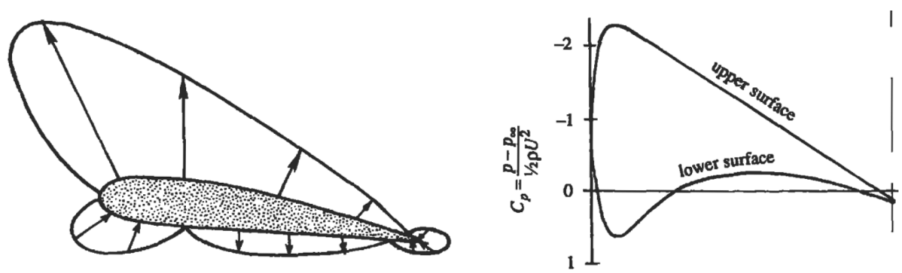
1. Fluid Dynamics Around Airfoils



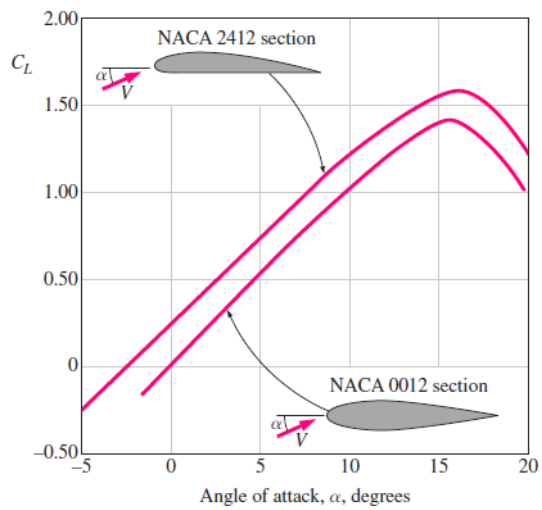
Two-dimensional flow around a streamlined shape





Foces on an airfoil

Distribution of pressue coefficient over an airfoil



The variation of the lift coefficient with the angle of attack for a symmetrical and non-symmetrical airfoil

2. Governing Equations

Conservation of mass: This equation describes the time rate of change of the fluid density at a fixed point in space.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

Conservation of momentum: Balance of Linear Momentum ($ma = \sum F$)

Momentum balance along the x-axis:

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \rho g_x$$

Momentum balance along the y-axis:

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \rho g_y$$

Constitutive laws: For a Newtonian fluid, the viscous stresses are proportional to the velocity gradients:

$$\begin{bmatrix} \tau_{xx} & \tau_{yx} \\ \tau_{xy} & \tau_{yy} \end{bmatrix} = \mu \begin{bmatrix} 2\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} & 2\frac{\partial v}{\partial y} \end{bmatrix}$$

Navier-Stokes Equations:

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \rho g_x$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \rho g_y$$

Or in vectorial form

$$\rho\left[\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V}\right] = -\vec{\nabla}p + \mu\nabla^2\vec{V} + \rho\vec{g}$$

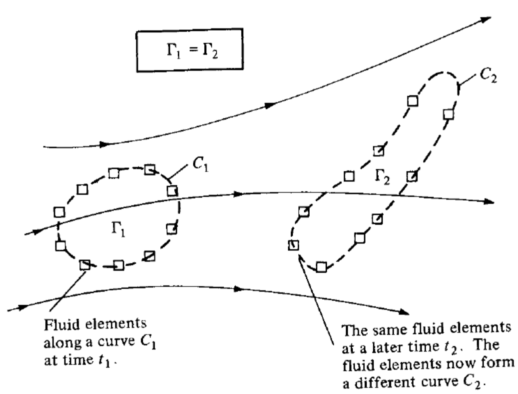
Using vector identities

$$\rho\left[\frac{\partial \vec{V}}{\partial t} + \vec{\nabla}\left(\frac{\vec{V}.\vec{V}}{2}\right) - \vec{V}\times\vec{\omega}\right] = -\vec{\nabla}p + \mu\left[\vec{\nabla}(\vec{\nabla}.\vec{V}) - \vec{\nabla}\times\vec{\omega}\right] + \rho\vec{g}$$

Bernoulli's Equation: Integrated forms of the simplified versions of the Navier-Stokes Equations, e.g. for unsteady irrational flows

$$\vec{\nabla}\left(\frac{\vec{V}.\vec{V}}{2}\right) + \frac{\vec{\nabla}p}{\rho} - \vec{g} = 0$$

Kelvin'sTheorem: In an incompressible inviscid flow with conservative body forces, the time rate of change of circulation around a closed curve consisting of the same fluid elements is zero, i.e, $\frac{D\Gamma}{Dt} = 0$



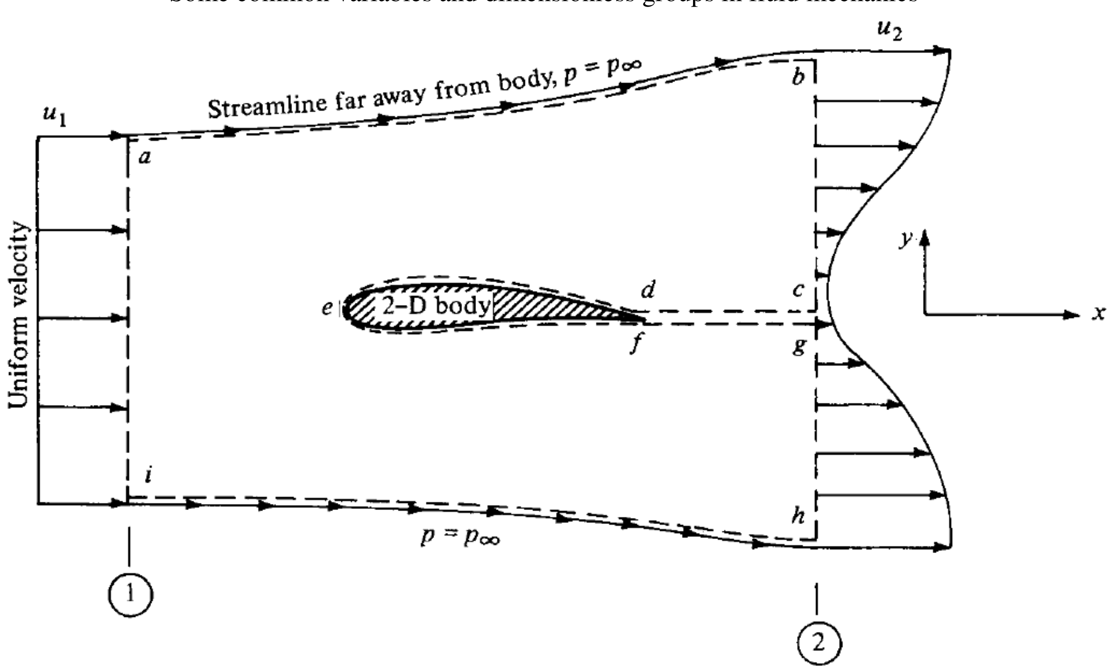
3. Dimensional Analysis and Control Volume Approach

Variables: Acceleration of gravity, g; Bulk modulus, E_v; Characteristic length, ℓ; Density, ρ; Frequency of oscillating flow, ω; Pressure, p (or Δp); Speed of sound, c; Surface tension, σ; Velocity, V; Viscosity, μ

Dimensionless Groups	Name	Interpretation (Index of Force Ratio Indicated)	Types of Applications
$\frac{\rho V \ell}{\mu}$	Reynolds number, Re	$\frac{\text{inertia force}}{\text{viscous force}}$	Generally of importance in all types of fluid dynamics problems
$\frac{V}{\sqrt{g\ell}}$	Froude number, Fr	$\frac{\text{inertia force}}{\text{gravitational force}}$	Flow with a free surface
$\frac{p}{\rho V^2}$	Euler number, Eu	$\frac{\text{pressure force}}{\text{inertia force}}$	Problems in which pressure, or pressure differences, are of interest
$\frac{\rho V^2}{E_v}$	Cauchy number,[a] Ca	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\frac{V}{c}$	Mach number,[a] Ma	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\frac{\omega \ell}{V}$	Strouhal number, St	$\frac{\text{inertia (local) force}}{\text{inertia (convective) force}}$	Unsteady flow with a characteristic frequency of oscillation
$\frac{\rho V^2 \ell}{\sigma}$	Weber number, We	$\frac{\text{inertia force}}{\text{surface tension force}}$	Problems in which surface tension is important

[a]The Cauchy number and the Mach number are related and either can be used as an index of the relative effects of inertia and compressibility. See accompanying discussion.

Some common variables and dimensionless groups in fluid mechanics

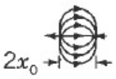
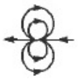
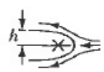


Control volume for obtaining drag on a two-dimensional body.

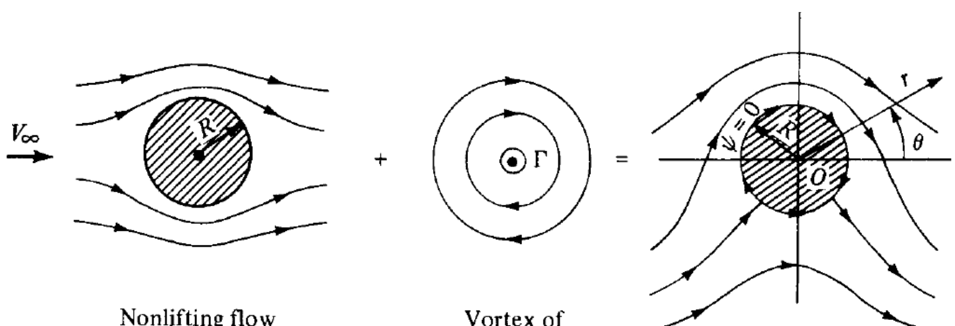
$$D = \rho u_1^2 \int_{y=h}^{y=b} \frac{u_2}{u_1}\left(1 - \frac{u_2}{u_1}\right) dy$$

The decrement of momentum flux is a direct measure of the body drag.

4. Potential Flow Theory

Flow	Streamlines	Stream Function ψ	Velocity Potential ϕ
Uniform flow in x direction		$V_\infty y$	$V_\infty x$
Source at the origin		$\frac{\Lambda}{2\pi}\theta$; $\quad \frac{\Lambda}{2\pi}\tan^{-1}\frac{y}{x}$	$\frac{\Lambda}{2\pi}\ln r$; $\quad \frac{\Lambda}{4\pi}\ln(x^2+y^2)$
Vortex at the origin		$\frac{\Gamma}{2\pi}\ln r$; $\quad \frac{\Gamma}{4\pi}\ln(x^2+y^2)$	$-\frac{\Gamma}{2\pi}\theta$; $\quad -\frac{\Gamma}{2\pi}\tan^{-1}\frac{y}{x}$
Source–sink pair		$-\frac{\Lambda}{2\pi}\tan^{-1}\frac{y/(x-x_0)-y/(x+x_0)}{1+[y^2/(x-x_0)(x+x_0)]}$;	$\frac{\Lambda}{4\pi}\ln\frac{(x+x_0)^2+y^2}{(x-x_0)^2+y^2}$
Doublet at the origin		$-\frac{\kappa}{2\pi}\frac{y}{x^2+y^2}$; $\quad -\frac{\kappa}{2\pi}\frac{\sin\theta}{r}$	$\frac{\kappa}{2\pi}\frac{x}{x^2+y^2}$; $\quad \frac{\kappa}{2\pi}\frac{\cos\theta}{r}$
Uniform flow past source		$V_\infty\left(\frac{h\theta}{\pi}-y\right)$; $\quad h=\frac{\Lambda}{2V_\infty}$	$V_\infty\left(\frac{h}{\pi}\ln r - x\right)$
Uniform flow past doublet		$V_\infty y\left(1-\frac{a^2}{r^2}\right)$	$V_\infty x\left(1+\frac{a^2}{r^2}\right)$
Uniform flow past circle with circulation		$V_\infty y\left(1-\frac{a^2}{r^2}\right)+\frac{\Gamma}{2\pi}\ln\left(\frac{r}{a}\right)$	$V_\infty x\left(1+\frac{a^2}{r^2}\right)-\frac{\Gamma}{2\pi}\theta$

Elementary flows, which can be superimposed to describe the flow around bodies of arbitrary shape.

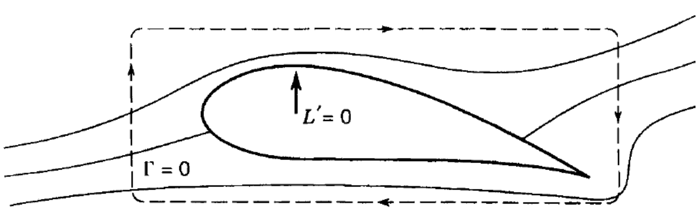


Nonlifting flow over a cylinder

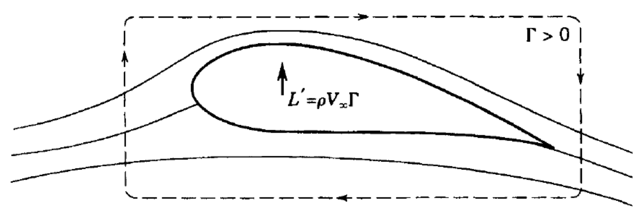
Vortex of strength Γ

Lifting flow over a cylinder

Doublet+vortex+uniform flow: synthesis of flow around circular cylinder with circulation



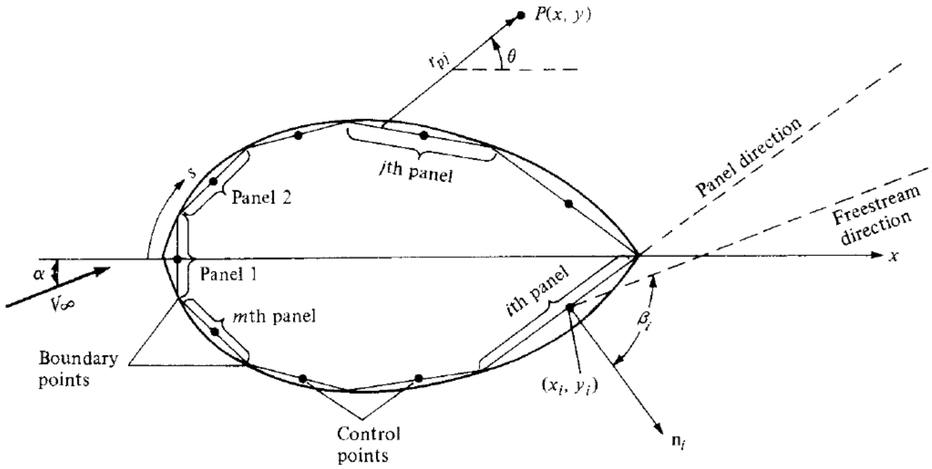
Irrotational flow around a nonsymetrical airfoil with zero circulation (zero lift)



Actual flow past a nonsymetrical airfoil with positive circulation (positive lift)

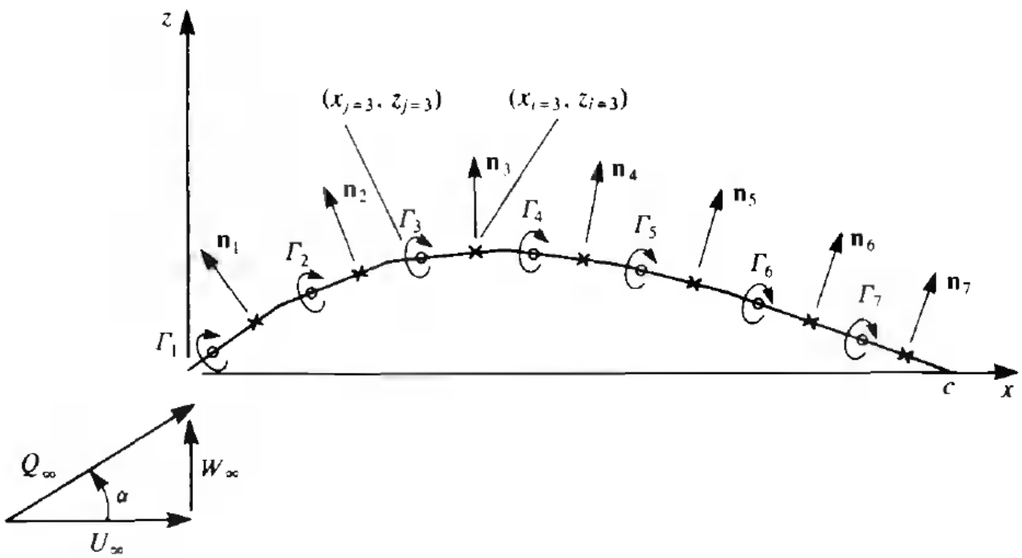
The Kutta-Joukowski theorem states that the force experienced by a body in a uniform stream is equal to the product of the fluid density, stream velocity, and circulation and has a direction perpendicular to the stream velocity, $L=\rho V_\infty\Gamma$.

5. Numerical (Panel) Method



Source panel distribution over the surface of a body of arbitary shape (for non-lifting bodies)

$$\frac{\lambda_i}{2} + \sum_{\substack{j=1 \\ (j \neq i)}}^{n} \frac{\lambda_j}{2\pi} \int_j \frac{\partial}{\partial n_i} (\ln r_{ij})\, ds_j + V_\infty \cos \beta_i = 0 \qquad (i = 1, 2, \ldots, n)$$

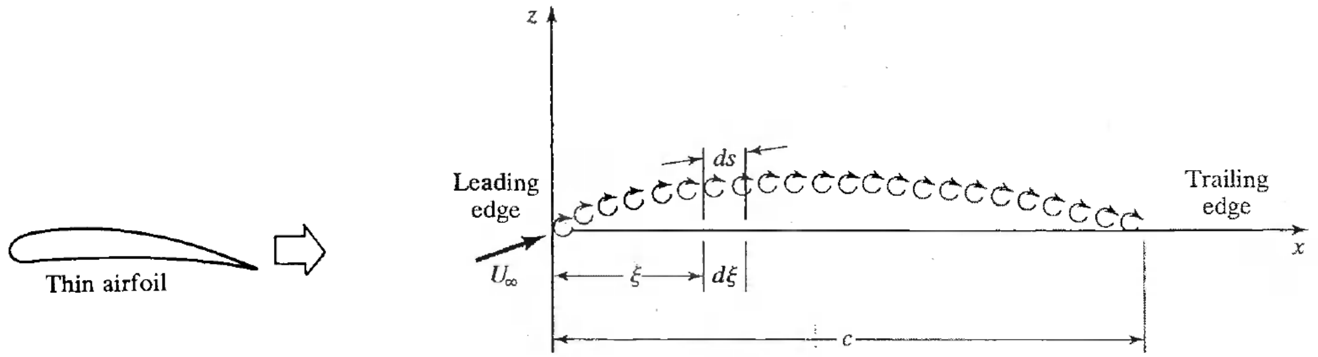


Discrete vortex representation of the thin, lifting airfoil model.

$$\mathbf{RHS}_i = -(U_\infty, W_\infty) \cdot \mathbf{n}_i$$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ a_{31} & a_{32} & \cdots & a_{3N} \\ \cdots & \cdots & \cdots & \cdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \cdots \\ \Gamma_N \end{pmatrix} = \begin{pmatrix} \mathrm{RHS}_1 \\ \mathrm{RHS}_2 \\ \mathrm{RHS}_3 \\ \cdots \\ \mathrm{RHS}_N \end{pmatrix}$$

6. Flow over Two-Dimensional Airfoil (Thin-Airfoil Theory)



Representation of the mean camber line by a vortex sheet whose filaments are of variable strength $\gamma(s)$

	Symmetric Airfoil	**Cambered Airfoil**
Chordwise circulation distribution, γ	$2\alpha V_\infty \dfrac{1+\cos\theta}{\sin\theta}$;	$2V_\infty\left[A_0 \dfrac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right]$
	$2\alpha V_\infty \sqrt{\dfrac{c-x}{x}}$	where $A_0 = \alpha - \dfrac{1}{\pi}\int_0^\pi \dfrac{dz}{dx}\,d\theta$
	where $x = \frac{1}{2}c(1-\cos\theta)$	$A_n = \dfrac{2}{\pi}\int_0^\pi \dfrac{dz}{dx}\cos n\theta\, d\theta$
Lift coefficient, c_l	$2\pi\alpha$	$2\pi\left[\alpha + \dfrac{1}{\pi}\int_0^\pi \dfrac{dz}{dx}(\cos\theta - 1)\,d\theta\right]$
Slope of c_l vs. α curve, m_0	2π	2π

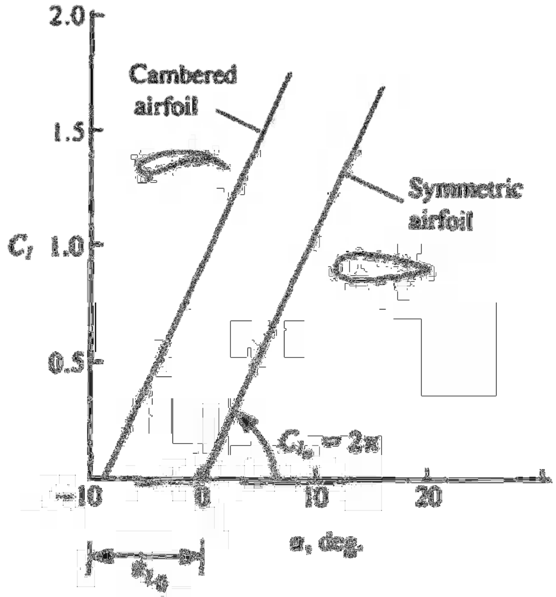
1. The lift slope of a two-dimensional airfoil is 2π.

2. The airfoil camber does not change the lift slope and can be viewed as an additional angle of attack effect.

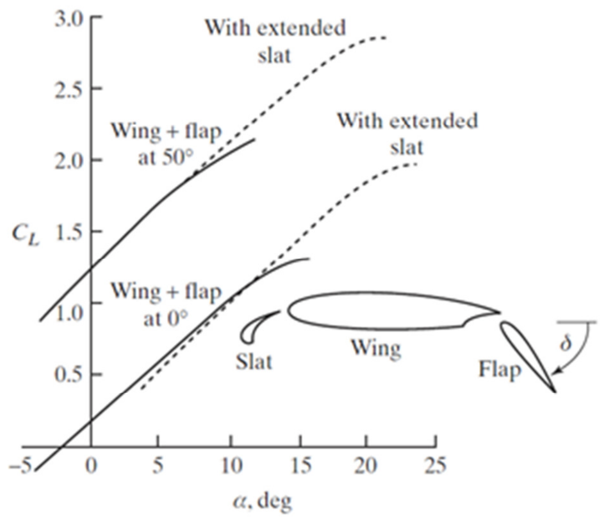
3. The trailing-edge section has a larger influence on the above camber effect. Therefore, if the lift of the airfoil needs to be changed without changing its angle of attack, then changing the chordline geometry (e.g. by flaps or slats) at the trailing-edge region is more effective than at the leading-edge region.

4. The effect of the thickness of the airfoil is not treated in a satisfactory manner by this approach.

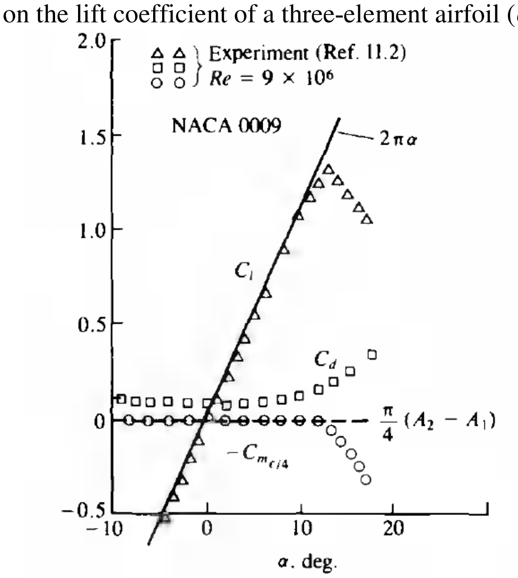
5. The two-dimensional drag coefficient obtained by this model is zero and there is no drag associated with the generation of two-dimensional lift. Experimental airfoil data, however, include drag due to viscous boundary layer on the airfoil.



Schematic description of airfoil camber effect on the lift coefficient

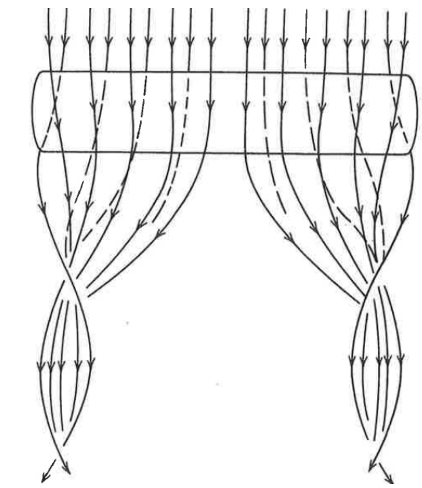


Effect of high-lift devices on the lift coefficient of a three-element airfoil (δ represents flap deflection)

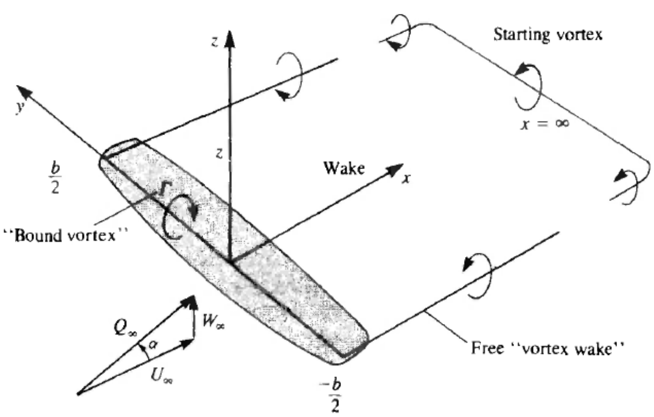


Lift and pitching moment of a NACA 0009 airfoil. The “zero-lift” drag coefficient is close to $C_d = 0.0055$.

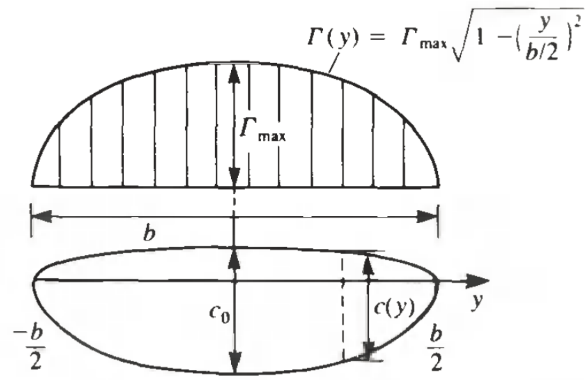
6. Flow Over Finite Wings (The Lifting Line Model)



Generation of vortex system by finite aspect ratio wing



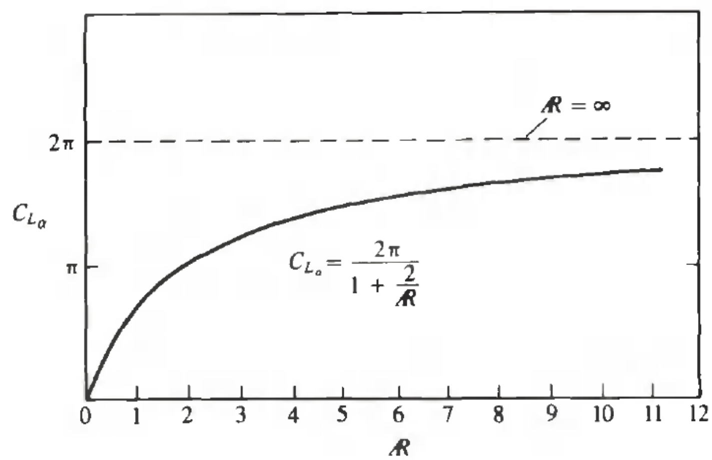
Far field horseshoe model of a finite wing



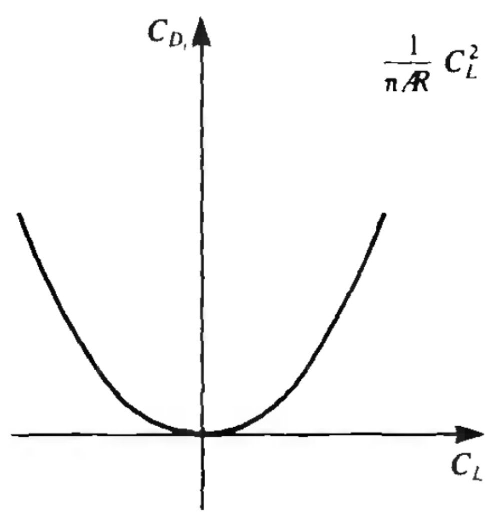
Chord and load distribution for a thin elliptic wing.

	Arbitrary Lift Distribution	**Elliptical Lift Distribution**
Spanwise circulation distribution,	$\frac{1}{2} m_{0s} c_s V_\infty \sum_{n=1}^{\infty} A_n \sin n\theta$	$\Gamma_s \sin\theta$; $\Gamma_s \sqrt{1-\left(\frac{y}{b/2}\right)^2}$
Sectional induced angle of attack, α_i	$-\frac{m_{0s} c_s}{4b} \sum_{n=1}^{\infty} nA_n \frac{\sin n\theta}{\sin\theta}$	$-\frac{C_L}{\pi AR}$
Sectional lift coefficient, c_l	$\frac{m_{0s} c_s}{c} \sum_{n=1}^{\infty} A_n \sin n\theta$	C_L
Sectional induced drag coefficient, c_{d_i}	$\frac{m_{0s}^2 c_s^2}{4bc} \left(\sum_{n=1}^{\infty} A_n \sin n\theta\right)\left(\sum_{k=1}^{\infty} kA_k \frac{\sin k\theta}{\sin\theta}\right)$	C_{D_i}
Sectional lift curve slope, m	$\frac{m_0}{1-\frac{m_0 \alpha_i}{c_l}}$	$\frac{m_0}{1+m_0/\pi AR}$
Wing lift coefficient, C_L	$\frac{m_{0s} c_s \pi b}{4S} A_1$	$\frac{\Gamma_s \pi b}{2V_\infty S}$
Wing induced drag coefficient, C_{D_i}	$\frac{m_{0s}^2 c_s^2 \pi}{16S} \sum_{n=1}^{\infty} nA_n^2$	$\frac{C_L^2}{\pi AR}$

1. The wing lift slope $dC_L/d\alpha$ decreases as wing aspect ratio becomes smaller.
2. The induced drag of a wing increases as wing aspect ratio decreases.
3. Using the results of this theory we must remember that the total drag D of a wing includes the induced drag D_i and the viscous drag D_0.

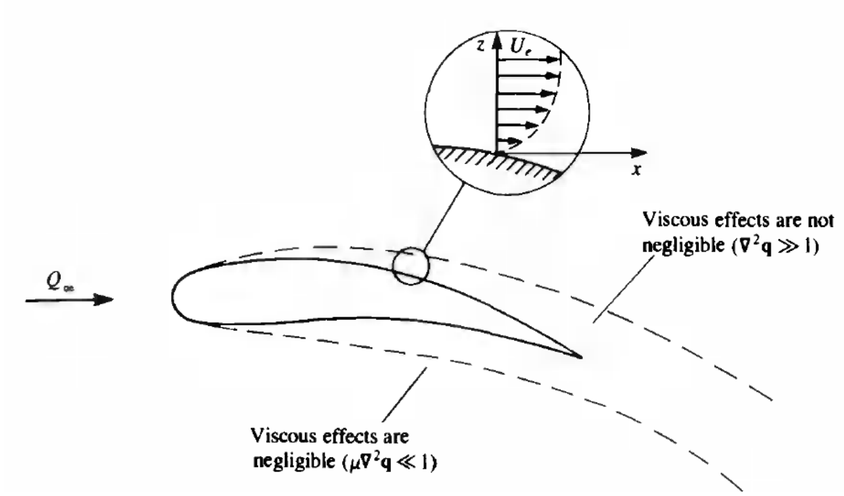


Variation of lift coefficient slope versus aspect ratio for thin elliptic wings.

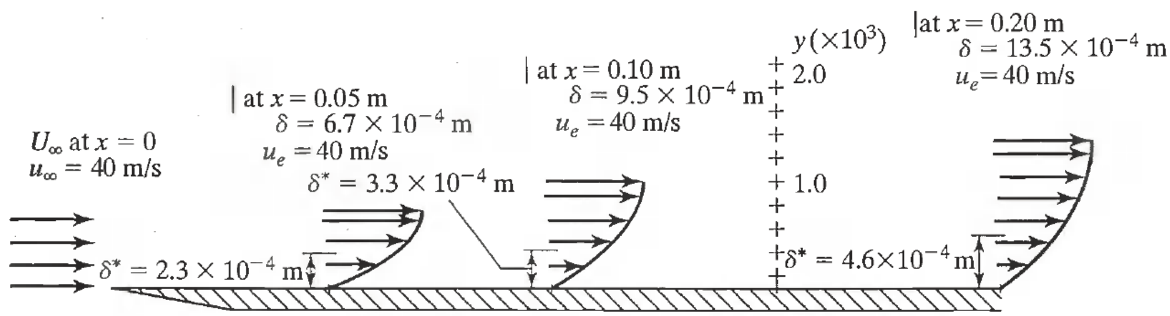


Induced drag for a finite elliptic wing versus C_L

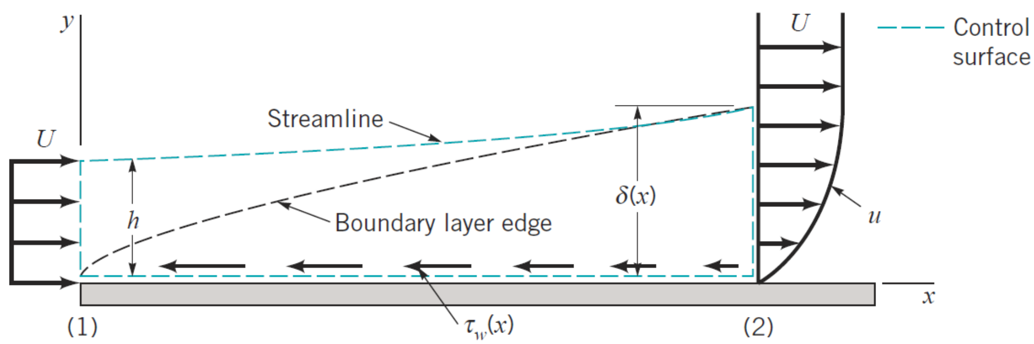
7. Viscous Flow and Boundary Layer Theory



Flow regions in a high Reynolds number flow



Velocity profile for the flat-plate laminar boundary layer $Re_c = 5.477 \times 10^5$



Control volume to derive the momentum integral equation for boundary layer flow.

Displacement thickness
$$\delta^*(x) = \int_0^\infty \left(1 - \frac{u(x,y)}{U}\right) dy$$

Momentum thickness
$$\Theta(x) = \int_0^\infty \frac{u(x,y)}{U}\left(1 - \frac{u(x,y)}{U}\right) dy$$

$$\mathrm{D}(x) = \rho U^2 \Theta(x)$$
$$\tau_w(x) = \rho U^2 \frac{d\Theta(x)}{dx}$$

Von Karman Momentum Integral: For an accelerating/decelerating boundary layer flow
$$\frac{d}{dx}\left(U(x)^2\Theta(x)\right) + U(x)\frac{dU(x)}{dx}\delta^*(x) = \frac{\tau_w(x)}{\rho}$$