Mergers in Innovative Industries: The Role of Product Market Competition*

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Abstract

We study how competition affects innovation (and welfare) when firms compete both in the product market and in innovation development. This relationship is complex and may lead to scenarios in which a lessening of competition increases R&D and consumer welfare in the long run, contradicting arguments provided by antitrust agencies in recent merger cases. We provide conditions for when a merger increases industry innovation and welfare, and when evaluating mergers based on static price effects is aligned with a fully dynamic merger evaluation. These conditions are based on properties of the product market payoffs.

JEL: D43, L40, L51, O31, O34, O38

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1 Introduction

Merger policy is based on the premise that a reduction in competition is likely to hurt consumers. In innovative industries, however, the role of competition on market outcomes is far less clear. For instance, Aghion et al. (2005) empirically find a non-monotonic relationship between competition and patenting, which raises the possibility that a lessening of competition may benefit consumers through enhanced innovation. In recent merger cases, however, the FTC and the DOJ have both argued that mergers would reduce incentives to innovate.[1] Since innovation is the engine of a growing economy, understanding how mergers affect R&D and welfare is critical.

In this paper we study how a merger—through its impact on product market competition—affects firms' incentives to innovate and, ultimately, consumer welfare. The motivation behind focusing on the role played by product market competition stems from the observation that firms invest in R&D because they wish to gain a product market advantage (e.g., a higher product quality increases demand, all else equal). Because mergers directly impact product market payoffs—hence, affecting the incentives to invest in R&D—accounting for the product market is crucial in assessing the real impact of a merger in an innovative industry. To this end, we propose a dynamic model of an innovative industry that accommodates arbitrary forms of product market competition (e.g., quantity competition with homogeneous goods, price competition with differentiated products, etc.) and study how competition affects innovation outcomes. In order to isolate the role of product market competition, in the baseline model we shut down forces that we acknowledge may be important for merger analysis (e.g., merger-specific R&D efficiencies) but interfere with the objective of understanding the role of product market competition on the firms' incentives to invest in R&D. In the extensions, we incorporate some of these forces to show that the main findings carry over to these richer environments.

In concrete terms, we develop a sequential extension to the classic patent-race models (Loury 1979, Lee and Wilde 1980, and Reinganum 1982) by allowing firms to compete both in developing a series of innovations and in the product market. We allow for *large* firms that compete in both developing innovations

[1] See, for instance, the complaint filed by the FTC concerning the merger between Pfizer Corporation and Wyeth Corporation, as well as the complaints filed by the DOJ concerning the merger between Regal Beloit Corporation and A.O. Smith Corporation and the merger between The Manitowoc Company, Inc. and Enodis plc.

and the product market as well as research *labs* that only compete in developing innovations. The distinction between large firms and labs captures the fact that firms are asymmetric in both size and scope in many innovative industries (e.g., pharmaceutical industry). Through successful innovation, a large firm becomes the market leader, replacing the previous leader. When a research lab successfully innovates, it auctions the innovation to a large firm, which results in a new industry leader. Being the leader provides a firm with an advantage in the product market—for instance, due to a cost or quality advantage—which creates a positive *profit gap* between the leader and the other firms. The incentives to innovate are precisely driven by this profit gap between the leader and the other firms. A merger between large firms is allowed to affect product market profits and, consequently, the profit gap between the leader and the other firms.

Mergers affect innovation through two channels. First, holding product market profits equal, a reduction in the number of firms performing R&D reduces the pace of innovation in the industry. Most of the patent race literature has focused on this first mechanism. Secondly, because a merger has a direct effect on the product market payoff and, consequently, the profit gap that exists between the leader and the other firms, mergers affects the incentives to innovate. Depending on the specifics of the product market competition, a merger may increase or decrease the profit gap between leaders and followers. This creates a potentially countervailing effect on the incentives to innovate, which may lead to a net increase in R&D outcomes despite a smaller number of firms performing R&D.

We find that the conjunction of these effects may generate a monotonic-increasing or non-monotonic relationship (e.g., inverted-U or N shaped) between R&D outcomes and the number of large firms. The potentially non-monotonic relationship between the number of large firms and R&D stands in contrast to the argument provided by the FTC and the DOJ that mergers reduce the incentives to innovate (see Footnote 1); and, implies that rejecting a merger due to a lessening of product market competition may not be appropriate. For instance, reduced competition in the product market may increase the firms' incentives to invest in R&D and also increase the rate at which innovations hit the product market. The increased arrival rate of innovations may more than compensate for the welfare loss that results from the static price effects created by a merger.

Using the profit gap between the leader and followers, we link the nature of product market competition with merger evaluation. We show that when the profit gap is *weakly increasing* in the number of large firms, a merger always reduces the industry's innovation rate.[2] In such a case, the negative impact of a merger on innovation reinforces any positive price effects created by the merger. Hence, rejecting a merger based on static price effects is aligned with a dynamic merger evaluation that rejects a merger if the merger decreases the present value of consumer surplus—where the change in the present value of consumer surplus is computed based on how the merger affects prices and innovation outcomes into the future.

We find that a profit gap between the leader and followers that is *decreasing* in the number of large firms is necessary but not sufficient for a merger to increase the industry's innovation rate. When the number of research labs is sufficiently large, however, a profit gap that is decreasing in the number of large firms is sufficient for a merger to increase the industry's innovation rate. Hence, when there are no concerns that a merger may increase prices, a decreasing profit gap—in the presence of a sufficiently large number of labs—is sufficient to establish that the merger is welfare improving, as it increases industry R&D and does not substantially impact prices. Thus, in this case, approving a merger based on static price effects is aligned with a dynamic merger evaluation.

These results highlight the importance of product market competition on the impact of a merger on innovation outcomes. They also show that the (commonly provided) argument that a merger reduces incentives to innovate does not always hold. It is only necessary to analyze properties of the product market payoffs in order to check for whether a merger increases R&D and for the alignment of static and dynamic merger evaluation. Analyzing properties of the product market payoffs is simple, as it does not require solving nor estimating a dynamic model where firms compete along multiple dimensions. Moreover, an empirical assessment of these properties requires no more information than what is commonly used for static merger simulations.

Finally, a merger may increase both the pace of innovation and prices in the short run, implying that evaluating a merger based on static price effects may not be aligned with a merger evaluation based on dynamic effects. For the merger to increase consumer surplus, the dynamic benefits of a greater rate of innovation must compensate for the short-run price effects created by the merger. To this end, we provide a sufficient condition for a merger to be consumer-surplus enhancing.

[2] See Section 3 for examples where the profit gap is weakly increasing or decreasing.

The existence of a profit gap between the leader and the followers that is decreasing in the number of firms is necessary for this condition to hold. We also provide numerical examples that show that a merger may enhance consumer surplus in the long run, even when prices increase in the short run.

The rest of the paper is organized as follows. Section 2 introduces the model and characterizes the equilibrium. Section 3 analyzes how market structure affects innovation and welfare outcomes and discusses implications for merger analysis. Section 4 extends the baseline model to study how the economic forces isolated in the baseline analysis carry over to richer environments. Section 5 provides numerical examples to illustrate the results. Lastly, Section 6 concludes.

1.1 Literature Review

The long-standing question of how competition affects the incentives to innovate stems from the work of Schumpeter (1942). Early work formalizing the ideas surrounding this question considered one-shot innovations, omitting both dynamic considerations and the role of product market competition (Loury 1979, Lee and Wilde 1980, and Reinganum 1982). An exception is Vives (2008), Ishida et al. (2011), and Motta and Tarantino (2016), who analyze the connection between product market competition and innovation incentives in the context of a static model with a deterministic innovation technology. Unlike the results in Vives (2008) and Motta and Tarantino (2016), we find that a reduction in the number of competitors can increase R&D expenditure per firm as well as total R&D in the market. Our results are aligned with Ishida et al. (2011), who find that the relationship between the industry-wide pace of innovation and competition can take various shapes in asymmetric environments.

Recent work has incorporated dynamics by assuming the existence of a sequence of innovations to answer various questions. Aghion et al. (2001), Aghion et al. (2005), and Aghion et al. (2015) study the impact of product market competition on R&D. These papers model the product market as a duopoly, where the intensity of competition is captured by the degree of substitution/collusion among the two firms. We build upon this literature by explicitly modelling the product market as competition between n firms and interpreting mergers as a reduction of competition. By doing so, we learn that the incentives to innovate not only depend of product substitution but also on how the firms compete. More so, this allows us to directly relate the R&D effects of a merger with observable market characteristics.

Segal and Whinston (2007) study how antitrust regulation shapes R&D outcomes by affecting the profit division between an innovating entrant and a stagnant incumbent. Acemoglu and Akcigit (2012) study the benefits of an IP policy that is contingent upon firms' relative progress in a step-by-step innovation framework. Parra (2016) studies optimal patent policy considering the nonstationary incentives of an incumbent who faces increased incentives to innovate as the patent expiration date approaches.

Our paper also relates to the horizontal merger literature. Farrell and Shapiro (1990) extend the ideas presented in Williamson (1968) and find sufficient conditions for mergers to enhance consumer surplus in a static framework. Gowrisankaran (1999) studies industry dynamics in a model with and without a process of endogenous mergers. Nocke and Whinston (2010) study conditions under which applying a static merger-review policy is optimal for a sequence of endogenous mergers. In contrast, we introduce innovation competition into the model and examine conditions under which a merger evaluation based on a static-price-effects criterion is aligned with a criterion considering both price and innovation effects from a dynamic standpoint. Nocke and Whinston (2013) study the optimal merger-review policy when the antitrust authority observes the characteristics of proposed mergers but cannot observe the characteristics nor the feasibility of mergers that are not proposed. Mermelstein et al. (2015) analyze the endogeneity between merger policy and investment decisions in a model where firms grow—either by accumulating capital or through mergers—to reduce their marginal cost of production.

Several authors have discussed, at a conceptual level, how innovation considerations should be incorporated into merger analysis (see, for instance, Gilbert and Sunshine 1995, Evans and Schmalansee 2002, Katz and Shelanski 2005, 2007). Igami and Uetake (2015) empirically study the impact of mergers on innovation in the hard-drive industry, analyzing how counterfactual merger policies would have affected market structure and performance. Hollenbeck (2015) incorporates innovation into the model developed in Mermelstein et al. (2015) and simulates the impact of mergers on R&D outcomes. Entezarkheir and Moshiri (2015) present cross-industry evidence on the impact of mergers on firms' patent citations, and Ornaghi (2009) presents a similar analysis for a richer set of R&D outcome variables but focused on the pharmaceutical industry. Lastly, Marshall and Parra (2015) analyze how the trade-offs isolated in this paper are affected by allowing for an endogenous market structure. It is shown that allowing for entry and exit creates a tension, as a merger with efficiencies may magnify market concentration by inducing non-merging firms to exit, resulting in amplified post-merger price effects.

2 A Model of Sequential Innovations with Product Market Competition

Consider a continuous-time infinitely lived industry where $n+m+1$ firms compete in developing new innovations (or products). Among these, $n+1$ firms are *large* in the sense that they also compete in the product market commercializing the innovations. The remaining m firms auction their innovations to the large firms; we call the latter set of firms *research labs*.

Competition in the product market is characterized by one technology leader and $n > 0$ symmetric followers (or competitors). For tractability purposes, we assume that the market leader is always one step ahead of the followers in terms of the technology to which they have access.[3] We relax this assumption in Section 4. The market leader obtains a profit flow $\pi_n^l > 0$, whereas each follower obtains a profit flow $\pi_n^f \in [0, \pi_n^l)$. Both π_n^l and π_n^f are weakly decreasing in the number of product market competitors in the industry (i.e., large firms), n, capturing that more intense product market competition decreases firm profits. For the purpose of reducing the dimensionality of the state space, we assume that the profit flows are stationary in the number of innovations. These assumptions allow for general forms of product market competition. For instance, firms can compete through prices, quantities, or qualities. They also allow for competition in various types of innovations. Firms may compete in developing process innovations, quality improvements, or products that leave previous vintages obsolete.[4]

Research labs do not compete in the product market and their only source of profits is the revenue they derive from selling their innovations to large firms. We assume that research labs sell their innovations using a second-price auction. In

[3] More precisely, this common assumption in the literature can be distilled as the conjunction of two independent assumptions about the nature of patent protection: a) a patent makes full disclosure of the patented technology, which allows followers to build upon the latest technology, leap-frogging the leader once they achieve an innovation; b) the legal cost of enforcing older patents more than exceeds the benefits of enforcing the patent.

[4] Sections 3 and 5 provide examples where all the assumptions of the model are satisfied.

case of a tie, we assume that the innovation is randomly assigned to one of the tying followers.[5] All firms discount their future payoffs at a rate of $r > 0$.

At each instant in time, every follower and research lab invests in R&D in order to achieve an innovation. Firm i chooses a Poisson innovation rate x_i at a cost of $c(x_i)$. We assume that $c(x_i)$ is strictly increasing, twice differentiable, strictly convex (i.e., $c''(x) > 0$ for all $x \geq 0$), and satisfies $c'(0) = 0$. The assumption that large firms and labs are equally productive along the R&D dimension is for notational ease. Introducing asymmetries does not impact our results in a significant way. We also assume that the Poisson processes are independent among firms, generating a stochastic process that is memoryless.

We focus on symmetric and stationary Markov perfect equilibria by using a continuous-time dynamic programming approach. Our assumptions guarantee the concavity of the value functions, implying equilibrium uniqueness.

Let $V_{n,m}$ represent the value of being the market leader, $W_{n,m}$ the value of being a follower, and $L_{n,m}$ the value of being a research lab when there are n followers and m labs in the industry. At time t, we can write the payoffs of the different types of firms as follows:

$$\begin{aligned}
V_{n,m} &= \int_t^\infty (\pi_n^l + \lambda_{n,m} W_{n,m}) e^{-(r+\lambda_{n,m})(s-t)} ds, \\
W_{n,m} &= \max_{x_i} \int_t^\infty (\pi_n^f + x_i V_{n,m} + x_{-i} W_{n,m} - c(x_i)) e^{-(r+\lambda_{n,m})(s-t)} ds, \\
L_{n,m} &= \max_{y_i} \int_t^\infty (y_i (V_{n,m} - W_{n,m} + L_{n,m}) + y_{-i} L_{n,m} - c(y_i)) e^{-(r+\lambda_{n,m})(s-t)} ds,
\end{aligned}$$

where $\lambda_{n,m} = \sum_i^n x_i + \sum_j^m y_j$ is the industry-wide *pace* or *speed* of innovation, $x_{-i} = \lambda_{n,m} - x_i$, and $y_{-i} = \lambda_{n,m} - y_i$. To understand the firms' payoffs, fix any instant of time $s > t$. With probability $\exp(-\lambda_{n,m}(s-t))$, no innovation has arrived between t and s. At that instant, the leader receives the flow payoff π_n^l and the expected value of becoming a follower, $\lambda_{n,m} W_{n,m}$. Each follower receives the flow payoff π_n^f; innovates at rate x_i; earns an expected payoff of $x_i V_{n,m}$; pays the flow cost of its R&D, $c(x_i)$; and faces innovation by other firms at rate x_{-i}. Note that since all large firms are symmetric, they value an innovation in $V_{n,m} - W_{n,m}$. These valuations, in conjunction with the auction format, imply that labs sell their innovations at price $V_{n,m} - W_{n,m}$ in equilibrium.[6] Labs obtain this revenue at rate

[5] This assumption simplifies exposition and does not affect the results of the paper.

[6] Since the winning bidder of an auction held by a lab earns zero surplus, we do not include

y_i; pay the flow cost of their R&D, $c(y_i)$; and face innovation by other firms at rate y_{-i}. All of these payoffs are discounted by $\exp(-r(s-t))$.

We solve the problem above by making use of the principle of optimality, which implies that, at every instant of time, the values must satisfy

$$rV_{n,m} = \pi_n^l - \lambda_{n,m}(V_{n,m} - W_{n,m}), \tag{1}$$

$$rW_{n,m} = \max_{x_i} \ \pi_n^f + x_i(V_{n,m} - W_{n,m}) - c(x_i), \tag{2}$$

$$rL_{n,m} = \max_{y_i} \ y_i(V_{n,m} - W_{n,m}) - c(y_i). \tag{3}$$

In words, the flow value of being the market leader at any instant of time, rV_n, is equal to the profit flow obtained at that instant plus the expected loss if an innovation occurs, $\lambda_{n,m}(W_{n,m} - V_{n,m})$. The instantaneous value of being a follower, $rW_{n,m}$, is equal to the profit flow plus the expected incremental value of becoming the leader, $x_i(V_{n,m} - W_{n,m})$, minus the flow cost of R&D. Finally, the flow value of being a research lab is equal to the expected payoff of successfully innovating and selling an innovation, $y_i(V_{n,m} - W_{n,m})$, minus the flow cost of R&D.

In the context of this model, the infinitely long patent protection and the assumption that a new innovation completely replaces the old technology implies that the incumbent has no incentives to perform R&D. That is, the leader's lack of R&D is an implication of our modeling choices rather than an assumption; see Parra (2016) for a formal proof. This result implies that a merger to monopoly—with the market leader being the only firm in the industry—reduces the pace of innovation to zero. In Section 4, we extend the model to allow for the leader to increase the quality of its innovation, attenuating the leader's replacement effect, inducing the leader to invest in R&D.

Maximizing value functions (2) and (3) and imposing symmetry among followers and research labs, we obtain $x_i = y_i = x^*_{n,m}$, where

$$c'(x^*_{n,m}) = V_{n,m} - W_{n,m} \tag{4}$$

or $x^*_{n,m} = 0$ if $c(0) > V_{n,m} - W_{n,m}$, where the subindices n and m capture how market structure affects R&D decisions. Equation (4) tells us that, at every instant of time, the followers and research labs invest until the marginal cost of increasing their arrival rate is equal to the incremental rent of achieving an innovation. Strict

auction payoffs in the value functions of the leader and followers.

convexity implies that condition (4) can be inverted so that $x^*_{n,m} = f(V_{n,m} - W_{n,m})$, where $f(z)$ is a strictly increasing function of z.[7] By replacing $x^*_{n,m}$ into equations (2) and (3), we can solve the game and prove the following proposition.

Proposition 1 (Market equilibrium)**.** *There is a unique symmetric equilibrium, which is determined by the solution of the system of equations (1–4).*

It can be easily verified that the payoffs in this model possess the expected comparative statics for given values of n and m. For instance, the value functions increase with larger profit flows or a lower interest rate (all else equal).

3 Mergers and Market Outcomes

To identify and characterize the basic trade-offs that arise when a merger in an innovative industry takes place, we study how a change in market structure affects R&D outcomes and, more generally, consumer welfare.

In the context of this model, a merger between large firms is interpreted as a lessening of product market competition and as a decrease in the number of firms performing R&D. While we recognize that merged firms may benefit from synergies when coordinating their research activities, the purpose of this work is to explore how product market competition affects firms' incentives to invest in R&D. Since the role of product market competition is independent of the existence of R&D synergies, we abstract away from this source of efficiency as a means of keeping the analysis tractable and limit our discussion of synergies to Section 4.

We note, however, that the lack of R&D synergies does not change the fact that firms may have incentives to merge. As illustrated by our examples in Section 5, mergers arise endogenously both because of the existence of (duplicated) R&D fixed costs and because of how the merger changes competition. We also show in Section 5 that incentives to merge do not imply that a merger will increase (or decrease) welfare. Because of this, one can interpret our analysis as an antitrust authority unexpectedly changing merger policy and allowing for one merger to happen.

In what follows, we define a merger as being desirable in the *static* sense if it increases (the flow of) consumer surplus at the very moment when the merger takes place. A merger is not desirable in the static sense, for instance, if prices

[7] This function is further characterized in Lemma 3 in the Appendix.

increase immediately after the merger. We define a merger as desirable in the *dynamic* sense if it increases the expected discounted consumer surplus. Likewise, we define a static (dynamic) merger-review criterion as one that approves a merger if and only if it is desirable in the static (dynamic) sense.

When two firms merge, we find that it affects dynamic incentives to invest in R&D through two channels: *product market competition* and *innovation competition*. We explore how these two forms of competition interact in determining the pace of innovation in the industry. We provide sufficient conditions under which a merger would decrease (increase) the pace of innovation, so that the rejection (approval) of a merger using a static merger-review criterion is further justified due to a lower (higher) pace of innovation. In such circumstances, we say that the static and dynamic merger-review criteria are aligned. The sufficient conditions for the static and dynamic merger-review criteria to be aligned are based on product market competition payoffs and, consequently, only require information for the estimation of a (static) demand. Finally, we provide a sufficient condition that guarantees that a merger is consumer-surplus enhancing from a dynamic standpoint. This last condition is also based on product market competition payoffs, and is of use when the sufficient conditions for the alignment of the static and dynamic merger-review criteria are not satisfied.

3.1 Pace of Innovation

We begin our analysis by considering how an *isolated* change in innovation competition or product market competition affects innovation outcomes. While mergers between large firms in practice affect both forms of competition simultaneously, this exercise gives us a first approach to understanding how each form of competition affects R&D outcomes. A key element in our analysis is the *profit gap* between the leader and a follower, $\Delta\pi_n \equiv \pi_n^l - \pi_n^f$. While most models of competition predict that both π_n^l and π_n^f are weakly decreasing in n; the profit gap can either increase or decrease with n even when both π_n^l and π_n^f are weakly decreasing in n (see examples in Table 1).

Proposition 2 (Product market and innovation competition)**.** *Competition affects innovation outcomes through two channels:*

i) Product market competition: *Fixing the number of firms, an increase in the profit gap between the leader and a follower, $\Delta\pi_n$, increases each firm's R&D investment, $x^*_{n,m}$, and the pace of innovation in the industry, $\lambda_{n,m}$.*

ii) Innovation competition: *A decrease in the number of research labs, m, decreases the overall pace of innovation in the industry, $\lambda_{n,m}$, but increases each firm's R&D investment, $x^*_{n,m}$.*

Firms' incentives to invest in R&D are driven by the incremental rent obtained from an innovation (see equation (4)). Proposition 2 tells us that a key object behind the incremental rent is the profit gap between the leader and the followers, as a larger profit gap increases the pace of innovation. Because a merger between large firms leads to product market concentration, the merger affects the relative profit earned by a market leader and its followers. This change in profits alters the profit gap and, ultimately, the incentives to invest in R&D.[8] As we shall see later, the profit gap is key to understand the impact of a merger on the pace of innovation.

From Proposition 2 we also learn that innovation competition affects the pace of innovation in the industry directly, through the number of firms performing R&D, and indirectly by altering the incremental rent of an innovation. To understand this last effect, suppose two research labs merge into one. Since research labs do not compete in the product market, the profit flows of the leader and followers are unaltered. This reduction in the number of firms performing R&D has a direct negative effect on the pace of innovation in the industry, $\lambda_{n,m}$ (i.e., fewer firms performing R&D). However, this reduction in $\lambda_{n,m}$ increases the expected time between innovations, extending the lifespan of a leader and raising the value of being a market leader, $V_{n,m}$. This increase in value induces the incremental rent of an innovation to rise, leading firms to invest more in R&D, which partially reverses the impact of the decrease in the number of firms performing R&D on the pace of innovation.[9] The following corollary is immediate from the previous discussion.

Corollary 1. *A merger between research labs decreases the pace of innovation.*

Proposition 2 illustrates how product market competition and innovation competition affect the incentives to innovate in isolation. As noted above, a merger

[8] It is through this channel that our analysis differs from the growth through innovation literature (e.g., Aghion et al. 2001), which has examined how the intensity of product market competition—captured by the degree of substitution among a fixed number of firms or the degree of collusion between firms—affects innovation. In our analysis, we explicitly study how a change in the number of competitors affects innovation through changes in product market payoffs. Our analysis encompasses substitution effects as well as various forms of competition and types of innovations.

[9] Note, of course, that the net effect of a decrease in the number of research labs on $\lambda_{n,m}$ must be negative, as it was the initial decrease in the pace of innovation that triggered the increase in the incremental rent of an innovation in the first place.

involving large firms affects both forms of competition simultaneously. The interaction between these forms of competition is quite complex. For instance, depending on how firms compete in the product market, these effects may either reinforce or collide with each other, making merger evaluation difficult. However, we can summarize the conjunction of these effects by studying the elasticity of a follower's R&D with respect to the number of large firms, $e_{x^*_{n,m},n} = -(dx^*_{n,m}/dn)(n/x^*_{n,m})$.

Lemma 1 (Pace of innovation)**.** *The pace of innovation, $\lambda_{n,m}$, decreases with a merger between two large large firms if*

$$e_{x^*_{n,m},n} < n/(n+m), \tag{5}$$

and increases otherwise.

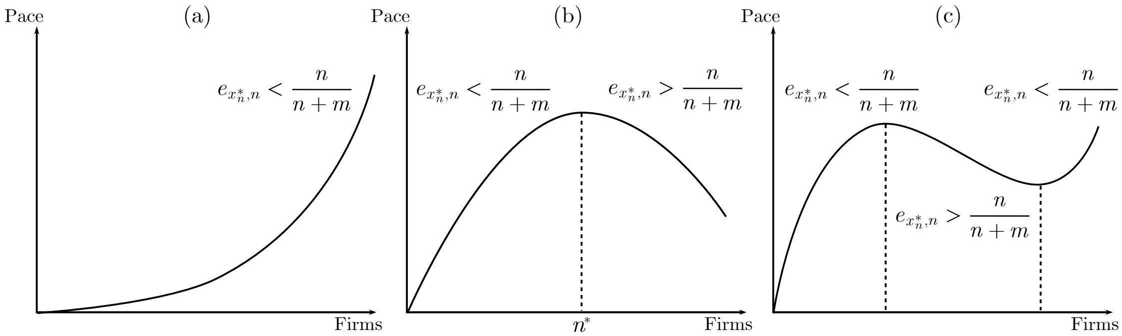
Lemma 1 tells us that we can summarize the total effect of a merger on R&D by comparing the relative importance of large firms in the market, $n/(n+m)$, with a firm's sensitivity to changes in R&D incentives, $e_{x^*_{n,m},n}$ (see Figure 1). In markets dominated by large firms or in markets where the incentives to innovate are not very responsive to changes in the number of product market competitors—for instance, due to long-term capacity constraints in R&D—a merger between large firms is likely to reduce the pace of innovation.

In what follows, we say that the product market payoffs have a *decreasing profit gap* between the leader and a follower when an increase in the number of large firms, n, *decreases* the profit gap, $\Delta\pi_n$. Likewise, we say that the product market payoffs have an *increasing profit gap* between the leader and a follower when an increase in the number of large firms, n, *increases* the profit gap, $\Delta\pi_n$.

Proposition 3 (Sufficiency of static desirability)**.** *A weakly increasing profit gap is sufficient for a merger to decrease the pace of innovation (i.e., $e_{x^*_{n,m},n} < n/(n+m)$). Hence, a weakly increasing profit gap is sufficient for a merger rejection based on a static merger-review criterion to be aligned with a dynamic merger-review criterion.*

Proposition 3 delivers a heuristic rule based on observable market characteristics to determine whether a merger rejection based on a static merger-review criterion is aligned with rejecting the merger using a dynamic merger-review criterion. The logic behind the result is as follows: if the profit gap between the leader and a follower, $\Delta\pi_n$, increases with the number of competitors, then a merger reduces the incentives to perform R&D both by reducing the profit gap in the

Figure 1: Industry's pace of innovation vs. number of competitors in the industry



product market and by reducing innovation competition. This effect is in addition to potential price effects caused by the merger. Thus, a rejection based on a static-merger review criterion is further justified by lower innovation outcomes. An example of product market competition with a weakly increasing profit gap is Bertrand competition in a market for homogeneous goods with symmetric followers and process innovations. In this context, increasing the number of followers (beyond one) does not affect the profit gap, as the market price equals the followers' marginal cost.

In general, the profit-gap analysis has to be performed on a case-by-case basis and needs no further information than that currently required for most merger simulations. Table 1 shows examples of different forms of market competition and the behavior of the profit gap. For instance, a constant-elasticity demand in a quantity competition game can deliver a profit gap that is increasing or decreasing in the number of firms depending on the value of the demand elasticity. Also, Cournot and Bertrand competition can, generally speaking, be associated with both an increasing or decreasing profit gap.

Proposition 4 (Necessity of a decreasing profit gap)**.** *A decreasing profit gap is necessary for a merger to increase the pace of innovation (i.e.,*$e_{x^*_{n,m},n} > n/(n+m)$*). If the number of research labs is large enough, a decreasing profit gap is also sufficient. In such a case, approving a merger using a static merger-review criterion is aligned with approving it using a dynamic merger-review criterion.*

When the profit gap decreases with competition, a merger between large firms creates a tension between the effects of product market competition and innovation competition. On the one hand, the decrease in product market competition increases the profit gap and, consequently, increases the incentives to perform R&D.

Table 1: Product market competition and the slope of the profit gap: examples

	Bertrand	Cournot I	Cournot II	Logit
Differentiation	No	No	No	Yes
Innovation type	Process	Process	Process	Quality ladder
Leader advantage	Marginal cost advantage: $mc_l = \beta mc_f$, $\beta \in (0,1)$			Quality gap: $\kappa > 0$
Demand	$Q = Q(P)$	$Q = a/P^{1/\sigma}$	$Q = a/P^{1/\sigma}$	$s_l = \frac{\exp\{\kappa - p_l\}}{\exp\{\kappa - p_l\} + n\exp\{-p_f\}}$ $s_f = \frac{\exp\{-p_f\}}{\exp\{\kappa - p_l\} + n\exp\{-p_f\}}$
Restrictions	None	$\frac{(1+\beta)}{(1-\beta)}\frac{\sigma(n-\sigma)}{(n-1)} < 1$	$\frac{(1+\beta)}{(1-\beta)}\frac{\sigma(n-\sigma)}{(n-1)} > 1$	Firm-level horizontal differentiation
Profit gap	Weakly increasing	Increasing	Decreasing	Decreasing

Notes: Subscripts l and f denote leader and follower, respectively. For simplicity, we assume that the horizontal differentiation in the logit model (i.e., the idiosyncratic taste shocks) is at the firm rather than the product level. The advantage of this assumption in this context is that same-firm products are homogeneous, eliminating a firm's incentives to keep separate products after a merger. See Marshall (2015) for an application with a closely related model.

On the other hand, the decrease in innovation competition has a negative effect on the pace of innovation. Although this tension may not always result in an increased pace of innovation, Proposition 4 shows that in industries in which research labs play an important role in total R&D, a decreasing profit gap between the leader and a follower is sufficient to increase the pace of innovation.[10] The intuition for this result follows from observing that the R&D incentives of research labs and large firms are aligned (see equation (4)). When market concentration increases R&D incentives, research labs magnify this effect, as more firms are affected by the enhanced incentives. This reduces the negative effect of having fewer firms performing R&D and potentially overcomes it, increasing the overall pace of innovation. As shown by our examples in Section 5, the number of research labs necessary to increase the pace of innovation can be quite small.

In summary, Proposition 3 and Proposition 4 show a direct link between product market competition and the impact of a merger on innovation outcomes. These results are encouraging in that they provide conditions based on product market

[10] The proof that a decreasing profit gap is sufficient for a merger to increase the pace of innovation for a sufficiently large m uses strict convexity of the cost function (i.e., $c''(x) > 0$ for all $x \geq 0$). We note, however, that the result applies for a broader set of cost functions. For instance, the result also applies for all cost functions satisfying $c(x) = x^\gamma/\gamma$ with $\gamma > 1$.

payoffs for when a merger may either increase or decrease the pace of innovation and for whether static and dynamic merger-review criteria are aligned. Since these conditions are based only on product market payoffs, they require the same information than what is commonly used for merger simulations.

Finally, our results also suggest the importance of using a flexible demand specification when performing an empirical assessment of a merger. A lack of flexibility in the demand model may prevent the data from showing the true relationship between the profit gap and the number of firms, which may lead the researcher to erroneously conclude that a merger will decrease (or increase) the pace of innovation.

3.2 Welfare Analysis

We have already provided sufficient conditions for instances when the rejection (approval) of a merger between large firms using a static merger-review criterion is aligned with the rejection (approval) using a dynamic criterion. However, the static and dynamic merger-review criteria are not always aligned, as a merger may increase both the pace of innovation and prices in the short run. Evaluating whether a merger between large firms is welfare enhancing requires understanding how it affects the path of prices faced by consumers and the pace of innovation. For this reason, we provide a sufficient condition for a merger between large firms to be consumer-surplus enhancing. To this end, we incorporate price effects into the analysis and study the trade-off between the price and innovation effects caused by a merger.

To establish conditions for the dynamic desirability of a merger, we impose further structure to the model.

Assumption 1. *Each innovation increases the consumer-surplus flow by* $\delta_n > 0$.

The term δ_n represents the increment in consumer surplus due to an innovation. If, for instance, firms compete in developing process innovations (i.e., cost-saving technologies), δ_n represents the decrease in cost that is passed on to consumers through lower prices and, consequently, higher consumer surplus. Table 2 provides examples of different demands with their respective expressions for the consumer surplus under different forms of competition. In all of these examples, a stronger version of Assumption 1 is satisfied: the increment in consumer-surplus flow δ_n is independent of the number of firms competing in the product market, n.

Table 2: Product market competition and consumer surplus: examples

	Bertrand	Cournot	Logit
Differentiation	No	No	Yes
Innovation type	Process	Process	Quality ladder
Leader advantage	Marginal cost advantage: $mc_l = \beta mc_f$, $\beta \in (0,1)$		Quality gap: $\kappa > 0$
Demand	$Q = a/P$ if $P < \bar{P}$		$s_l = \frac{\exp\{\kappa - p_l\}}{\exp\{\kappa - p_l\} + n\exp\{-p_f\}}$ $s_f = \frac{\exp\{-p_f\}}{\exp\{\kappa - p_l\} + n\exp\{-p_f\}}$
Consumer-surplus flow (cs_n)	$a\log\bar{P} - a\log p_n$		$\log\left(\exp\{\kappa - p_l\} + n\exp\{-p_f\}\right) + \gamma$
Innovation effect on CS (δ_n)	$-a\log\beta$		κ
Restrictions	None	None	Firm-level horizontal differentiation

Notes: Subscripts l and f denote leader and follower, respectively. The γ parameter in the logit-model consumer surplus is Euler's constant.

Given Assumption 1, the discounted expected consumer surplus, CS_n, which incorporates the dynamic benefits of future innovations, is given by

$$rCS_n = cs_n + \lambda_{n,m}\delta_n/r, \tag{6}$$

where cs_n is the consumer-surplus flow at the moment the merger takes place and when there are n product market competitors.[11] Observe that the discounted expected consumer surplus is greater than cs_n and that it is increasing in both the pace of innovation and the magnitude with which each innovation enhances consumer surplus, δ_n. The discounted expected consumer surplus also decreases with the interest rate, as future breakthroughs are discounted at a higher rate.

From equation (6), we can note that a merger affects the discounted expected consumer surplus through three mechanisms. First, market concentration has a direct effect on spot prices, affecting the consumer surplus cs_n from the very moment the merger takes place and into the future. Concentration also affects the discounted expected consumer surplus by potentially changing the pass-through of innovations on consumer welfare, δ_n. Finally, as discussed in the previous subsec-

[11] See Lemma 4 for the derivation of equation (6).

tion, market concentration has an effect on the pace of innovation, $\lambda_{n,m}$. Therefore, it is not clear ex-ante that a merger that increases the pace of innovation will necessarily increase consumer welfare, as the lessening of product market competition caused by the merger also has an effect on the path of prices.

Lemma 2 (Dynamic-merger analysis)**.** *A merger is desirable in the dynamic sense iff*

$$\psi_{n,m} \equiv \frac{rn}{\delta_n \lambda_{n,m}} \frac{dcs_n}{dn} + \frac{d\delta_n}{dn}\frac{n}{\delta_n} + \frac{n}{n+m} < e_{x^*_{n,m},n}, \tag{7}$$

where dcs_n/dn is the derivative of the consumer-surplus flow (at the moment when the merger takes place) with respect to n, and $d\delta_n/dn$ the derivative of the effect of an innovation on the consumer-surplus flow with respect to n.

From Proposition 4 we know that a decreasing profit gap between the leader and a follower is necessary for a merger to increase the speed of innovation (i.e., for $e_{x^*_{n,m},n} > n/(n+m)$ to hold). When a merger increases the market price (i.e., $dcs_n/dn > 0$) and reduces the innovation pass-through on consumer surplus (i.e., $d\delta_n/dn \geq 0$), however, the left-hand side of inequality (7) is larger than $n/(n+m)$, meaning that an increase in the speed of innovation is necessary but not sufficient for the merger to increase welfare. Condition (7) further requires that the innovation effects of a merger to more than compensate for its price effects. These observations combined imply that a decreasing profit gap is necessary (but not sufficient) for a merger to be desirable in the dynamic sense, and allow us to make the following predictions about the impact of a merger on consumer welfare.

Corollary 2. *When a merger increases the market price (i.e., $dcs_n/dn > 0$) and reduces the innovation pass-through on consumer surplus (i.e., $d\delta_n/dn \geq 0$), then*

- *i) a decreasing profit gap between the leader and a follower is necessary (but not sufficient) for the merger to be desirable in the dynamic sense;*
- *ii) an increasing profit gap between the leader and a follower implies that the merger is not desirable in the dynamic sense.*

While Corollary 2 helps identify scenarios where a merger is not desirable in the dynamic sense (i.e., increasing profit gap), it does not provide sufficient conditions for when a merger increases the discounted expected consumer surplus. We show that for a sufficiently large number of labs and under a restriction on how competition impacts the pass-through of innovations on consumer welfare (i.e., $d\delta_n/dn$), a decreasing profit gap between the leader and a follower becomes sufficient for the merger to be desirable from a dynamic standpoint. The driver of the result is that when market concentration increases R&D incentives, research labs magnify the effect of the merger on the pace of innovation, as more firms are affected by the enhanced incentives. It is noteworthy that this sufficient condition only depends on the number of firms and on properties of the product market payoffs.

Proposition 5. *Suppose a merger keeps the innovation pass-through on consumer surplus constant (i.e., $d\delta_n/dn = 0$). A decreasing profit gap between the leader and a follower is sufficient for the merger to be desirable in the dynamic sense if the number of research labs is large enough.*

4 Extensions

In this section, we extend the model in two separate directions to show how our results carry over to richer environments. We first consider a version of the model where the leader can extend its lead through R&D, and then a version of the model that allows for merger-specific R&D efficiencies. To simplify exposition, we assume throughout this section that there are no labs (i.e., $m = 0$).

4.1 Leader Innovation

The previous section abstracted away from the possibility that the leader invests in R&D by assuming that old patents were not enforceable—enabling followers to imitate them—and thus keeping the leader only one step ahead of all followers. This extension shows that the profit gap remains important when market leaders can invest in R&D to increase their technological lead. In particular, a weakly increasing profit gap is still sufficient for a merger to decrease the pace of innovation, and a decreasing profit gap is still necessary but not sufficient for a merger to lead to higher levels of R&D

Following Acemoglu and Akcigit (2012), we modify the baseline model by assuming that followers make radical innovations, making the replaced leader's product obsolete and available to unsuccessful followers; and, that market leaders invest in R&D to increase the quality of their product, which increases their profit flow. In concrete terms, we assume that the leader may be k steps ahead of the followers, receiving a profit flow of π_n^k. We assume $\pi_n^{k+1} > \pi_n^k$, so that a larger technological gap leads to a higher profit flow. As before, each follower innovates at a rate x_n^f at a flow cost of $c(x_n^f)$. Similarly, the leader can now achieve an innovation at a rate x_f^l at a flow cost $c(x_n^l)$. For this extension, we also assume $c'''(x) \geq 0$.

Although our results will apply to environments in which the leader may improve the quality of its product multiple times, for illustration purposes, we examine a situation in which the leader can increase the quality of its product only once (i.e., $k \in \{1, 2\}$). In the model, we also assume that the followers' profit flow remains constant independently of how many steps ahead the leader is. Then, the followers value function is still represented by equation (2). Let V_n^k be the value of being a leader that has innovated $k \in \{1, 2\}$ times. The leader's value equations are represented by

$$\begin{aligned} rV_n^1 &= \max_{x_n^l} \pi_n^1 + x_n^l \left(V_n^2 - V_n^1\right) - c(x_n^l) + nx_n^f(W_n - V_n^1) && (8) \\ rV_2 &= \pi_n^2 + nx_n^f(W_n - V_n^2), && (9) \end{aligned}$$

The first equation describes the value of a being a leader that has innovated only once and that is investing in R&D to increase the quality of its product. The second equation describes the value of a leader that has already increased the quality of its innovation, enjoying a profit flow π_n^2. Note that because we assume it is infeasible for the leader to increase the product quality a second time and because developing a radical innovation replaces the current technology that the leader possess, the leader chooses not to invest in R&D when it is two steps ahead (replacement effect).

The first order condition for the followers is given by equation (4), whereas the first order condition for the leader that is one step ahead is given by

$$c_x\left(\hat{x}_n^l\right) = V_n^2 - V_n^1. \tag{10}$$

Similar to the followers in the baseline model, the leader will invest in R&D when the marginal cost of R&D equals the incremental rent of achieving an innovation, $V_n^2 - V_n^1$.

Define $\Delta_n^f = \pi_n^1 - \pi_n^f$ and $\Delta_n^l = \pi_n^2 - \pi_n^1$ to be the profit gap that exists between a one-step ahead leader and its followers, and the profit gap that exists between being a two-step ahead leader and a one-step ahead leader. Let $\lambda_n^2 = nx_n^f$ and $\lambda_n^1 = nx_n^f + x_n^l$ be the pace of innovation when the leader is two and one step ahead, respectively. We start by showing that the profit gap has a similar role to that in the baseline model.

Proposition 6 (Innovating leader)**.** *There exists a unique symmetric equilibrium, which is characterized by the solution of equations* (2)*,* (4)*,* (8)*,* (9)*, and* (10)*. An increase in the profit gap of the leader* Δ_n^l *increases R&D investments of the leader and followers; consequently, it increases the pace of innovation in the economy. An increase in the profit gap of the followers* Δ_n^f *increases the followers' R&D, but decreases the R&D of the leader. The pace of innovation, however, increases with* Δ_n^f *regardless of whether the leader is one or two-steps ahead.*

An increase of the profit gap of any firm that is ahead in the quality ladder increases the reward to innovate for all the firms that lag behind. This increase in reward, thus, increases the R&D incentives of every firm aiming to reach that state. For instance, an increase in the profit gap of a one-step ahead leader increases not only its R&D incentives but also the incentives of followers aiming to become a one-step ahead leader.

In contrast, an increase in the profit gap of firms that are behind in the quality ladder does not lead to higher rewards for innovation for the firm ahead. On the contrary, the increase in profit gap of laggard firms induces them to perform more R&D, increasing the competition of the firm ahead. In turn, the increased competition faced by the firm ahead, decreases its incremental rent and incentives to perform R&D. This countervailing effect is, however, of second order as the pace of innovation increases with a larger profit gap of the followers.

Proposition 7 (Innovating leader II)**.** *Profit gaps* Δ_n^f *and* Δ_n^l *that are weakly increasing in* n *are sufficient to guarantee that market concentration leads to a slower pace of innovation. Similarly, decreasing profits gaps are necessary but not sufficient for market concentration to lead to higher innovation pace.*

Although this formulation abstracts away from research labs, it is not hard to see that the sufficiency result presented in Proposition 4 can be extended to this framework. Research labs mimic the incentives of the followers, magnifying their response in R&D investments due to changes in market concentration. Because a profit gap Δ_n^f that is decreasing in n tends to increase the followers' R&D when the product market concentrates, mergers can lead to higher R&D outcomes when there is a sufficiently large number of research labs and there are decreasing profit gaps. In other words, approval of a merger based only on static merger-review criterion would be aligned with its approval based on a dynamic merger-review criterion.

4.2 Merger-specific R&D Efficiencies

In this extension, we allow for merger-specific R&D efficiencies or synergies. We assume that the R&D efficiencies of the merged firm only last until the merged firm achieves its first post-merger innovation. One interpretation of this assumption is that while firms may benefit from sharing information about current projects—in particular, if they are working towards producing the same innovation—the synergies seize to exist for future projects that the merging firms have not started to work on. We capture these synergies by assuming that the merged firm, denoted by M, enjoys an innovation arrival rate that is $\phi > 1$ times greater than that of the rest of the firms for any given level of investment.[12]

To analyze the equilibrium of this industry, we have to consider two separate industry phases. In the first phase, we have that the merged firm enjoys the merger-specific efficiencies and, in the second phase, we consider the scenario where the merger-specific R&D efficiencies have expired. Once the R&D efficiencies have expired, the equilibrium analysis is identical to that of Section 2, as all followers are symmetric. The incentives of firms change, however, during the period when the merged firm still enjoys the merger-specific R&D efficiencies. In this interim period, the value functions of all firms must satisfy

$$
\begin{aligned}
r\tilde{V}_n &= \pi_n^l - \sum_{i \neq M} x_i(\tilde{V}_n - \tilde{W}_n) - \phi x_M(\tilde{V}_n - W_n) && (11)\\
r\tilde{W}_n &= \max_{x_i} \pi_n^f + x_i(\tilde{V}_n - \tilde{W}_n) + \phi x_M(W_n - \tilde{W}_n) - c(x_i) && (12)\\
rM_n &= \max_{x_M} \pi_n^f + \phi x_M(V_n - M_n) - c(x_M), && (13)
\end{aligned}
$$

where $\tilde{V}_n$ is the value of an unmerged firm leader, $\tilde{W}_n$ the value of an unmerged firm follower, M_n the value of the merged firm. V_n and W_n are the values of being a follower and a leader after the merged firm's efficiencies have expired; these value functions are the same as those in the baseline model (see equations (1) and (2)). Equations (11) to (13) possess the same structure as the equations in the baseline model, but with the important difference that the firms face asymmetric incentives due to the presence of efficiencies. The value functions also capture that once the merged firm successfully innovates—which happens at rate ϕx_M—the incentives of all firms return to the case of no efficiencies. That is, at rate ϕx_M an

[12] This modeling approach is equivalent to assuming that the flow cost of producing an innovation decreases by a factor of $\alpha = \phi^{-1} \in (0, 1]$ with the merger; i.e., $c_M(x) = \alpha c(x)$.

unmerged leader gains the incremental value $W_n - \tilde{V}_n$, and an unmerged follower gains $W_n - \tilde{W}_n$.[13]

In equilibrium, the followers invest according to the incremental value of becoming the industry leader[14]

$$c'(x^*) = (\tilde{V}_n - \tilde{W}_n), \quad c'(x_M^*) = \phi(V_n - M_n).$$

Lemma 6 and Lemma 7 in the Appendix show that x_M^* is increasing in ϕ, while x^* is decreasing in ϕ. These results suggest that, on the one hand, efficiencies make each unit of effort more productive for the merged firm, inducing the merged firm to increase its investment for greater efficiency levels. On the other hand, the efficiencies discourage the unmerged followers because the greater efficiency of the merged firm is as if the unmerged firms faced greater competition (see Proposition 2). While the merger-specific efficiencies increase the arrival rate of innovations for the merged firm and decrease it for the unmerged firms, we show that the overall pace of innovation increases in ϕ. That is, relative to the equilibrium without efficiencies, innovations arrive faster on average in the equilibrium with efficiencies. We also find that the role played by product market payoffs in understanding the impact of a merger on the pace of innovation also extends to this environment.

Proposition 8. *The pace of innovation during the period when the merged firm enjoys the efficiencies, $\tilde{\lambda}_n \equiv (n-1)x^* + \phi x_M^*$, is increasing in both ϕ and $\Delta\pi_n$.*

Proposition 8 has several implications. First, efficiencies and a decreasing profit gap—i.e., a profit gap that increases with a merger—complement each other in increasing the post-merger pace of innovation. That is, even in presence of merger-specific efficiencies, the properties of the product market payoffs continue to impact the pace of innovation. Second, since the merger-specific efficiencies increase the post-merger pace of innovation, we have that the sufficient conditions for a merger to be welfare improving in Lemma 2 are also sufficient for the merger to be welfare improving when in presence of merger-specific R&D efficiencies. That is, if a merger were to be approved without considering efficiencies—for instance, because these are not verifiable—it also should be approved if efficiencies exist.

[13] In Lemma 5, we show that when $\phi = 1$, $\tilde{W}_n = W_n = M_n$ and $\tilde{V}_n = V_n$.

[14] Equilibrium existence and uniqueness arguments follow closely those in Proposition 1 and Proposition 6, for space consideration they are omitted.

5 An Illustrative Example

In this section we parameterize the baseline model and simulate the effect of mergers on market outcomes. The purpose of this exercise is to show, first, that the relationship between market structure and the pace of innovation is complex; and second, that mergers can enhance consumer surplus despite short-run price effects. In many of the examples we provide, reduced competition and the elimination of duplicated R&D fixed costs make mergers profitable for the merging parties. That is, in the numerical examples it will often be true that $W_{n-1} > 2W_n$ or $V_{n-1} \geq W_n + V_n$, which are necessary conditions for there to be incentives to merge. We also show that incentives to merge do not imply that a merger will necessarily benefit (or hurt) consumers. We consider the case without labs, $m = 0$, unless otherwise noted. Henceforth, we drop the m subscript for ease of notation.

5.1 Parameters

We consider a market for a homogeneous good, where firms compete in quantity (Cournot competition), and market demand is given by $Q = a/P$, with $a > 0$ and $P \leq \bar{P}$. Firms also compete developing a sequence of cost-saving innovations. Each innovation provides the innovating firm with a marginal cost advantage, reducing the leader's marginal cost by a factor of $\beta \in (0, 1)$. The R&D cost function is given by $c(x_i) = \gamma_0 + \gamma_1^{-1} x_i^{\gamma_1}$, where $\gamma_0 \geq 0$ represents the fixed costs of performing R&D and $\gamma_1 > 1$.

We denote, at any instant of time, the marginal cost of the followers by mc and the marginal cost of the leader by $\beta \cdot mc$. The equilibrium market price is $p_n = mc(\beta + n)/n$, which depends on the follower's marginal cost of production, the size of the leader's cost advantage, and the number of followers in the market. As expected, the equilibrium market price is decreasing in n and increasing in both β and mc. Similarly, profits are given by

$$\pi_n^l = a\frac{(n(1-\beta)+\beta)^2}{(\beta+n)^2}, \qquad \pi_n^f = a\frac{\beta^2}{(\beta+n)^2},$$

which do not depend on the current marginal cost, nor the number of innovations that have taken place. Profits do depend, however, on the number of followers and the size of the leader's cost advantage, β. These equilibrium profits imply that the profit gap is positive, $\Delta\pi_n \equiv \pi_n^l - \pi_n^f > 0$; decreasing in the number

Table 3: Market-outcome comparison for different numbers of followers and parameter values

a) $\beta = 0.85$, $\gamma_0 = 0.55$, $\gamma_1 = 1.07$, $m = 0$

n	$e_{x_n^*,n}$	ψ_n	λ_n	CS_n	V_n	W_n
1	0.877	1.019	4.543	39.050	415.999	414.887
2	1.117	1.012	4.527	39.363	165.853	164.794
3	1.202	1.010	4.237	36.725	83.333	82.309
4	1.235	1.008	3.977	34.285	46.264	45.264
5	1.246	1.007	3.768	32.312	26.482	25.501
6	1.247	1.006	3.602	30.738	14.690	13.725
7	1.242	1.006	3.468	29.470	7.097	6.145
8	1.234	1.005	3.360	28.435	1.919	0.978

b) $\beta = 0.85$, $\gamma_0 = 0.55$, $\gamma_1 = 1.37$, $m = 0$

n	$e_{x_n^*,n}$	ψ_n	λ_n	CS_n	V_n	W_n
1	0.573	1.030	2.865	22.682	443.416	441.940
2	0.807	1.016	3.531	29.645	180.412	179.178
3	0.893	1.011	3.747	31.939	92.445	91.360
4	0.929	1.008	3.842	32.971	52.601	51.616
5	0.944	1.007	3.896	33.562	31.198	30.286
6	0.949	1.006	3.934	33.977	18.366	17.511
7	0.948	1.005	3.965	34.314	10.059	9.248
8	0.944	1.004	3.994	34.617	4.365	3.591

c) $\beta = 0.21$, $\gamma_0 = 0.095$, $\gamma_1 = 1.0125$, $m = 0$

n	$e_{x_n^*,n}$	ψ_n	λ_n	CS_n	V_n	W_n
1	0.976	1.000	37.048	3464.601	74.071	73.025
2	1.001	1.000	37.258	3484.460	23.881	22.844
3	1.003	1.000	37.219	3480.852	11.694	10.662
4	1.002	1.000	37.191	3478.311	6.772	5.744
5	1.000	1.000	37.181	3477.375	4.246	3.221
6	0.999	1.000	37.182	3477.487	2.753	1.729
7	0.998	1.000	37.190	3478.233	1.784	0.762
8	0.997	1.000	37.202	3479.359	1.112	0.093

d) $\beta = 0.85$, $\gamma_0 = 0.55$, $\gamma_1 = 1.37$, $m = 2$

n	$e_{x_n^*,n}$	ψ_n	λ_n	CS_n	V_n	W_n
1	0.223	1.020	4.315	36.824	419.829	418.685
2	0.507	1.012	4.456	38.670	171.046	170.005
3	0.639	1.009	4.410	38.412	87.689	86.734
4	0.711	1.007	4.356	37.984	49.791	48.903
5	0.754	1.006	4.315	37.645	29.366	28.530
6	0.780	1.005	4.287	37.421	17.086	16.292
7	0.797	1.005	4.271	37.295	9.117	8.358
8	0.807	1.004	4.263	37.246	3.645	2.916

Notes: Fixed parameter values are $r = 0.03$, $a = 60$, and $mc = 10$. n is the number of followers, $e_{x_n^*,n}$ is the elasticity of a firm's R&D level with respect to n, ψ_n is defined in (7), λ_n is the pace of innovation, CS_n is the expected discounted consumer surplus, V_n is the value of being the leader, and W_n is the value of being a follower.

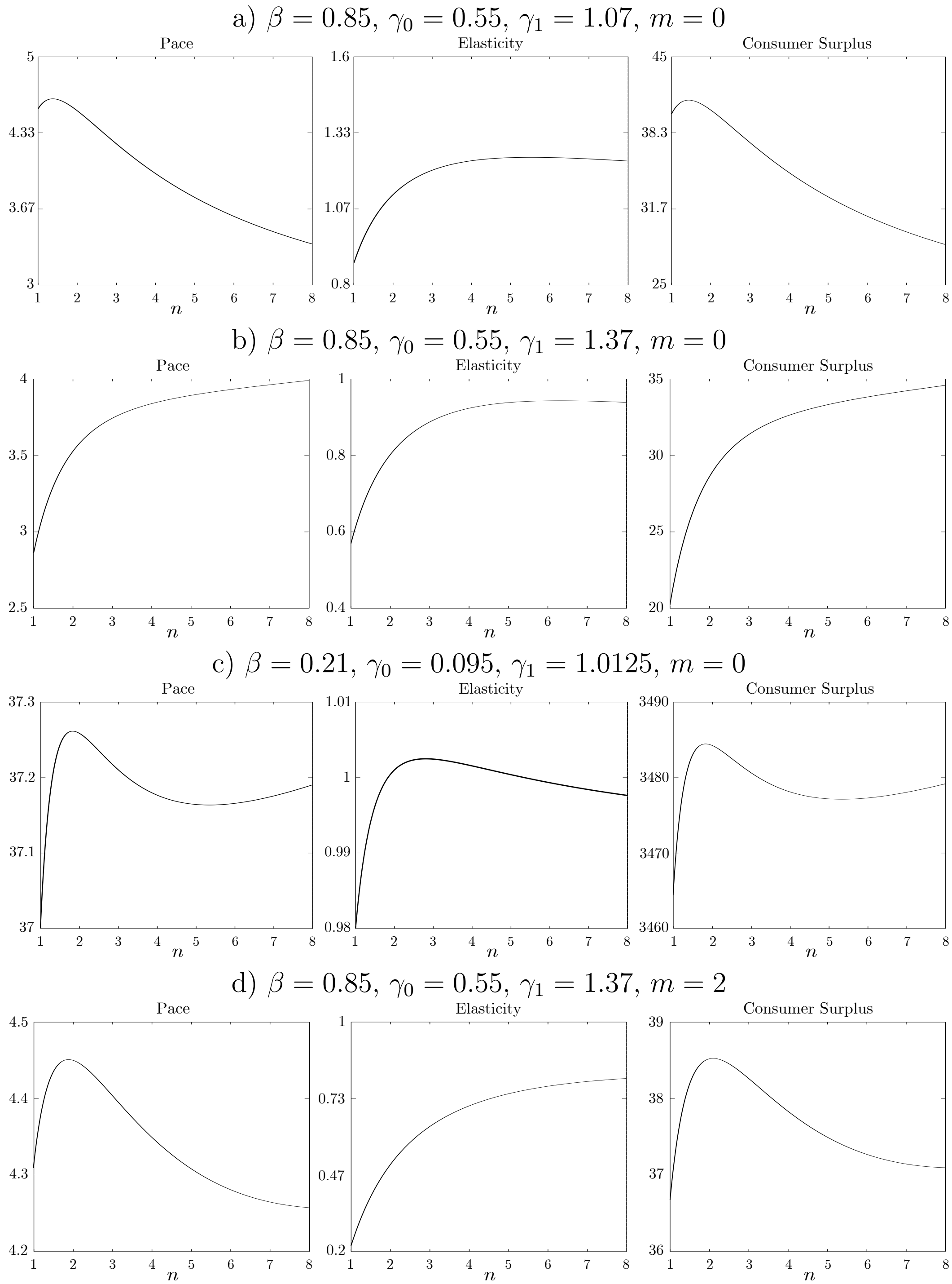
of followers, $d\Delta\pi_n/dn < 0$; and increasing in the cost advantage of the leader, $d\Delta\pi_n/d\beta < 0$. As discussed above, a decreasing profit gap suggests that a merger may potentially increase consumer surplus if it increases the pace of innovation by a sufficient amount (see Lemma 2 and Corollary 2).

Finally, to capture the role of the pace of innovation on the path of prices faced by consumers, we make use of the expected discounted consumer surplus defined in equation (6). The flow of consumer surplus when the market price is p_n is given by $cs_n = a \log \bar{P} - a \log p_n$, and an innovation increases the flow of consumer surplus by $\delta \equiv -a \log \beta > 0$.

5.2 Results

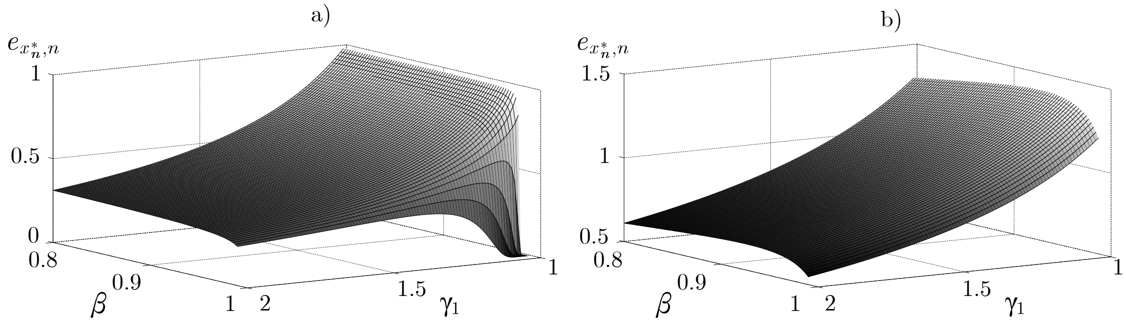
Using this setup, we provide four numerical examples to illustrate our results. In Table 3.a (see Figure 2.a) we show market outcomes for a set of parameters that create an inverted-U relationship between the pace of innovation and the number of followers. A similar inverted-U relationship is found for the expected discounted consumer surplus. This example shows that a merger may enhance consumer surplus by increasing the pace of innovation—for instance, when going

Figure 2: Market-outcome comparison for different numbers of followers and parameter values



Notes: Fixed parameter values are $r = 0.03$, $a = 60$, and $mc = 10$. "n" is the number of large firms, "Pace" is the pace of innovation (λ_n), "Elasticity" is the elasticity of a firm's R&D level with respect to n ($e_{x_n^*,n}$), and "Consumer Surplus" is the discounted expected consumer surplus (CS_n).

Figure 3: Elasticity of R&D: a comparative static analysis



Notes: Common parameters: $a = 30$, $m = 0$ and $mc = 10$. Parameters in panel a): $n = 1$, $r = 0.3$, and $\gamma_0 = 0$. Parameters in panel b): $n = 5$, $r = 0.03$, and $\gamma_0 = 0.2$.

from $n = 3$ to $n = 2$—even though the merger reduces competition in the product market and, consequently, increases prices in the short run. The gains in consumer surplus arise from consumers enjoying more frequent price reductions caused by the impact of the merger on the pace of innovation. The positive effect of a merger on consumer surplus implies that the increased frequency of these price reductions more than compensates for the short-run price effects due to reduced product market competition.

Result 1. *A merger may enhance consumer surplus even if it increases prices in the short run.*

In Tables 3.b and 3.c (see Figures 2.b and 2.c, respectively), we show examples where the pace of innovation varies monotonically (Table 3.b) or non-monotonically (N-shaped in Table 3.c) with respect to the number of followers. These examples illustrate the complex relationship that exists between the number of firms and the pace of innovation. As discussed in Section 3, the shape of this relationship is given by the relative importance of two separate effects created by a merger. On the one hand, a merger may increase the profit gap between the leader and followers—increasing the incentives to innovate; on the other hand, it reduces the number of firms performing R&D. Figures 2.b and 2.c show that the dominance of one effect over the other may change as a function of the number of firms, creating an inverted-U- or even an N shaped relationship between the pace of innovation and the number of firms.

Result 2. *The relationship between the pace of innovation and the number of firms can be monotonic or non-monotonic (e.g., inverted-U or N shaped).*

Table 3.b shows an example where a profit gap that decreases in the number of firms is insufficient for a merger to increase the pace of innovation. In Proposition 4, however, we argue that for a sufficiently large number of research labs, m, a decreasing profit gap becomes sufficient for a merger to increase the pace of innovation. Using the same parameters as in Table 3.b, we find that $m = 2$ research labs are sufficient for a merger to increase the pace of innovation whenever the number of large firms ranges between 2 and 8. We report these results in Table 3.d (see Figure 2.d). This result suggests that even a small number of labs may transform the relationship between the number of firms and the pace of innovation. Therefore, even in concentrated industries, a decreasing profit gap may be sufficient to invalidate the argument that a merger will reduce the pace of innovation.

Result 3. *A small number of labs may be sufficient for a profit gap that decreases in the number of firms to increase the pace of innovation with a merger.*

Table 3.b and Table 3.d show two scenarios where firms have incentives to merge (e.g., consider in both cases a change from $n = 8$ to $n = 7$ firms). In the first case, a merger would decrease consumer surplus, while in the second, it would increase consumer surplus. These examples illustrate that incentives to merge do not imply that consumers will necessarily win (or lose) with a merger. Changes in competition that hurt consumers in the short run incentivize firms to merge but, as discussed above, will only benefit consumers in the long run if the product market payoffs have the right properties.

Result 4. *Incentives to merge do not imply that the merger will increase (or decrease) consumer surplus.*

Finally, Figure 3 shows the relationship between the elasticity of a firm's R&D curve and some of the key parameters of the model, β and γ_1. Even though the profit gap increases with the cost advantage of the leader β, the elasticity of a follower's R&D with respect to the number of firms, $e_{x_n^*,n}$, does not have a monotone comparative static in β. Similarly, more inelastic R&D cost technologies (i.e., a higher γ_1) also affect $e_{x_n^*,n}$ non-monotonically. This non-monotonicity captures the complex interaction that exists between the different components of the model and further highlights the value of our conditions for the alignment of the static and dynamic merger-review criteria.

6 Concluding Remarks

We studied the impact of mergers in innovative industries. We found that a merger between two large firms affects R&D outcomes both directly by reducing the number of firms performing R&D and indirectly by changing the product market profits. The relationship among these effects is complex and may lead to scenarios where a merger increases an industry's pace of innovation and consumer surplus in the long run.

Based on properties of the product market competition game, we provide conditions for when a merger increases or decreases the pace of innovation. These conditions are based on product market payoffs and provide valuable information on whether the (common) argument that a merger reduces incentives to innovate really applies.[15] Moreover, these conditions are simple to check—in the sense that they only require information that is commonly used for merger simulations or demand estimation. Based on these results, we provide conditions for when rejecting or approving a merger using a static merger-review criterion (i.e., based on static price effects) is aligned with a dynamic merger-review criterion, which considers effects on both the price and innovation processes. Finally, we provide a sufficient condition for when a merger benefits consumers in the long run despite any short run price effects.

Our theoretical results together with empirical evidence suggesting that reduced product market competition may increase innovation rates—e.g., see Aghion et al. (2005)—stress the relevance and importance of analyzing the dynamic effects of mergers in innovative industries. As mentioned above, checking our sufficient conditions for whether a merger increases innovation rates does not require estimating or solving a dynamic model. We believe these conditions are simple enough to be easily brought into merger evaluation.

Finally, our results also highlight the importance of product market payoffs for the analysis of the impact of mergers on R&D outcomes. For this reason, empirical studies should carefully specify demand models and the rules of the product market competition game. A lack of flexibility in the model may prevent the data from showing the true relationship between the profit gap and the number of firms, which may lead the researcher to erroneously conclude that a merger will decrease (or increase) the pace of innovation.

[15] See footnote 1.

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Appendix

A Preliminary Results

Lemma 3. *The function $f(z)$ implicitly defined by $c'(f(z)) = z$ satisfies:*

1. *$f(z) > 0$ for all $z > 0$ and $f(0) = 0$.*
2. *$f'(z) > 0$ for all $z \geq 0$. Also, if $c'''(x) \geq 0$, $f''(z) \leq 0$, i.e. f is concave.*
3. *Let $h(z) = (n+1)zf(z) - c(f(z))$ for $z \geq 0$. Then $h'(z) = (n+1)f(z) + nzf'(z) > 0$ for all $z \geq 0$.*

Proof. 1. $c(x)$ being strictly increasing and differentiable implies $c'(x) > 0$ for all $x > 0$. $c(x)$ being strictly convex implies $c''(x) > 0$ for all $x \geq 0$. Thus, $c'(x)$ is unbounded above and for each z there exists a unique value of $x = f(z) > 0$ such that $c'(x) = z$. Moreover, because $c'(0) = 0$, then $f(0) = 0$.
2. The first result follows from the derivative of the inverse function being equal to $f'(z) = 1/c''(f(z))$ in conjunction with the strict convexity of $c(x)$. The second from $f''(z) = -c'''(f(z))/(c''(f(z))^3)$ and the assumption $c'''(x) \geq 0$.
3. Differentiating h and using $c'(f(z)) = z$ delivers $h'(z) = (n+1)f(z) + nzf'(z)$, which is positive by claims 1 and 2. □

Lemma 4. *The discounted expected consumer surplus is given by equation (6).*

Proof. Consider an asset that pays the consumer surplus flow at every instant of time. Starting from a consumer surplus cs_n, the value of this asset is given by

$$rA(cs_n) = cs_n + \lambda_{n,m}(A(cs'_n) - A(cs_n)) \tag{14}$$

where cs'_n is the consumer surplus after an innovation arrives. Using the condition that $cs'_n = cs_n + \delta_n$, we guess and verify that equation (6) solves equation (14), i.e., $A(cs_n) = CS_n$, proving the result. □

Lemma 5. *If $\phi = 1$, then $\tilde{V}_n = V_n$ and $M_n = \tilde{W}_n = W_n$ (see equations (1)-(2) and (11)-(13)).*

Proof. First, observe that $M_n = W_n$ when $\phi = 1$, as they both solve the same equation. Second, given the definition of $Z_n \equiv V_n - W_n$, Z_n satisfies the implicit expression for $\tilde{Z}_n \equiv \tilde{V}_n - \tilde{W}_n$. Third, one can write $r\tilde{W}_n = rW_n + \tilde{Z}_n(W_n - \tilde{W}_n)$, which implies $W_n = \tilde{W}_n$ since both $r > 0$ and $\tilde{Z}_n > 0$. Lastly, $Z_n = \tilde{Z}_n$ and $W_n = \tilde{W}_n$ imply $V_n = \tilde{V}_n$. □

Lemma 6. *M, x^*_M, and ϕx^*_M are increasing in ϕ.*

Proof. Implicit differentiation of M_n and x^*_M (see equation (13)) yields

$$\frac{dM_n}{d\phi} = \frac{(V_n - M_n)f(\phi(V_n - M_n))}{r + \phi f(\phi(V_n - M_n))} > 0,$$
$$\frac{df(\phi(V_n - M_n))}{d\phi} = f'(\phi(V_n - M_n))\frac{r(V_n - M_n)}{r + \phi f(\phi(V_n - M_n))} > 0.$$

The fact that ϕx_M^* is also increasing in ϕ follows from the last equation. □

Lemma 7. *$\tilde{V}_n - \tilde{W}_n$ (see equations (11) and (12)) is decreasing in ϕ.*

Proof. Implicity define $\tilde{Z}_n \equiv \tilde{V}_n - \tilde{W}_n$ as

$$r\tilde{Z}_n = \Delta\pi - n\tilde{Z}f(\tilde{Z}_n) - \phi f(\phi(V_n - M_n))\tilde{Z}_n + c(f(\tilde{Z}_n)).$$

Implicit differentiation of $\tilde{Z}_n$ yields

$$\frac{d\tilde{Z}_n}{d\phi} = \frac{-\tilde{Z}_n \frac{d\phi f(\phi(V_n - M_n))}{d\phi}}{r + nf(\tilde{Z}) + (n-1)\tilde{Z}_n f'(\tilde{Z}_n) + \phi f(\phi(V_n - M_n))} < 0.$$

□

B Proofs

Proof of Proposition 1. Using the first order condition (see equation (4)), we find that the equilibrium values for the leader and followers are given by

$$rV_{n,m} = \pi_n^l - (n+m)(V_{n,m} - W_{n,m})f(V_{n,m} - W_{n,m})$$
$$rW_{n,m} = \pi_n^f + (V_{n,m} - W_{n,m})f(V_{n,m} - W_{n,m}) - c(f(V_{n,m} - W_{n,m})).$$

Subtracting these equations and defining $Z_{n,m} \equiv V_{n,m} - W_{n,m}$ we obtain

$$rZ_{n,m} = \Delta\pi_n - (n+m+1)Z_{n,m}f(Z_{n,m}) + c(f(Z_{n,m})). \tag{15}$$

To prove existence and uniqueness of an equilibrium with $Z_{n,m} > 0$, note that the left-hand side of equation (15) is strictly increasing in $Z_{n,m}$ and ranges from 0 to ∞. Lemma 3.1 implies that the right-hand side of equation (15) is strictly decreasing in $Z_{n,m}$, taking the value of $\Delta\pi_n + c(0) > 0$ when $Z_{n,m} = 0$. Thus, the two functions intersect once at a positive value of $Z_{n,m}$, proving the result. ■

Proof of Proposition 2. Using implicit differentiation in equation (15), we reach the following results:

i) The derivative of $Z_{n,m}$ with respect to $\Delta\pi_n$ is given by

$$\frac{dZ_{n,m}}{d\Delta\pi_n} = \frac{1}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})} > 0.$$

Since $x_{n,m}^* = f(Z_{n,m})$ and $\lambda_{n,m} = (n+m)f(Z_{n,m})$, Lemma 3.2 implies that both are increasing in $\Delta\pi_n$.

ii) The derivative of $Z_{n,m}$ with respect to m is given by

$$\frac{dZ_{n,m}}{dm} = \frac{-Z_{n,m}f(Z_{n,m})}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})} < 0.$$

Thus, an increase in m decreases a firm's R&D investment. The derivative of the pace of innovation with respect to m is

$$\begin{aligned}\frac{d\lambda_{n,m}}{dm} &= f(Z_{n,m}) + (n+m)f'(Z_{n,m})\frac{dZ_{n,m}}{dm} \\ &= \frac{rf(Z_{n,m}) + (n+m+1)f(Z_{n,m})^2}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})} > 0.\end{aligned}$$

proving that the pace of innovation increases with m. ■

Proof of Lemma 1. Using implicit differentiation in equation (15), we find that the derivative of the pace of innovation with respect to n is

$$\frac{d\lambda_{n,m}}{dn} = f(Z_{n,m}) + (n+m)f'(Z_{n,m})\frac{dZ_{n,m}}{dn}. \tag{16}$$

This derivative is positive when

$$\frac{n}{n+m} < -\frac{n}{f(Z_{n,m})}\frac{df(Z_{n,m})}{dZ_{n,m}}\frac{dZ_{n,m}}{dn} = -\frac{dx^*_{n,m}/x^*_n}{dn/n} \equiv e_{x^*_{n,m},n}, \tag{17}$$

which proves the result. ■

Proof of Proposition 3. Using implicit differentiation in equation (15) we obtain $dZ_{n,m}/dn$. Replacing it in (16), we find

$$\frac{d\lambda_{n,m}}{dn} = \frac{rf(Z_{n,m}) + (n+m+1)f(Z_{n,m})^2 + (n+m)f'(Z_{n,m})\frac{d\Delta_n}{dn}}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})}. \tag{18}$$

If Δ_n satisfies $d\Delta_n/dn > 0$ (i.e., if Δ_n has an increasing profit gap), then the derivative is positive. Hence, a reduction in the number of large firms leads to a reduction in the pace of innovation. ■

Proof of Proposition 4. A necessary condition for equation (18) to be negative is $d\Delta_n/dn < 0$. For sufficiency, we need to show that there exists an $\bar{m}$ such that $m > \bar{m}$ implies $d\lambda_{n,m}/dn < 0$. Since the denominator of (18) is positive, $d\lambda_{n,m}/dn < 0$ is equivalent to

$$\frac{r}{n+m}\frac{f(Z_{n,m})}{f'(Z_{n,m})} + \frac{n+m+1}{n+m}\frac{f(Z_{n,m})^2}{f'(Z_{n,m})} < -\frac{d\Delta\pi_n}{dn}.$$

$d\Delta\pi_n/dn < 0$ guarantees that right-hand side of the inequality is always positive. Given that $f(0) = 0$ and $f'(0) > 0$ (see Lemma 3), and $dZ_{n,m}/dm < 0$, it is sufficient to show that $\lim_{m\to\infty} Z_{n,m} = 0$ for the inequality to hold.

For any small $\epsilon > 0$, pick $Z_\epsilon \in (0, \epsilon)$. By Proposition 1, equation (15) has a unique solution. Using (15), define m_ϵ to be

$$m_\epsilon = \frac{\Delta\pi_n + c(f(Z_\epsilon)) - (r + (n+1)f(Z_\epsilon))Z_\epsilon}{f(Z_\epsilon)Z_\epsilon},$$

which is always well defined (but possibly negative). Thus, take any decreasing sequence of Z_ϵ converging to zero. For each element of the sequence, there exists an increasing sequence m_ϵ that delivers Z_ϵ as an equilibrium. Thus, $\lim_{m\to\infty} Z_{n,m} = 0$ and the result follows. ■

Proof of Lemma 2. The derivative of CS_n with respect to n is given by

$$\frac{dCS_n}{dn} = \frac{dcs_n}{dn} + \frac{1}{r}\left(\frac{d\lambda_{n,m}}{dn}\delta_n + \lambda_{n,m}\frac{d\delta_n}{dn}\right).$$

Using equation (18), we note that

$$(n+m)\frac{d\lambda_{n,m}}{dn} = \lambda_{n,m}\left(1 - \frac{m+n}{n}e_{x_n^*,n}\right).$$

By replacing this expression into dCS_n/dn, we find that a merger increases consumer surplus if and only if condition (7) holds. ■

Proof of Proposition 5. Using the definition $\lambda_{n,m} = (n+m)x^*_{n,m}$ and the assumption that $d\delta_n/dn = 0$, we re-write condition (7) as:

$$\frac{dcs_n}{dn} < -\frac{\delta_n}{r}\left((n+m)\frac{dx^*_{n,m}}{dn} + x^*_{n,m}\right).$$

We show that when m is sufficiently large, a profit gap that is decreasing in the number of firms is sufficient to guarantee that the parenthesis in the expression above goes to $-\infty$, which ensures that the condition holds, as dcs_n/d_n is finite. From Proposition 2, we know that $x^*_{n,m}$ decreases with m. Now, observe

$$(n+m)\frac{dx^*_{n,m}}{dn} = \frac{\frac{d\Delta_n}{dn} - Z_{n,m}f(Z_{n,m})}{\frac{r}{n+m} + \frac{n+m+1}{n+m}f(Z_{n,m}) + Z_{n,m}f'(Z_{n,m})}.$$

From the proof of Proposition 4 we know that $\lim_{m\to\infty} Z_{n,m} = 0$. From Lemma 3, we also know that that $f(0) = 0$ and $f'(0) > 0$. Therefore, when the profit gap is decreasing in the number of firms (i.e., $d\Delta_n/dn < 0$) we have

$$\lim_{m\to\infty}(n+m)\frac{dx^*_{n,m}}{dn} = -\infty,$$

and the result follows. ■

Proof of Proposition 6. Define the incremental rent of the leader to be $H_n = V_n^2 - V_n^1$ and the incremental rent of followers $Z_n = V_n^1 - W_n$. Using the inversion defined in Lemma 3 we write $\hat{x}_n^l = f(H_n)$ and $\hat{x}_n^f = f(Z_n)$. Subtracting (8) from (9) delivers

$$rH_n = \Delta_n^l - f(H_n) H_n + c(f(H_n)) - nf(Z_n) H_n.$$

Similarly, subtracting (8) and (2) delivers:

$$rZ_n = \Delta_n^f + f(H_n) H_n - c(f(H_n)) - (n+1) f(Z_n) Z_n + c(f(Z_n))$$

We need to show that there exists unique positive values of H_n and Z_n that simultaneously solve the equations above. Rewrite the first equation as:

$$f(Z_n) = \frac{\Delta_n^l - (f(H_n) + r) H_n + c(f(H_n))}{nH_n}$$

Using Lemma 3 we can show that this expression defines a negative, monotonic and continuous relation between Z_n and H_n. In particular, observe that if $H_n \to 0$, then $Z_n \to \infty$. Also, if $H_n \to \infty$, then $Z_n < 0$. Rewrite the expression for rZ_n as:

$$rZ_n + (n+1) f(Z_n) Z_n - c(f(Z_n)) = \Delta_n^f + f(H_n) H_n - c(f(H_n))$$

Lemma 3 implies a increasing, monotonic and continuous relation between Z_n and H_n. Observe that $H_n = 0$ implies $Z_n > 0$. Also, $H_n \to \infty$ implies $Z_n \to \infty$. Therefore, the relation described by both equations must intercept and, because both expressions are monotonic, there is a unique intersection. Thus, an equilibrium exists and is unique.

To study the relation between the profit gaps and firms investments and pace of innovation we need to understand the impact of the gaps in the incremental rent, i.e., $\frac{dH_n}{d\Delta_n^k}$ and $\frac{dZ_n}{d\Delta_n^k}$ for $k\{l, f\}$. For this we make use of the implicit function theorem. Define $g : \mathbb{R}^2 \to \mathbb{R}^2$ where

$$\begin{aligned} g_1(H_n, Z_n) &= \Delta_n^l - (f(H_n) + r) H_n + c(f(H_n)) - nf(Z_n) H_n \\ g_2(H_n, Z_n) &= \Delta_n^f + f(H_n) H_n - c(f(H_n)) - ((n+1) f(Z_n) + r) Z_n + c(f(Z_n)). \end{aligned}$$

Then, an equilibrium is defined by $g(H_n, Z_n) = 0$ and the implicit function theorem implies (in matrix notation):

$$\left[\frac{dH_n}{d\Delta_n^f}, \frac{dH_n}{d\Delta_n^l}; \frac{dZ_n}{d\Delta_n^f}, \frac{dZ_n}{d\Delta_n^l}\right] = -\left(A^{-1}\right) B \tag{19}$$

where

$$A = \begin{bmatrix} \frac{\partial g_1}{\partial H_n} & \frac{\partial g_1}{\partial Z_n} \\ \frac{\partial g_2}{\partial H_n} & \frac{\partial g_2}{\partial Z_n} \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{\partial g_1}{\partial \Delta_n^f} & \frac{\partial g_1}{\partial \Delta_n^l} \\ \frac{\partial g_2}{\partial \Delta_n^f} & \frac{\partial g_2}{\partial \Delta_n^l} \end{bmatrix}. \tag{20}$$

Using Lemma 3, we find that

$$A = -\begin{bmatrix} r + nf(Z_n) + f(H_n) & nf'(Z_n)H_n \\ -f(H_n) & r + (n+1)f(Z_n) + nf'(Z_n)Z_n \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

The inverse of A is given by

$$A^{-1} = -\frac{1}{|A|}\begin{bmatrix} r + (n+1)f(Z_n) + nf'(Z_n)Z_n & -nf'(Z_n)H_n \\ f(H_n) & r + nf(Z_n) + f(H_n) \end{bmatrix}$$

where $|A|$ is equal to

$$(r + nf(Z_n) + f(H_n))(r + (n+1)f(Z_n) + nf'(Z_n)Z_n) + nf'(Z_n)f(H_n)H_n,$$

which is positive. Then, using equation (19), we compute the derivatives:

$$\begin{bmatrix} \frac{dH_n}{d\Delta_n^f} & \frac{dH_n}{d\Delta_n^l} \\ \frac{dZ_n}{d\Delta_n^f} & \frac{dZ_n}{d\Delta_n^l} \end{bmatrix} = \frac{1}{|A|}\begin{bmatrix} -nf'(Z_n)H_n & r + (n+1)f(Z_n) + nf'(Z_n)Z_n \\ r + nf(Z_n) + f(H_n) & f(H_n) \end{bmatrix},$$

proving the statements with respect to firms' R&D investments and that an increase of Δ_n^l leads to a higher innovation pace. To show the relation between the profit gap of the followers and λ_n^2 observe

$$\begin{aligned} \frac{d\lambda_n^2}{d\Delta_n^f} &= nf'(Z_n)\frac{dZ_n}{d\Delta_n^f} + f'(H_n)\frac{dH_n}{d\Delta_n^f} \\ &= nf'(Z_n)\frac{r + nf(Z_n) + f(H_n) - f'(H_n)H_n}{|A|}. \end{aligned}$$

By Lemma 3 the function $f(z)$ is concave and $f(0) = 0$. Together they imply $f(z) \geq f'(z)z$; thus, the derivative is positive, and the result follows. ■

Proof of Proposition 7. As in the previous proof, we make use of the implicit function theorem. Let $g(H_n, Z_n)$ be the function defined in the proof of Proposition 6. Then, the implicit function theorem implies (in matrix notation)

$$\left[\frac{dH_n}{dn}; \frac{dZ_n}{dn}\right] = -\left(A^{-1}\right)B \tag{21}$$

where A is the matrix defined in (20) and

$$B = \left[\frac{\partial g_1}{\partial n}; \frac{\partial g_2}{\partial n}\right] = \left[\frac{d\Delta_n^l}{dn} - f(Z_n)H_n; \frac{d\Delta_n^f}{dn} - f(Z_n)Z_n\right].$$

Using equation (21) we compute the derivatives

$$\frac{dH_n}{dn} = \frac{\psi_{n+1}\left(\frac{d\Delta_n^l}{dn} - f\left(Z_n\right)H_n\right) + nf'\left(Z_n\right)\left(Z_n\frac{d\Delta_n^l}{dn} - H_n\frac{d\Delta_n^f}{dn}\right)}{|A|}$$
$$\frac{dZ_n}{dn} = \frac{f\left(H_n\right)\left(\frac{d\Delta_n^l}{dn} + \frac{d\Delta_n^f}{dn} - f\left(Z_n\right)\left(Z_n + H_n\right)\right) + \psi_n\left(\frac{d\Delta_n^f}{dn} - f\left(Z_n\right)Z_n\right)}{|A|},$$

where $\psi_x = r + xf\left(Z_n\right) > 0$ for all $x > 0$. With these computations we can now prove that the pace of innovation increases in n under increasing profit gaps. Let's start studying the situation in which the leader is two steps ahead, the derivative of λ_n^2 with respect n is given by

$$\begin{aligned}\frac{d\lambda_n^2}{dn} &= f\left(Z_n\right) + nf'\left(Z_n\right)\frac{dZ_n}{d_n} \\ &= \frac{\left(\psi_n + f\left(H_n\right)\right)\left(f\left(Z_n\right)\psi_{n+1} + nf'\left(Z_n\right)\frac{d\Delta_n^f}{dn}\right) + nf'\left(Z_n\right)f\left(H_n\right)\frac{d\Delta_n^l}{dn}}{|A|}.\end{aligned}$$

which is positive whenever $\frac{d\Delta_n^l}{dn}, \frac{d\Delta_n^f}{dn} \geq 0$. Also, we can see that $\frac{d\Delta_n^l}{dn}, \frac{d\Delta_n^f}{dn} < 0$ are necessary but not sufficient for $\frac{d\lambda_n^2}{dn}$ to be negative.

When the leader performs R&D, i.e., the leader is one step ahead of the followers, the derivative of the pace of innovation is given by:

$$\begin{aligned}\frac{d\lambda_n^1}{dn} &= \frac{d\lambda_n^2}{dn} + f'\left(H_n\right)\frac{dH_n}{dn} \\ &= \frac{f\left(Z_n\right)\psi_n\psi_{n+1}}{|A|} + \frac{nf'\left(Z_n\right)\left(f\left(H_n\right) + f'\left(H_n\right)Z_n\right) + f'\left(H_n\right)\psi_{n+1}}{|A|}\frac{d\Delta_n^l}{dn} \\ &+ \frac{nf'\left(Z_n\right)\psi_n}{|A|}\frac{d\Delta_n^f}{dn} + \left(f\left(H_n\right) - f'\left(H_n\right)H_n\right)\frac{nf'\left(Z_n\right)\frac{d\Delta_n^f}{dn} + f\left(Z_n\right)\psi_{n+1}}{|A|}.\end{aligned}$$

By Lemma 3 the function $f(z)$ is concave and $f(0) = 0$; these two conditions imply $f(z) \geq f'(z)z$. Then the derivatives are positive whenever $\frac{d\Delta_n^l}{dn}, \frac{d\Delta_n^f}{dn} \geq 0$, and $\frac{d\Delta_n^l}{dn}, \frac{d\Delta_n^f}{dn} < 0$ are necessary but not sufficient for $\frac{d\lambda_n^1}{dn}$ to be negative. ■

Proof of Proposition 8. Differentiation of $\tilde{\lambda}_n \equiv (n-1)x^* + \phi x_M^*$ with respect to ϕ yields

$$\begin{aligned}\frac{d\tilde{\lambda}_n}{d\phi} &= (n-1)f'(\tilde{Z}_n)\frac{d\tilde{Z}_n}{d\phi} + \frac{d\phi f(\phi(V_n - M_n))}{d\phi} \\ &= \frac{d\phi f(\phi(V_n - M_n))}{d\phi} \\ &\quad \times \left(1 - \frac{(n-1)\tilde{Z}_n f'(\tilde{Z}_n)}{r + nf(\tilde{Z}_n) + (n-1)\tilde{Z}_n f'(\tilde{Z}_n) + \phi f(\phi(V_n - M_n))}\right) > 0,\end{aligned}$$

where we make use of the results in Lemma 6 and Lemma 7.

Differentiation of $\tilde{Z}_n \equiv \tilde{V}_n - \tilde{W}_n$ and $U_n = V_n - M_n$ with respect to $\Delta\pi_n$ (see equations (1), (11), (12), and (13)) yield

$$\begin{aligned}\frac{d\tilde{Z}_n}{d\Delta\pi_n} &= \frac{1 - \phi^2 f'(\phi U_n)\tilde{Z}_n dU_n/d\Delta\pi_n}{r + nf(\tilde{Z}_n) + (n-1)f'(\tilde{Z}_n)\tilde{Z}_n + \phi f(\phi U_n)} > 0, \\ \frac{dU_n}{d\Delta\pi_n} &= \frac{r + f(Z_n)}{r + \phi f(\phi U_n)}\frac{dZ_n}{d\Delta\pi_n} > 0,\end{aligned}$$

where Z_n is defined in equation (15). We use a result in Proposition 2 for the second inequality above. Hence, we have that

$$\begin{aligned}\frac{d\tilde{\lambda}_n}{d\Delta\pi_n} = &\frac{(n-1)f'(\tilde{Z}_n)}{r + nf(\tilde{Z}_n) + (n-1)f'(\tilde{Z}_n)\tilde{Z}_n + \phi f(\phi U_n)} + \\ &\phi^2 f'(\phi U_n)\frac{dU_n}{d\Delta\pi_n}\left(1 - \frac{(n-1)f'(\tilde{Z}_n)\tilde{Z}_n}{r + nf(\tilde{Z}_n) + (n-1)f'(\tilde{Z}_n)\tilde{Z}_n + \phi f(\phi U_n)}\right) > 0.\end{aligned}$$

■