Innovation and Competition: The Role of the Product Market*

Guillermo Marshall† Álvaro Parra‡

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Abstract

We study how competition impacts innovation (and welfare) when firms compete both in the product market and in innovation development. This relationship is complex and may lead to scenarios in which a lessening of competition increases R&D and consumer welfare in the long run, contradicting arguments provided by antitrust agencies in recent merger cases. We provide conditions for when competition increases or decreases industry innovation and welfare. These conditions are based on properties of the product market payoffs.

JEL: D43, L40, L51, O31, O34, O38

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†Department of Economics, University of Illinois at Urbana-Champaign, 214 David Kinley Hall, 1407 W Gregory St, Urbana, IL 61801. gmarshll@illinois.edu

‡Sauder School of Business, University of British Columbia, 2053 Main Mall, Vancouver, BC, V6T1Z2, Canada. alvaro.parra@sauder.ubc.ca

I Introduction

Merger policy is based on the premise that a lessening of competition is likely to hurt consumers. This view has guided the analysis of mergers in innovative industries on both sides of the Atlantic despite a lack of consensus on how competition impacts innovation outcomes.[1] For example, Aghion *et al.* (2005) empirically find a non-monotonic relationship between competition and patenting, which raises the possibility that a lessening of competition may benefit consumers through enhanced innovation. Because innovation drives economic progress, studying how competition affects R&D and welfare is key for antitrust and public policy.

In this paper, we analyze how competition affects firms' incentives to innovate and consumer welfare, focusing on the role played by the product market. To this end, we propose a *dynamic* model of an innovative industry that accommodates arbitrary product market games (e.g., quantity competition with homogeneous goods, price competition with differentiated products, etc.), and study how the product market game being played by the firms shapes the relationship between competition and innovation. The motivation behind examining the role played by the product market stems from the observation that firms invest in R&D because they wish to gain a product market advantage (e.g., a greater product quality or a lower marginal cost). Because competition impacts product market payoffs, competition impacts the incentives to invest in R&D through the product market. Understanding the role played by the product market game has important policy implications, as it can help design a merger policy that is tailored to the specifics of each market.

In concrete terms, we develop a sequential extension to the classic patent-race models (Loury 1979, Lee and Wilde 1980, and Reinganum 1982) by allowing firms to compete both in developing a series of innovations and in the product market. We allow for *large* firms that compete in developing innovations and in the product market as well as research *labs* that only compete in developing innovations. The distinction between large firms and labs captures the fact that

[1] See, for instance, the complaint filed by the Federal Trade Commission (FTC) concerning the merger between Pfizer Corporation and Wyeth Corporation, as well as the complaints filed by the Department of Justice (DOJ) concerning the merger between Regal Beloit Corporation and A.O. Smith Corporation and the merger between The Manitowoc Company, Inc. and Enodis plc. A similar argument was provided by the European Commission (EC) in its investigations of Qualcomm's proposed acquisition of NXP, Bayer's proposed acquisition of Monsanto, and the proposed merger between Dow and DuPont. In fact, the Dow and DuPont merger was cleared by the European Commission subject to a divestiture of DuPont's R&D organization.

firms are asymmetric in both size and scope in many innovative industries (e.g., pharmaceutical industry). Through successful innovation, a large firm becomes the market leader, replacing the previous leader. When a research lab successfully innovates, it auctions the innovation to a large firm, which results in a new industry leader. Being the leader provides a firm with an advantage in the product market—for instance, due to a cost or quality advantage—which creates a positive *profit gap* between the leader and the followers.

In the model, competition affects innovation through two channels. First, holding product market profits equal, a reduction in the number of firms performing R&D reduces the pace of innovation in the industry (Reinganum, 1985). Most of the patent race literature has focused on this first mechanism. Secondly, because competition has a direct effect on the product market payoffs and, consequently, the profit gap that exists between the leader and the followers, competition affects the incentives to innovate through the product market. Depending on the specifics of the product market game, a lessening of competition may increase or decrease the profit gap between leaders and followers. This creates a potentially countervailing effect on the incentives to innovate, which may generate a monotonic-increasing or non-monotonic relationship (e.g., inverted-U or N shaped) between innovation outcomes and the number of large firms.

Although the relationship between competition and innovation may in principle take various shapes, the product market game being played by the firms puts restrictions on this relationship. Product market games can be categorized according to the properties of an equilibrium object: the profit gap between the leader and followers.

We show that when the profit gap between the leader and followers is *weakly increasing* in the number of large firms, competition always increases the industry's innovation rate. Because competition also (weakly) decreases equilibrium prices, competition unequivocally increases the discounted expected consumer surplus in this case. Product market games that feature a weakly increasing profit gap include some parameterizations of price and quantity competition games with homogeneous goods.

We also show that a profit gap between the leader and followers that is *decreasing* in the number of large firms is necessary but not sufficient for competition to decrease the industry's innovation rate. When the number of research labs is sufficiently large, however, a profit gap that is decreasing in the number of large firms is sufficient for competition to decrease the industry's innovation rate. Some parameterizations of quantity competition games with homogeneous goods and price competition games with differentiated products are examples that feature a decreasing profit gap.

Because competition decreases equilibrium prices, a negative relationship between competition and innovation is not one-to-one with a negative relationship between competition and consumer welfare. However, we show that when the number of research labs is sufficiently large, a profit gap that is decreasing in the number of large firms suffices for competition to decrease the discounted expected consumer surplus. That is, there are scenarios in which a lessening of competition may increase consumer welfare in the long run. In these scenarios, the increased arrival rate of innovations more than compensates for the welfare loss that results from static price effects.

These results highlight the importance of the product market for analyzing the impact of mergers (or more generally, competition) on R&D outcomes. Our results are constructive in that they isolate a specific property of the product market payoffs that is key for understanding the relationship between innovation and competition. From a policy perspective, our results suggest caution when analyzing mergers in innovative industries, as the argument commonly invoked by antitrust agencies—a merger reduces incentives to innovate—is not true in general.[2] As well, our analysis calls for the use of flexible demand systems when conducting model-based competition analyses in innovative industries. A lack of model flexibility may restrict the product market payoffs in ways that prevent the data from showing the true relationship between competition and innovation.

The rest of the paper is organized as follows. Section II introduces the model and characterizes the equilibrium. Section III analyzes how market structure affects innovation and welfare outcomes. Section IV provides numerical examples to illustrate the results. Lastly, Section V concludes.

I(i) Literature Review

The question of how competition affects the incentives to innovate stems from the work of Schumpeter (1942).[3] The literature has taken two approaches to modeling the relationship between R&D investments and its returns. The first

[2]See Footnote 1.

[3]See Gilbert (2006) and Cohen (2010) for surveys of the literature.

branch assumes that R&D investments deliver deterministic returns. The second branch—the patent race literature—assumes a stochastic link in which greater investments lead to greater innovation rates.

In a deterministic-R&D model, Dasgupta and Stiglitz (1980a) (henceforth, DS) study the role of product market competition in a scenario in which symmetric firms compete á la Cournot and in developing process innovations. Under an isoelastic demand assumption, the authors show that an increase in the number of firms decreases each firms' investments, but increases aggregate investments. More recently, Vives (2008) generalizes these findings by allowing for a broader set of demand functions and price competition games. Ishida *et al.* (2011) shows that the assumption of symmetric firms is critical for the results in DS: in quantity competition models with high- and low-costs firms, an increase in the number of high-cost firms only leads to DS's result among high-cost firms, as low-cost firms experience enhanced incentives to innovate. In price competition models, Motta and Tarantino (2017) identify conditions under which DS's result holds when the reduction of competition is due to a merger. In contrast to these papers, we show that once dynamics are incorporated, the relationship between competition and innovation is richer than previously described. We illustrate this in Section IV, where we present an example satisfying DS's original assumptions about the product market game—Cournot competition with isoelastic demand—in which a lessening of competition may lead to enhanced or reduced innovation incentives both at the individual and aggregate level.

Early work in the patent race literature often omitted the role of product market competition and dynamic considerations (Loury 1979, Lee and Wilde 1980, and Reinganum 1982).[4] In a single-innovation model, Dasgupta and Stiglitz (1980b) modeled payoffs as the result of product market competition, but did not study how competition affects innovation outcomes. Reinganum (1985) incorporated dynamics by studying a sequence of patent races where firms compete through a ladder of innovations. She finds results analogous to DS in a context where product market payoffs are unaffected by the number of competitors. Aghion *et al.* (2001), Aghion *et al.* (2005), and other follow-up papers have examined the impact

[4]Although the patent race literature has a dynamic dimension—the expected arrival time of innovations—we use the term dynamic to incorporate the intertemporal tradeoffs that arise when firms compete in developing a sequence of innovations and in the product market. For example, the tradeoff that arises when a lessening of competition increases prices in the short run but enhances the rate at which innovations reach the market is an intertemporal tradeoff that is absent in the early patent race literature.

of product market competition on innovation decisions in duopolistic markets. In these models, the duopolists compete in prices, and competition is captured by the degree of substitution between the products sold by the firms. A key observation in these papers is that innovation is driven by the "escape competition" effect, i.e., the difference between the payoffs before and after the introduction of an innovation. We build upon these ideas by extending the model to an arbitrary number of firms and allowing for general product market games. We directly link the escape of competition effect (and, thus, R&D outcomes) with product market outcomes. A key finding in our paper is that the relation between competition and innovation is determined by a combination of how firms compete (quantity or prices) and the shape of the demand function. This finding has consequences for empirical work, as some parametric choices may lead to empirical models that are not flexible enough to capture the effects of competition on innovation.

Our analysis is built upon a standard dynamic model of innovation. Versions of the model have been used by Aghion and Howitt (1992) to study endogenous growth; Segal and Whinston (2007) to study the impact of antitrust regulation on innovation outcomes; Acemoglu and Akcigit (2012) to study an IP policy contingent on the technology gap among firms; Denicolò and Zanchettin (2012) to study leadership cycles; Acemoglu *et al.* (2013) to study productivity growth and firm reallocation; and by Parra (2017) to study the dynamics of the Arrow's Replacement Effect and its impact on patent design.

Our paper also contributes to the horizontal merger literature.[5] Several authors have discussed at a conceptual level how innovation considerations should be incorporated into merger analysis (see, for instance, Gilbert and Sunshine 1995, Evans and Schmalansee 2002, Katz and Shelanski 2005, 2007), and a number of recent papers have explored this issue empirically.[6] Closer to our work is Mermelstein *et al.* (2015) and Hollenbeck (2015), who use computational methods to study optimal merger policy in a dynamic oligopoly model with endogenous capital and R&D investments. Although we do not model merger decisions (or merger-specific synergies), we contribute to this literature by providing analytic results that clarify the role played by market concentration on firms' investments decisions.

[5]See, for example, Williamson (1968), Farrell and Shapiro (1990), Gowrisankaran (1999), Nocke and Whinston (2010, 2013), Federico *et al.* (2017).

[6]Ornaghi (2009) studies mergers in the pharmaceutical industry and their impact in R&D. Igami and Uetake (2016) studies the relation between mergers and innovation in the hard-drive industry and Entezarkheir and Moshiri (2015) performs a cross-industry analysis.

II A Model of Sequential Innovations with Product Market Competition

Consider a continuous-time infinitely lived industry where $n+m+1$ firms compete in developing new innovations (or products). Among these, $n+1$ firms are *large* in the sense that they also compete in the product market selling final products. The remaining m firms auction their innovations to the large firms; we call the latter set of firms *research labs*.

Competition in the product market is characterized by one technology leader and $n > 0$ symmetric followers (or competitors). For tractability purposes, we assume that the market leader is always one step ahead of the followers in terms of the technology to which they have access.[7] We relax this assumption in the Online Appendix, where we allow the leader to invest in increasing its technological lead relative to the followers.

The market leader obtains a profit flow $\pi_n^l > 0$, whereas each follower obtains a profit flow $\pi_n^f \in [0, \pi_n^l)$. Both π_n^l and π_n^f are weakly decreasing in the number of product market competitors in the industry (i.e., large firms), n, capturing that more intense product market competition decreases firm profits. For the purpose of reducing the dimensionality of the state space, we assume that the profit flows are stationary in the number of innovations. These assumptions allow for general forms of product market competition. For instance, firms can compete through prices, quantities, or qualities. They also allow for competition in various types of innovations. Firms may compete in developing process innovations, quality improvements, or products that leave previous vintages obsolete.[8]

Research labs do not compete in the product market and their only source of profits is the revenue they derive from selling their innovations to large firms. We assume that research labs sell their innovations using a second-price auction. In case of a tie, we assume that the innovation is randomly assigned to one of the tying followers.[9] All firms discount their future payoffs at a rate of $r > 0$.

At each instant in time, every follower and research lab invests in R&D in order

[7]More precisely, this common assumption in the literature can be distilled as the conjunction of two independent assumptions about the nature of patent protection: a) a patent makes full disclosure of the patented technology, which allows followers to build upon the latest technology, leap-frogging the leader once they achieve an innovation; b) the legal cost of enforcing older patents more than exceeds the benefits of enforcing the patent.

[8]Sections III and IV provide examples where all the assumptions of the model are satisfied.

[9]This assumption simplifies exposition and does not affect the results of the paper.

to achieve an innovation. Firm i chooses a Poisson innovation rate x_i at a cost of $c(x_i)$. We assume that $c(x_i)$ is strictly increasing, twice differentiable, strictly convex (i.e., $c''(x) > 0$ for all $x \geq 0$), and satisfies $c'(0) = 0$. The assumption that large firms and labs are equally productive along the R&D dimension is for notational ease. Introducing asymmetries does not impact our results in a significant way. We also assume that the Poisson processes are independent among firms, generating a stochastic process that is memoryless.

We focus on symmetric and stationary Markov perfect equilibria by using a continuous-time dynamic programming approach. Our assumptions guarantee the concavity of the value functions, implying equilibrium uniqueness.

Let $V_{n,m}$ represent the value of being the market leader, $W_{n,m}$ the value of being a follower, and $L_{n,m}$ the value of being a research lab when there are n followers and m labs in the industry. At time t, we can write the payoffs of the different types of firms as follows:

$$
\begin{aligned}
V_{n,m} &= \int_t^\infty (\pi_n^l + \lambda_{n,m} W_{n,m}) e^{-(r+\lambda_{n,m})(s-t)} ds, \\
W_{n,m} &= \max_{x_i} \int_t^\infty (\pi_n^f + x_i V_{n,m} + x_{-i} W_{n,m} - c(x_i)) e^{-(r+\lambda_{n,m})(s-t)} ds, \\
L_{n,m} &= \max_{y_i} \int_t^\infty (y_i(V_{n,m} - W_{n,m} + L_{n,m}) + y_{-i} L_{n,m} - c(y_i)) e^{-(r+\lambda_{n,m})(s-t)} ds,
\end{aligned}
$$

where $\lambda_{n,m} = \sum_i^n x_i + \sum_j^m y_j$ is the industry-wide *pace* or *speed* of innovation, $x_{-i} = \lambda_{n,m} - x_i$, and $y_{-i} = \lambda_{n,m} - y_i$. To understand the firms' payoffs, fix any instant of time $s > t$. With probability $\exp(-\lambda_{n,m}(s-t))$, no innovation has arrived between t and s. At that instant, the leader receives the flow payoff π_n^l and the expected value of becoming a follower, $\lambda_{n,m} W_{n,m}$. Each follower receives the flow payoff π_n^f; innovates at rate x_i; earns an expected payoff of $x_i V_{n,m}$; pays the flow cost of its R&D, $c(x_i)$; and faces innovation by other firms at rate x_{-i}. Note that since all large firms are symmetric, they value an innovation in $V_{n,m} - W_{n,m}$. These valuations, in conjunction with the auction format, imply that labs sell their innovations at price $V_{n,m} - W_{n,m}$ in equilibrium.[10] Labs obtain this revenue at rate y_i; pay the flow cost of their R&D, $c(y_i)$; and face innovation by other firms at rate y_{-i}. All of these payoffs are discounted by $\exp(-r(s-t))$.

We solve the problem above by making use of the principle of optimality, which

[10]Since the winning bidder of an auction held by a lab earns zero surplus, we do not include auction payoffs in the value functions of the leader and followers.

implies that, at every instant of time, the values must satisfy

$$rV_{n,m} = \pi_n^l - \lambda_{n,m}(V_{n,m} - W_{n,m}), \tag{1}$$

$$rW_{n,m} = \max_{x_i} \ \pi_n^f + x_i(V_{n,m} - W_{n,m}) - c(x_i), \tag{2}$$

$$rL_{n,m} = \max_{y_i} \ y_i(V_{n,m} - W_{n,m}) - c(y_i). \tag{3}$$

In words, the flow value of being the market leader at any instant of time, rV_n, is equal to the profit flow obtained at that instant plus the expected loss if an innovation occurs, $\lambda_{n,m}(W_{n,m} - V_{n,m})$. The instantaneous value of being a follower, $rW_{n,m}$, is equal to the profit flow plus the expected incremental value of becoming the leader, $x_i(V_{n,m} - W_{n,m})$, minus the flow cost of R&D. Finally, the flow value of being a research lab is equal to the expected payoff of successfully innovating and selling an innovation, $y_i(V_{n,m} - W_{n,m})$, minus the flow cost of R&D.

In the context of this model, the infinitely long patent protection and the assumption that a new innovation completely replaces the old technology implies that the incumbent has no incentives to perform R&D. That is, the leader's lack of R&D is an implication of our modeling choices rather than an assumption; see Parra (2017) for a formal proof. In the Online Appendix, we extend the model to allow for the leader to increase the quality of its innovation, attenuating the leader's replacement effect, inducing the leader to invest in R&D.

Maximizing value functions (2) and (3), and imposing symmetry among followers and research labs, we obtain $x_i = y_i = x^*_{n,m}$, where

$$c'(x^*_{n,m}) = V_{n,m} - W_{n,m} \tag{4}$$

or $x^*_{n,m} = 0$ if $c(0) > V_{n,m} - W_{n,m}$; with the subindices n and m capturing how market structure affects R&D decisions. Equation (4) tells us that, at every instant of time, the followers and research labs invest until the marginal cost of increasing their arrival rate is equal to the incremental rent of achieving an innovation. The incremental rent of achieving an innovation relates to the "escape competition" effect in Aghion *et al.* (2001), as $V_{n,m} - W_{n,m}$ represents the benefits of escaping competition through an innovation.

Strict convexity implies that condition (4) can be inverted so that $x^*_{n,m} = f(V_{n,m} - W_{n,m})$, where $f(z)$ is a strictly increasing function of z.[11] By replacing

[11] This function is further characterized in Lemma 1 in the Appendix.

$x^*_{n,m}$ into equations (2) and (3), we can solve the game and prove the following proposition.

Proposition 1 (Market equilibrium)**.** *There is a unique symmetric equilibrium, which is determined by the solution of the system of equations (1–4).*

It can be easily verified that the payoffs in this model possess the expected comparative statics for given values of n and m. For instance, the value functions increase with larger profit flows or a lower interest rate (all else equal).

III Market Structure and Performance

We next study how market structure affects R&D outcomes and, more generally, consumer welfare. Market structure affects dynamic incentives to invest in R&D through two channels: *product market competition* and *innovation competition*. We explore how these two forms of competition interact and determine market outcomes.

III(i) Pace of Innovation

We begin our analysis by considering how an *isolated* change in innovation competition or product market competition affects innovation outcomes. Although a change in the number of large firms—i.e., firms competing in innovation development and in the product market—affects both forms of competition simultaneously, this exercise gives us a first approach to understanding how each form of competition affects R&D outcomes. A key object in our analysis is the *profit gap* between the leader and a follower, $\Delta\pi_n \equiv \pi^l_n - \pi^f_n$, which measures the (static) product market benefit of being the market leader. While most models of product market competition predict that both π^l_n and π^f_n are weakly decreasing in n; the profit gap can either increase or decrease with n even when both π^l_n and π^f_n are weakly decreasing in n (see examples in Table I).

Proposition 2 (Product market and innovation competition)**.** *Competition affects innovation outcomes through two channels:*

i) Product market competition: *Fixing the number of firms, an increase in the profit gap between the leader and a follower, $\Delta\pi_n$, increases each firm's R&D investment, $x^*_{n,m}$, and the pace of innovation in the industry, $\lambda_{n,m}$.*

ii) Innovation competition: *A decrease in the number of research labs, m, decreases the overall pace of innovation in the industry, $\lambda_{n,m}$, but increases each firm's R&D investment, $x^*_{n,m}$.*

Firms' incentives to invest in R&D are driven by the incremental rent obtained from an innovation (see equation (4)). Proposition 2 tells us that the incremental rent is increasing in the profit gap between the leader and the followers, and a greater profit gap increases the pace of innovation. This result implies that because product market concentration changes product market payoffs—and, consequently, the profit gap—product market concentration has an impact on the incentives to invest in R&D.[12] As we shall see later, a specific property of the profit gap determines the shape of the relationship between competition and the pace of innovation.

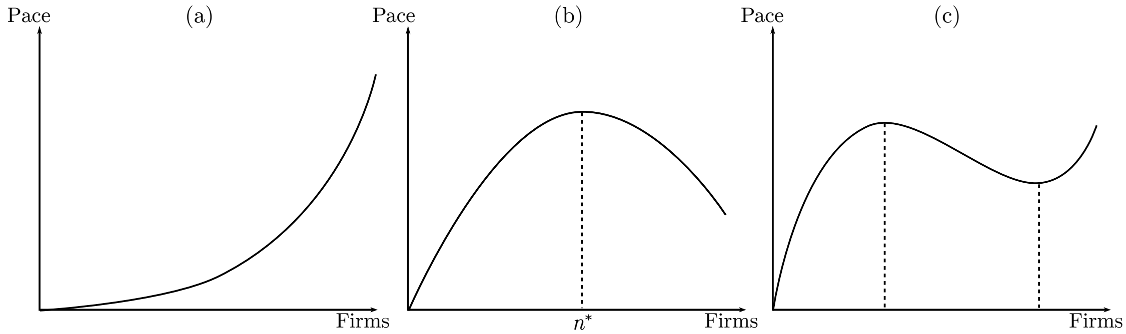
From Proposition 2 we also learn that innovation competition affects the pace of innovation in two ways. To understand these effects, suppose we decrease the number of research labs by one. Varying the number of labs is convenient because it allows us to abstract away from product market effects, as labs do not compete in the product market. First, the reduction in the number of firms performing R&D has a direct negative effect on the pace of innovation in the industry, $\lambda_{n,m}$ (i.e., fewer firms performing R&D). Second, this reduction in $\lambda_{n,m}$ increases the expected time between innovations, extending the lifespan of a leader and raising the value of being a market leader, $V_{n,m}$. This causes an increase in the incremental rent of an innovation, incentivizing the remaining firms to invest more in R&D. Although each remaining firm increases its R&D investment, the first effect dominates, and the lessening of innovation competition leads to a decrease in the industry's pace of innovation.[13] A similar result is discussed in Reinganum (1985).

Proposition 2 illustrates how product market competition and innovation competition affect the incentives to innovate in isolation. A change in the number of large firms, however, affects both forms of competition simultaneously. The

[12]It is through this channel that our analysis differs from the growth through innovation literature (e.g., Aghion *et al.* 2001), which has examined how the intensity of product market competition—captured by the degree of substitution among a fixed number of firms or the degree of collusion between firms—affects innovation. In our analysis, we explicitly study how a change in the number of competitors affects innovation through changes in product market payoffs. Our analysis encompasses substitution effects as well as various forms of competition and types of innovations.

[13]The net effect of a decrease in the number of research labs on $\lambda_{n,m}$ must be negative, as it was the initial decrease in the pace of innovation that triggered the increase in the incremental rent of an innovation in the first place.

Figure 1: Industry's pace of innovation vs. number of competitors in the industry



interaction between these forms of competition is complex, as these effects may either reinforce or collide with each other. Because of the interaction of these effects, the relationship between competition and innovation may be monotonic or non-monotonic. Figure 1 shows some examples based on parameterizations of the model that we discuss in Section IV.

Although the relationship between competition and innovation may in principle take various shapes, the product market game being played by the firms puts restrictions on this relationship. Product market games can be categorized according to an equilibrium object: the profit gap between the leader and followers. We say that the product market payoffs have a *decreasing profit gap* between the leader and a follower when an increase in the number of large firms, n, *decreases* the profit gap, $\Delta\pi_n$. Likewise, we say that the product market payoffs have an *increasing profit gap* between the leader and a follower when an increase in the number of large firms, n, *increases* the profit gap, $\Delta\pi_n$. Table I shows examples of product market games, and provides information about the shape of the profit gap in each example. For instance, a constant-elasticity demand in a quantity competition game can deliver a profit gap that is increasing or decreasing in the number of firms depending on the value of the elasticity of demand. In what follows, we explore how the shape of the profit gap between the leader and followers puts restrictions on the relationship between competition and innovation.

Proposition 3 (Competition increases innovation)**.** *A weakly increasing profit gap is sufficient for competition to increase the pace of innovation.*

Proposition 3 shows a sufficient condition that guarantees an increasing relationship between competition and innovation. The logic behind the result is as

Table I: Product market competition and the slope of the profit gap: examples

	Bertrand	Cournot I	Cournot II	Logit
Differentiation	No	No	No	Yes
Innovation type	Process	Process	Process	Quality ladder
Leader advantage	Marginal cost advantage: $mc_l = \beta mc_f$, $\beta \in (0,1)$			Quality gap: $\kappa > 0$
Demand	$Q = Q(P)$	$Q = a/P^{1/\sigma}$	$Q = a/P^{1/\sigma}$	$s_l = \frac{\exp\{\kappa - p_l\}}{\exp\{\kappa - p_l\} + n\exp\{-p_f\}}$ $s_f = \frac{\exp\{-p_f\}}{\exp\{\kappa - p_l\} + n\exp\{-p_f\}}$
Restrictions	None	$\frac{(1+\beta)}{(1-\beta)}\frac{\sigma(n-\sigma)}{(n-1)} < 1$	$\frac{(1+\beta)}{(1-\beta)}\frac{\sigma(n-\sigma)}{(n-1)} > 1$	Firm-level horizontal differentiation
Profit gap	Weakly increasing	Increasing	Decreasing	Decreasing

Notes: Subscripts l and f denote leader and follower, respectively. For simplicity, we assume that the horizontal differentiation in the logit model (i.e., the idiosyncratic taste shocks) is at the firm rather than the product level. See Marshall (2015) for an application with a closely related model.

follows: if the profit gap between the leader and a follower, $\Delta\pi_n$, increases with the number of competitors, then greater competition increases the incentives to perform R&D (see Proposition 2.i). This product-market effect on innovation is reinforced by a larger number of firms performing R&D (see Proposition 2.ii). An example of a product market game with a weakly increasing profit gap is a price competition in a market for homogeneous goods where firms develop process innovations.[14]

Proposition 4 (Competition decreases innovation)**.** *A decreasing profit gap is necessary for competition to decrease the pace of innovation. If the number of research labs is large enough, a decreasing profit gap is sufficient.*

When the profit gap decreases in the number of firms, a lessening of competition creates a tension between the effects of product market competition and innovation competition. On the one hand, the decrease in product market competition increases the profit gap and, consequently, increases the incentives to perform R&D (see Proposition 2.i). On the other hand, a lessening of innovation competition has

[14] In this price competition example, increasing the number of followers (beyond one) does not affect the profit gap, as the market price equals the followers' marginal cost.

a negative effect on the pace of innovation (see Proposition 2.ii). Although this tension may result in an increased pace of innovation (see Section IV for examples), Proposition 4 shows that in industries in which research labs play an important role in total R&D, a decreasing profit gap between the leader and a follower is sufficient for a lessening of competition to increase the pace of innovation.[15]

The intuition for the sufficiency result in Proposition 4 follows from observing that the R&D incentives of research labs and large firms are aligned (see equation (4)). When market concentration increases R&D incentives, research labs magnify this effect, as more firms are affected by the enhanced incentives to perform R&D. Although the auction mechanism used by labs simplifies the analysis, it is not necessary for these results to go through. As long as the labs' incentives are aligned with those of large firms, it follows that labs will magnify the impact of product market competition on R&D outcomes.

In summary, our results show that the product market game played by the firms determines the relationship between competition and innovation. Our results are constructive in that they isolate a specific property of the product market payoffs that is key for understanding this relationship. These findings have several implications. First, they call for a merger policy that is tailored to the specifics of each market, as the argument that a lessening of competition causes a decrease in the pace of innovation is not true in general. Second, our results suggest that model-based research on the impact of competition on innovation should specify product market games that do not ex-ante restrict the relationship between competition and innovation. This is particularly relevant for empirical work, as restrictive empirical models may prevent the analysis from showing the true relationship between competition and innovation.

III(ii) Welfare Analysis

We have already provided sufficient conditions for instances when competition increases or decreases the pace of innovation. Evaluating whether an increase in competition is welfare enhancing, however, requires understanding how it affects both the path of prices faced by consumers and the pace of innovation. To this end, we incorporate price effects into the analysis and study the trade-off between

[15] The proof that a decreasing profit gap is sufficient for competition to decrease the pace of innovation for a sufficiently large m uses strict convexity of the cost function (i.e., $c''(x) > 0$ for all $x \geq 0$). We note, however, that the result applies for a broader set of cost functions. For instance, the result also applies for all cost functions satisfying $c(x) = x^\gamma/\gamma$ with $\gamma > 1$.

the price and innovation effects caused by a change in competition.

To make statements about the relationship between competition and consumer welfare, we impose further structure to the model.

Assumption 1. *Each innovation increases the consumer-surplus flow by* $\delta_n > 0$.

The term δ_n represents the increment in consumer surplus due to an innovation. If, for instance, firms compete in developing process innovations (i.e., cost-saving technologies), δ_n represents the decrease in cost that is passed on to consumers through lower prices and, consequently, higher consumer surplus. Table II provides examples of different demand functions with their respective expressions for the consumer surplus. In all of these examples, a stronger version of Assumption 1 is satisfied: the increment in consumer-surplus flow δ_n is independent of the number of firms competing in the product market, n.

Given Assumption 1, the discounted expected consumer surplus, CS_n, which incorporates the dynamic benefits of future innovations, is given by

$$rCS_n = cs_n + \lambda_{n,m}\delta_n/r, \tag{5}$$

where cs_n is the consumer-surplus flow when there are n product market competitors.[16] Observe that the discounted expected consumer surplus is greater than cs_n and that it is increasing in both the pace of innovation and the magnitude with which each innovation enhances consumer surplus, δ_n. The discounted expected consumer surplus also decreases with the interest rate, as future breakthroughs are discounted at a higher rate.

From equation (5), we can note that competition affects the discounted expected consumer surplus through three mechanisms. First, market concentration has a direct effect on spot prices, affecting the consumer-surplus flow cs_n. Market concentration also affects the discounted expected consumer surplus by potentially changing the pass-through of innovations on consumer welfare, δ_n. Finally, as discussed in the previous subsection, market concentration has an effect on the pace of innovation, $\lambda_{n,m}$. Because a lessening of competition may increase the pace of innovation at the same time that it increases prices, the relationship between competition and innovation is not one-to-one with the relationship between competition and consumer welfare.

[16] See Lemma 2 for the derivation of equation (5).

TABLE II: PRODUCT MARKET COMPETITION AND CONSUMER SURPLUS: EXAMPLES

	Bertrand	Cournot	Logit
Differentiation	No	No	Yes
Innovation type	Process	Process	Quality ladder
Leader advantage	Marginal cost advantage: $mc_l = \beta mc_f$, $\beta \in (0,1)$		Quality gap: $\kappa > 0$
Demand	$Q = a/P$ if $P < \bar{P}$		$s_l = \frac{\exp\{\kappa - p_l\}}{\exp\{\kappa - p_l\} + n\exp\{-p_f\}}$ $s_f = \frac{\exp\{-p_f\}}{\exp\{\kappa - p_l\} + n\exp\{-p_f\}}$
Consumer-surplus flow (cs_n)	$a\log\bar{P} - a\log p_n$		$\log\left(\exp\{\kappa - p_l\} + n\exp\{-p_f\}\right) + \gamma$
Innovation effect on CS (δ_n)	$-a\log\beta$		κ
Restrictions	None	None	Firm-level horizontal differentiation

Notes: Subscripts l and f denote leader and follower, respectively. The γ parameter in the logit-model consumer surplus is Euler's constant.

Equation (5) shows that when a lessening of competition increases the market price (i.e., $dcs_n/dn > 0$) and decreases the innovation pass-through on consumer surplus (i.e., $d\delta_n/dn \geq 0$), an increase in the speed of innovation is necessary for a lessening of competition to increase welfare. Based on these observations and our propositions on the relationship between competition and innovation, we can establish the following results on the impact of competition on consumer welfare.

Proposition 5 (Competition and consumer welfare)**.**

i) Suppose competition decreases the market price (i.e., $dcs_n/dn > 0$) and increases the innovation pass-through on consumer surplus (i.e., $d\delta_n/dn \geq 0$). An increasing profit gap between the leader and a follower is sufficient for competition to increase the discounted expected consumer surplus.

ii) Suppose competition decreases the market price (i.e., $dcs_n/dn > 0$) and keeps the innovation pass-through on consumer surplus constant (i.e., $d\delta_n/dn = 0$). A decreasing profit gap between the leader and a follower is sufficient for a lessening of competition to increase the expected discounted consumer surplus if the number of research labs is large enough.

Proposition 5 first shows that a profit gap that is increasing the number of firms is sufficient for competition to increase consumer welfare. This implication is straightforward since, in this case, competition increases the pace of innovation (Proposition 3) at the same that it decreases prices in the short run. The proposition also shows that for a sufficiently large number of labs, and under a restriction on how competition impacts the pass-through of innovations on consumer welfare (i.e., $d\delta_n/dn$), a decreasing profit gap becomes sufficient for a lessening of competition to increase consumer welfare. The driver of the result is that when market concentration increases R&D incentives, research labs magnify the effect of competition on the pace of innovation, as more firms are affected by the enhanced incentives. It is noteworthy that these sufficient conditions only depend on the number of firms and on properties of the product market payoffs.

IV An Illustrative Example

In this section we parameterize the model and simulate the effect of competition on market outcomes. The purpose of this exercise is to illustrate our results by providing examples that show, first, that the relationship between market structure and the pace of innovation is complex; and second, that a lessening of competition can enhance consumer surplus despite short-run price effects.[17] We consider the case without labs, $m = 0$, unless otherwise noted. Henceforth, we drop the m subscript for ease of notation.

IV(i) Parameters

We consider a market for a homogeneous good, where firms compete in quantity (Cournot competition), and market demand is given by $Q = a/P$, with $a > 0$ and $P \leq \bar{P}$. Firms also compete developing a sequence of cost-saving innovations. Each innovation provides the innovating firm with a marginal cost advantage, reducing the leader's marginal cost by a factor of $\beta \in (0, 1)$. The R&D cost function is given by $c(x_i) = \gamma_0 + \gamma_1^{-1} x_i^{\gamma_1}$, where $\gamma_0 \geq 0$ represents the fixed costs of performing R&D and $\gamma_1 > 1$.

[17] In many of the examples we provide, reduced competition and the elimination of duplicated R&D fixed costs create incentives to merge. That is, in the numerical examples it will often be true that $W_{n-1} > 2W_n$ or $V_{n-1} \geq W_n + V_n$, which are necessary conditions for a merger to be profitable. In these examples, incentives to merge do not imply that a lessening of competition will necessarily benefit (or hurt) consumers.

We denote, at any instant of time, the marginal cost of the followers by mc and the marginal cost of the leader by $\beta \cdot mc$. The equilibrium market price is $p_n = mc(\beta + n)/n$, which depends on the follower's marginal cost of production, the size of the leader's cost advantage, and the number of followers in the market. As expected, the equilibrium market price is decreasing in n and increasing in both β and mc. Similarly, equilibrium profits are given by

$$\pi_n^l = a\frac{(n(1-\beta)+\beta)^2}{(\beta+n)^2}, \qquad \pi_n^f = a\frac{\beta^2}{(\beta+n)^2},$$

which do not depend on the current marginal cost, nor the number of innovations that have taken place. Profits do depend, however, on the number of followers and the size of the leader's cost advantage, β. These equilibrium profits imply that the profit gap is positive, $\Delta\pi_n \equiv \pi_n^l - \pi_n^f > 0$; decreasing in the number of followers, $d\Delta\pi_n/dn < 0$; and increasing in the cost advantage of the leader, $d\Delta\pi_n/d\beta < 0$. As discussed above, a decreasing profit gap may lead to scenarios in which a lessening of competition increases consumer surplus (see Proposition 5).

Finally, to capture the role of the pace of innovation on the path of prices faced by consumers, we make use of the expected discounted consumer surplus defined in equation (5). The flow of consumer surplus when the market price is p_n is given by $cs_n = a\log\bar{P} - a\log p_n$, and an innovation increases the flow of consumer surplus by $\delta \equiv -a\log\beta > 0$.

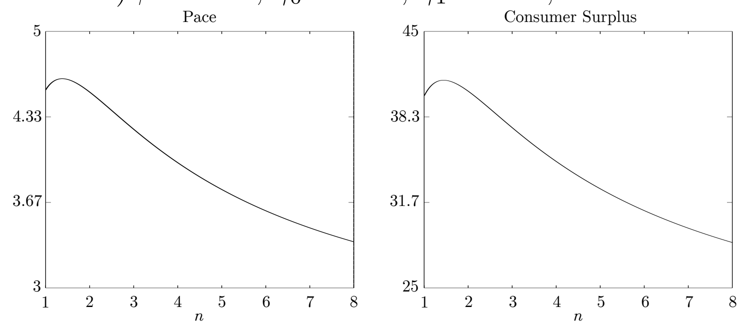
IV(ii) Results

Using this setup, we provide four numerical examples to illustrate our results. In Table III.a (see Figure 2.a) we show market outcomes for a set of parameters that create an inverted-U relationship between the pace of innovation and the number of followers. A similar inverted-U relationship is found for the expected discounted consumer surplus. This example shows that a lessening of competition may enhance consumer surplus by increasing the pace of innovation—for instance, when going from $n = 3$ to $n = 2$—even though market concentration increases prices in the short run. The gains in consumer surplus arise from consumers enjoying more frequent price reductions—caused by a greater pace of innovation—that more than compensate for the short-run price effects.

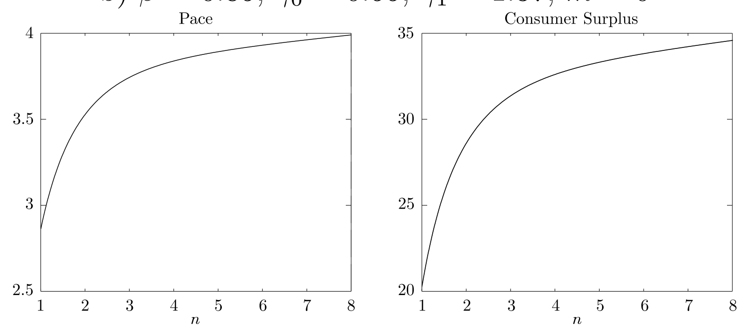
Result 1. *A lessening of competition may enhance consumer surplus even if it increases prices in the short run.*

Figure 2: Market-outcome comparison for different numbers of followers and parameter values

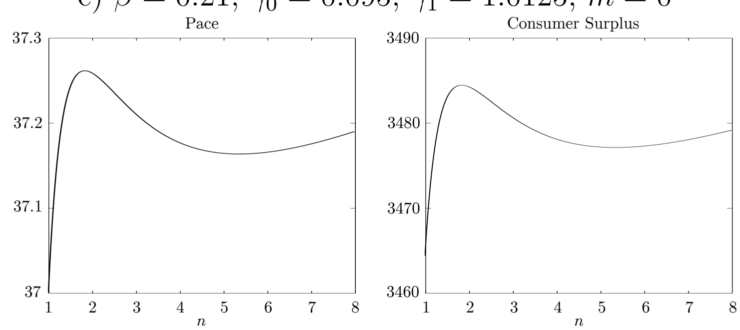
a) $\beta = 0.85$, $\gamma_0 = 0.55$, $\gamma_1 = 1.07$, $m = 0$



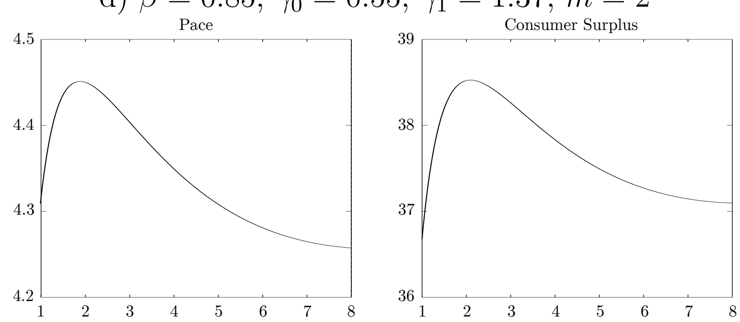
b) $\beta = 0.85$, $\gamma_0 = 0.55$, $\gamma_1 = 1.37$, $m = 0$



c) $\beta = 0.21$, $\gamma_0 = 0.095$, $\gamma_1 = 1.0125$, $m = 0$



d) $\beta = 0.85$, $\gamma_0 = 0.55$, $\gamma_1 = 1.37$, $m = 2$



Notes: Fixed parameter values are $r = 0.03$, $a = 60$, and $mc = 10$. "n" is the number of large firms, "Pace" is the pace of innovation (λ_n), and "Consumer Surplus" is the discounted expected consumer surplus (CS_n).

Table III: Market-outcome comparison for different numbers of followers and parameter values

a) $\beta = 0.85$, $\gamma_0 = 0.55$, $\gamma_1 = 1.07$, $m = 0$

n	λ_n	CS_n	V_n	W_n
1	4.543	39.050	415.999	414.887
2	4.527	39.363	165.853	164.794
3	4.237	36.725	83.333	82.309
4	3.977	34.285	46.264	45.264
5	3.768	32.312	26.482	25.501
6	3.602	30.738	14.690	13.725
7	3.468	29.470	7.097	6.145
8	3.360	28.435	1.919	0.978

b) $\beta = 0.85$, $\gamma_0 = 0.55$, $\gamma_1 = 1.37$, $m = 0$

n	λ_n	CS_n	V_n	W_n
1	2.865	22.682	443.416	441.940
2	3.531	29.645	180.412	179.178
3	3.747	31.939	92.445	91.360
4	3.842	32.971	52.601	51.616
5	3.896	33.562	31.198	30.286
6	3.934	33.977	18.366	17.511
7	3.965	34.314	10.059	9.248
8	3.994	34.617	4.365	3.591

c) $\beta = 0.21$, $\gamma_0 = 0.095$, $\gamma_1 = 1.0125$, $m = 0$

n	λ_n	CS_n	V_n	W_n
1	37.048	3464.601	74.071	73.025
2	37.258	3484.460	23.881	22.844
3	37.219	3480.852	11.694	10.662
4	37.191	3478.311	6.772	5.744
5	37.181	3477.375	4.246	3.221
6	37.182	3477.487	2.753	1.729
7	37.190	3478.233	1.784	0.762
8	37.202	3479.359	1.112	0.093

d) $\beta = 0.85$, $\gamma_0 = 0.55$, $\gamma_1 = 1.37$, $m = 2$

n	λ_n	CS_n	V_n	W_n
1	4.315	36.824	419.829	418.685
2	4.456	38.670	171.046	170.005
3	4.410	38.412	87.689	86.734
4	4.356	37.984	49.791	48.903
5	4.315	37.645	29.366	28.530
6	4.287	37.421	17.086	16.292
7	4.271	37.295	9.117	8.358
8	4.263	37.246	3.645	2.916

Notes: Fixed parameter values are $r = 0.03$, $a = 60$, and $mc = 10$. n is the number of followers, λ_n is the pace of innovation, CS_n is the expected discounted consumer surplus, V_n is the value of being the leader, and W_n is the value of being a follower.

In Tables III.b and III.c (see Figures 2.b and 2.c, respectively), we show examples in which the pace of innovation varies monotonically (Table III.b) or non-monotonically (N-shaped in Table III.c) with respect to the number of followers. These examples illustrate the complex relationship that exists between the number of firms and the pace of innovation. As discussed in Section III, the shape of this relationship is given by the relative importance of two separate effects created by a change in competition. On the one hand, a lessening of competition may increase the profit gap between the leader and followers—increasing the incentives to innovate; on the other hand, it reduces the number of firms performing R&D.

Result 2. *The relationship between the pace of innovation and the number of firms can be monotonic or non-monotonic (e.g., inverted-U or N shaped).*

Lastly, in Proposition 4, we argue that a decreasing profit gap becomes sufficient for competition to decrease the pace of innovation when the number of research labs, m, is sufficiently large. In these examples, however, we find that the number of research labs needed to generate a decreasing relationship between competition and the pace of innovation can be as small as zero (e.g., Table III.a and Table III.c). In some cases, however, we do find that a a profit gap that decreases in the number of firms is insufficient for competition to decrease the pace of innovation (e.g., Table III.b). To illustrate how the presence of research labs can impact market outcomes, Table III.d (see Figure 2.d) uses the same parameters as Table III.b, but adds two research labs to the analysis ($m = 2$). The table shows that it may only take a small number of labs ($m = 2$ in this case) to transform the relationship between the number of firms and the pace of innovation.

Result 3. *A profit gap that decreases in the number of firms in conjunction with a small number of labs may generate a decreasing relationship between competition and innovation.*

V Concluding Remarks

We studied the impact of competition on market outcomes in innovative industries. A lessening of competition affects R&D outcomes both directly by reducing the number of firms performing R&D and indirectly by changing the product market profits. The relationship among these effects is complex and may lead to scenarios in which a lessening of competition increases an industry's pace of innovation and consumer surplus in the long run.

Although the relationship between competition and innovation may take various shapes, the product market game being played by firms puts shape restrictions on this relationship. We provided conditions for when competition increases or decreases the pace of innovation as well as consumer welfare. These conditions are based on product market payoffs, and highlight the importance of the product market for analyzing the impact of mergers (or more generally, competition) on R&D outcomes.

Our results have two broader implications. First, the results suggest that a lessening of competition may enhance innovation, which contradicts arguments provided by antitrust agencies in recent merger cases.[18] Second, the shape restrictions that product market payoffs put on the relationship between competition and innovation threaten the credibility of model-based empirical studies that do not specify flexible demand systems. Product market games that ex-ante restrict the relationship between competition and innovation should be avoided by researchers conducting empirical studies.

[18] See Footnote 1.

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Appendix

A Preliminary Results

Lemma 1. *The function $f(z)$ implicitly defined by $c'(f(z)) = z$ satisfies:*

1. *$f(z) > 0$ for all $z > 0$ and $f(0) = 0$.*
2. *$f'(z) > 0$ for all $z \geq 0$. Also, if $c'''(x) \geq 0$, $f''(z) \leq 0$, i.e. f is concave.*
3. *Let $h(z) = (n+1)zf(z) - c(f(z))$ for $z \geq 0$. Then $h'(z) = (n+1)f(z) + nzf'(z) > 0$ for all $z \geq 0$.*

Proof. 1. $c(x)$ being strictly increasing and differentiable implies $c'(x) > 0$ for all $x > 0$. $c(x)$ being strictly convex implies $c''(x) > 0$ for all $x \geq 0$. Thus, $c'(x)$ is unbounded above and for each z there exists a unique value of $x = f(z) > 0$ such that $c'(x) = z$. Moreover, because $c'(0) = 0$, then $f(0) = 0$.
2. The first result follows from the derivative of the inverse function being equal to $f'(z) = 1/c''(f(z))$ in conjunction with the strict convexity of $c(x)$. The second from $f''(z) = -c'''(f(z))/(c''(f(z))^3)$ and the assumption $c'''(x) \geq 0$.
3. Differentiating h and using $c'(f(z)) = z$ delivers $h'(z) = (n+1)f(z) + nzf'(z)$, which is positive by claims 1 and 2. □

Lemma 2. *The discounted expected consumer surplus is given by equation (5).*

Proof. Consider an asset that pays the consumer surplus flow at every instant of time. Starting from a consumer surplus cs_n, the value of this asset is given by

$$rA(cs_n) = cs_n + \lambda_{n,m}(A(cs'_n) - A(cs_n)) \tag{6}$$

where cs'_n is the consumer surplus after an innovation arrives. Using the condition that $cs'_n = cs_n + \delta_n$, we guess and verify that equation (5) solves equation (6), i.e., $A(cs_n) = CS_n$, proving the result. □

B Proofs

Proof of Proposition 1. Using the first order condition (see equation (4)), we find that the equilibrium values for the leader and followers are given by

$$\begin{aligned} rV_{n,m} &= \pi^l_n - (n+m)(V_{n,m} - W_{n,m})f(V_{n,m} - W_{n,m}) \\ rW_{n,m} &= \pi^f_n + (V_{n,m} - W_{n,m})f(V_{n,m} - W_{n,m}) - c(f(V_{n,m} - W_{n,m})). \end{aligned}$$

Subtracting these equations and defining $Z_{n,m} \equiv V_{n,m} - W_{n,m}$ we obtain

$$rZ_{n,m} = \Delta\pi_n - (n+m+1)Z_{n,m}f(Z_{n,m}) + c(f(Z_{n,m})). \tag{7}$$

To prove existence and uniqueness of an equilibrium with $Z_{n,m} > 0$, note that the left-hand side of equation (7) is strictly increasing in $Z_{n,m}$ and ranges from 0 to ∞.

Lemma 1.1 implies that the right-hand side of equation (7) is strictly decreasing in $Z_{n,m}$, taking the value of $\Delta\pi_n + c(0) > 0$ when $Z_{n,m} = 0$. Thus, the two functions intersect once at a positive value of $Z_{n,m}$, proving the result. ■

Proof of Proposition 2. Using implicit differentiation in equation (7), we reach the following results:

i) The derivative of $Z_{n,m}$ with respect to $\Delta\pi_n$ is given by

$$\frac{dZ_{n,m}}{d\Delta\pi_n} = \frac{1}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})} > 0.$$

Since $x^*_{n,m} = f(Z_{n,m})$ and $\lambda_{n,m} = (n+m)f(Z_{n,m})$, Lemma 1.2 implies that both are increasing in $\Delta\pi_n$.

ii) The derivative of $Z_{n,m}$ with respect to m is given by

$$\frac{dZ_{n,m}}{dm} = \frac{-Z_{n,m}f(Z_{n,m})}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})} < 0.$$

Thus, an increase in m decreases a firm's R&D investment. The derivative of the pace of innovation with respect to m is

$$\begin{aligned}\frac{d\lambda_{n,m}}{dm} &= f(Z_{n,m}) + (n+m)f'(Z_{n,m})\frac{dZ_{n,m}}{dm} \\ &= \frac{rf(Z_{n,m}) + (n+m+1)f(Z_{n,m})^2}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})} > 0.\end{aligned}$$

proving that the pace of innovation increases with m. ■

Proof of Proposition 3. Using implicit differentiation in equation (7) we obtain $dZ_{n,m}/dn$. By replacing it in

$$\frac{d\lambda_{n,m}}{dn} = f(Z_{n,m}) + (n+m)f'(Z_{n,m})\frac{dZ_{n,m}}{dn}, \tag{8}$$

we find

$$\frac{d\lambda_{n,m}}{dn} = \frac{rf(Z_{n,m}) + (n+m+1)f(Z_{n,m})^2 + (n+m)f'(Z_{n,m})\frac{d\Delta_n}{dn}}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})}. \tag{9}$$

If Δ_n satisfies $d\Delta_n/dn > 0$ (i.e., if Δ_n has an increasing profit gap), then the derivative is positive. Hence, a reduction in the number of large firms leads to a reduction in the pace of innovation. ■

Proof of Proposition 4. A necessary condition for equation (9) to be negative is $d\Delta_n/dn < 0$. For sufficiency, we need to show that there exists an $\bar{m}$ such that $m > \bar{m}$ implies $d\lambda_{n,m}/dn < 0$. Since the denominator of (9) is positive,

$d\lambda_{n,m}/dn < 0$ is equivalent to

$$\frac{r}{n+m}\frac{f(Z_{n,m})}{f'(Z_{n,m})} + \frac{n+m+1}{n+m}\frac{f(Z_{n,m})^2}{f'(Z_{n,m})} < -\frac{d\Delta\pi_n}{dn}.$$

$d\Delta\pi_n/dn < 0$ guarantees that right-hand side of the inequality is always positive. Given that $f(0) = 0$ and $f'(0) > 0$ (see Lemma 1), and $dZ_{n,m}/dm < 0$, it is sufficient to show that $\lim_{m\to\infty} Z_{n,m} = 0$ for the inequality to hold.

For any small $\epsilon > 0$, pick $Z_\epsilon \in (0, \epsilon)$. By Proposition 1, equation (7) has a unique solution. Using (7), define m_ϵ to be

$$m_\epsilon = \frac{\Delta\pi_n + c(f(Z_\epsilon)) - (r + (n+1)f(Z_\epsilon))Z_\epsilon}{f(Z_\epsilon)Z_\epsilon},$$

which is always well defined (but possibly negative). Thus, take any decreasing sequence of Z_ϵ converging to zero. For each element of the sequence, there exists an increasing sequence m_ϵ that delivers Z_ϵ as an equilibrium. Thus, $\lim_{m\to\infty} Z_{n,m} = 0$ and the result follows. ■

Proof of Proposition 5. i) See text.
ii) Using the definition $\lambda_{n,m} = (n+m)x^*_{n,m}$ and the assumption that $d\delta_n/dn = 0$, we re-write $\frac{dCS_n}{dn} < 0$ as:

$$\frac{dcs_n}{dn} < -\frac{\delta_n}{r}\left((n+m)\frac{dx^*_{n,m}}{dn} + x^*_{n,m}\right).$$

We show that when m is sufficiently large, a profit gap that is decreasing in the number of firms is sufficient to guarantee that the parenthesis in the expression above goes to $-\infty$, which ensures that the condition holds, as dcs_n/d_n is finite. From Proposition 2, we know that $x^*_{n,m}$ decreases with m. Now, observe

$$(n+m)\frac{dx^*_{n,m}}{dn} = \frac{\frac{d\Delta_n}{dn} - Z_{n,m}f(Z_{n,m})}{\frac{r}{n+m} + \frac{n+m+1}{n+m}f(Z_{n,m}) + Z_{n,m}f'(Z_{n,m})}.$$

From the proof of Proposition 4 we know that $\lim_{m\to\infty} Z_{n,m} = 0$. From Lemma 1, we also know that that $f(0) = 0$ and $f'(0) > 0$. Therefore, when the profit gap is decreasing in the number of firms (i.e., $d\Delta_n/dn < 0$) we have

$$\lim_{m\to\infty}(n+m)\frac{dx^*_{n,m}}{dn} = -\infty,$$

and the result follows. ■

Online Appendix: Not For Publication

C Leader Innovation

Our baseline model abstracted away from the possibility that the leader invests in R&D by assuming that old patents were not enforceable—enabling followers to imitate them—and thus keeping the leader only one step ahead of all followers. This extension shows that the profit gap remains important when market leaders can invest in R&D to increase their technological lead. In particular, a weakly increasing profit gap is still sufficient for competition to increase the pace of innovation, and a decreasing profit gap is still necessary but not sufficient for competition to lead to lower levels of R&D

Following Acemoglu and Akcigit (2012), we modify the baseline model by assuming that followers make radical innovations, making the replaced leader's product obsolete and available to unsuccessful followers; and, that market leaders invest in R&D to increase the quality of their product, which increases their profit flow. In concrete terms, we assume that the leader may be k steps ahead of the followers, receiving a profit flow of π_n^k. We assume $\pi_n^{k+1} > \pi_n^k$, so that a larger technological gap leads to a higher profit flow. As before, each follower innovates at a rate x_n^f at a flow cost of $c(x_n^f)$. Similarly, the leader can now achieve an innovation at a rate x_f^l at a flow cost $c(x_n^l)$. For this extension, we also assume $c'''(x) \geq 0$.

Although our results will apply to environments in which the leader may improve the quality of its product multiple times, for illustration purposes, we examine a situation in which the leader can increase the quality of its product only once (i.e., $k \in \{1, 2\}$). In the model, we also assume that the followers' profit flow remains constant independently of how many steps ahead the leader is. Then, the followers value function is still represented by equation (2). Let V_n^k be the value of being a leader that has innovated $k \in \{1, 2\}$ times. The leader's value equations are represented by

$$rV_n^1 = \max_{x_n^l} \pi_n^1 + x_n^l \left(V_n^2 - V_n^1\right) - c(x_n^l) + nx_n^f(W_n - V_n^1) \quad (10)$$

$$rV_2 = \pi_n^2 + nx_n^f(W_n - V_n^2), \quad (11)$$

The first equation describes the value of a being a leader that has innovated only once and that is investing in R&D to increase the quality of its product. The second equation describes the value of a leader that has already increased the quality of its innovation, enjoying a profit flow π_n^2. Note that because we assume it is infeasible for the leader to increase the product quality a second time and because developing a radical innovation replaces the current technology that the leader possess, the leader chooses not to invest in R&D when it is two steps ahead (replacement effect).

The first order condition for the followers is given by equation (4), whereas the first order condition for the leader that is one step ahead is given by

$$c_x\left(\hat{x}_n^l\right) = V_n^2 - V_n^1. \tag{12}$$

Similar to the followers in the baseline model, the leader will invest in R&D when the marginal cost of R&D equals the incremental rent of achieving an innovation, $V_n^2 - V_n^1$.

Define $\Delta_n^f = \pi_n^1 - \pi_n^f$ and $\Delta_n^l = \pi_n^2 - \pi_n^1$ to be the profit gap that exists between a one-step ahead leader and its followers, and the profit gap that exists between being a two-step ahead leader and a one-step ahead leader. Let $\lambda_n^2 = nx_n^f$ and $\lambda_n^1 = nx_n^f + x_n^l$ be the pace of innovation when the leader is two and one step ahead, respectively. We start by showing that the profit gap has a similar role to that in the baseline model.

Proposition 6 (Innovating leader)**.** *There exists a unique symmetric equilibrium, which is characterized by the solution of equations* (2)*,* (4)*,* (10)*,* (11)*, and* (12)*. An increase in the profit gap of the leader Δ_n^l increases R&D investments of the leader and followers; consequently, it increases the pace of innovation in the economy. An increase in the profit gap of the followers Δ_n^f increases the followers' R&D, but decreases the R&D of the leader. The pace of innovation, however, increases with Δ_n^f regardless of whether the leader is one or two-steps ahead.*

An increase of the profit gap of any firm that is ahead in the quality ladder increases the reward to innovate for all the firms that lag behind. This increase in reward, thus, increases the R&D incentives of every firm aiming to reach that state. For instance, an increase in the profit gap of a one-step ahead leader increases not only its R&D incentives but also the incentives of followers aiming to become a one-step ahead leader.

In contrast, an increase in the profit gap of firms that are behind in the quality ladder does not lead to higher rewards for innovation for the firm ahead. On the contrary, the increase in profit gap of laggard firms induces them to perform more R&D, increasing the competition of the firm ahead. In turn, the increased competition faced by the firm ahead, decreases its incremental rent and incentives to perform R&D. This countervailing effect is, however, of second order as the pace of innovation increases with a larger profit gap of the followers.

Proposition 7 (Innovating leader II)**.** *Profit gaps Δ_n^f and Δ_n^l that are weakly increasing in n are sufficient to guarantee that market concentration leads to a slower pace of innovation. Similarly, decreasing profits gaps are necessary but not sufficient for market concentration to lead to higher innovation pace.*

Although this formulation abstracts away from research labs, it is not hard to see that the sufficiency result presented in Proposition 4 can be extended to this framework. Research labs mimic the incentives of the followers, magnifying their response in R&D investments due to changes in market concentration. Because a profit gap Δ_n^f that is decreasing in n tends to increase the followers' R&D when the product market concentrates, competition can lead to decreased R&D outcomes when there is a sufficiently large number of research labs and there are decreasing profit gaps.

Omitted Proofs

Proof of Proposition 6. Define the incremental rent of the leader to be $H_n = V_n^2 - V_n^1$ and the incremental rent of followers $Z_n = V_n^1 - W_n$. Using the inversion defined in Lemma 1 we write $\hat{x}_n^l = f(H_n)$ and $\hat{x}_n^f = f(Z_n)$. Subtracting (10) from (11) delivers:

$$rH_n = \Delta_n^l - f(H_n) H_n + c(f(H_n)) - nf(Z_n) H_n.$$

Similarly, subtracting (10) and (2) delivers:

$$rZ_n = \Delta_n^f + f(H_n) H_n - c(f(H_n)) - (n+1) f(Z_n) Z_n + c(f(Z_n))$$

We need to show that there exists unique positive values of H_n and Z_n that simultaneously solve the equations above. Rewrite the first equation as:

$$f(Z_n) = \frac{\Delta_n^l - (f(H_n) + r) H_n + c(f(H_n))}{nH_n}$$

Using Lemma 1 we can show that this expression defines a negative, monotonic and continuous relation between Z_n and H_n. In particular, observe that if $H_n \to 0$, then $Z_n \to \infty$. Also, if $H_n \to \infty$, then $Z_n < 0$. Rewrite the expression for rZ_n as:

$$rZ_n + (n+1) f(Z_n) Z_n - c(f(Z_n)) = \Delta_n^f + f(H_n) H_n - c(f(H_n))$$

Lemma 1 implies a increasing, monotonic and continuous relation between Z_n and H_n. Observe that $H_n = 0$ implies $Z_n > 0$. Also, $H_n \to \infty$ implies $Z_n \to \infty$. Therefore, the relation described by both equations must intercept and, because both expressions are monotonic, there is a unique intersection. Thus, an equilibrium exists and is unique.

To study the relation between the profit gaps and firms investments and pace of innovation we need to understand the impact of the gaps in the incremental rent, i.e., $\frac{dH_n}{d\Delta_n^k}$ and $\frac{dZ_n}{d\Delta_n^k}$ for $k\{l, f\}$. For this we make use of the implicit function theorem. Define $g : \mathbb{R}^2 \to \mathbb{R}^2$ where

$$g_1(H_n, Z_n) = \Delta_n^l - (f(H_n) + r) H_n + c(f(H_n)) - nf(Z_n) H_n$$
$$g_2(H_n, Z_n) = \Delta_n^f + f(H_n) H_n - c(f(H_n)) - ((n+1) f(Z_n) + r) Z_n + c(f(Z_n)).$$

Then, an equilibrium is defined by $g(H_n, Z_n) = 0$ and the implicit function theorem implies (in matrix notation):

$$\left[\frac{dH_n}{d\Delta_n^f}, \frac{dH_n}{d\Delta_n^l}; \frac{dZ_n}{d\Delta_n^f}, \frac{dZ_n}{d\Delta_n^l}\right] = -\left(A^{-1}\right) B \tag{13}$$

where

$$A = \begin{bmatrix} \frac{\partial g_1}{\partial H_n} & \frac{\partial g_1}{\partial Z_n} \\ \frac{\partial g_2}{\partial H_n} & \frac{\partial g_2}{\partial Z_n} \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{\partial g_1}{\partial \Delta_n^f} & \frac{\partial g_1}{\partial \Delta_n^l} \\ \frac{\partial g_2}{\partial \Delta_n^f} & \frac{\partial g_2}{\partial \Delta_n^l} \end{bmatrix}. \tag{14}$$

Using Lemma 1, we find that

$$A = -\begin{bmatrix} r + nf(Z_n) + f(H_n) & nf'(Z_n) H_n \\ -f(H_n) & r + (n+1) f(Z_n) + nf'(Z_n) Z_n \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

The inverse of A is given by

$$A^{-1} = -\frac{1}{|A|} \begin{bmatrix} r + (n+1) f(Z_n) + nf'(Z_n) Z_n & -nf'(Z_n) H_n \\ f(H_n) & r + nf(Z_n) + f(H_n) \end{bmatrix}$$

where $|A|$ is equal to

$$(r + nf(Z_n) + f(H_n))(r + (n+1) f(Z_n) + nf'(Z_n) Z_n) + nf'(Z_n) f(H_n) H_n,$$

which is positive. Then, using equation (13), we compute the derivatives:

$$\begin{bmatrix} \frac{dH_n}{d\Delta_n^f} & \frac{dH_n}{d\Delta_n^l} \\ \frac{dZ_n}{d\Delta_n^f} & \frac{dZ_n}{d\Delta_n^l} \end{bmatrix} = \frac{1}{|A|} \begin{bmatrix} -nf'(Z_n) H_n & r + (n+1) f(Z_n) + nf'(Z_n) Z_n \\ r + nf(Z_n) + f(H_n) & f(H_n) \end{bmatrix},$$

proving the statements with respect to firms' R&D investments and that an increase of Δ_n^l leads to a higher innovation pace. To show the relation between the profit gap of the followers and λ_n^2 observe

$$\begin{aligned} \frac{d\lambda_n^2}{d\Delta_n^f} &= nf'(Z_n) \frac{dZ_n}{d\Delta_n^f} + f'(H_n) \frac{dH_n}{d\Delta_n^f} \\ &= nf'(Z_n) \frac{r + nf(Z_n) + f(H_n) - f'(H_n) H_n}{|A|}. \end{aligned}$$

By Lemma 1 the function $f(z)$ is concave and $f(0) = 0$. Together they imply $f(z) \geq f'(z)z$; thus, the derivative is positive, and the result follows. ■

Proof of Proposition 7. As in the previous proof, we make use of the implicit function theorem. Let $g(H_n, Z_n)$ be the function defined in the proof of Proposition 6. Then, the implicit function theorem implies (in matrix notation)

$$\left[\frac{dH_n}{dn}; \frac{dZ_n}{dn}\right] = -\left(A^{-1}\right) B \tag{15}$$

where A is the matrix defined in (14) and

$$B = \left[\frac{\partial g_1}{\partial n}; \frac{\partial g_2}{\partial n}\right] = \left[\frac{d\Delta_n^l}{dn} - f\left(Z_n\right) H_n; \frac{d\Delta_n^f}{dn} - f\left(Z_n\right) Z_n\right].$$

Using equation (15) we compute the derivatives

$$\frac{dH_n}{dn} = \frac{\psi_{n+1}\left(\frac{d\Delta_n^l}{dn} - f\left(Z_n\right) H_n\right) + nf'\left(Z_n\right)\left(Z_n \frac{d\Delta_n^l}{dn} - H_n \frac{d\Delta_n^f}{dn}\right)}{|A|}$$
$$\frac{dZ_n}{dn} = \frac{f\left(H_n\right)\left(\frac{d\Delta_n^l}{dn} + \frac{d\Delta_n^f}{dn} - f\left(Z_n\right)\left(Z_n + H_n\right)\right) + \psi_n\left(\frac{d\Delta_n^f}{dn} - f\left(Z_n\right) Z_n\right)}{|A|},$$

where $\psi_x = r + xf\left(Z_n\right) > 0$ for all $x > 0$. With these computations we can now prove that the pace of innovation increases in n under increasing profit gaps. Let's start studying the situation in which the leader is two steps ahead, the derivative of λ_n^2 with respect n is given by

$$\begin{aligned}\frac{d\lambda_n^2}{dn} &= f\left(Z_n\right) + nf'\left(Z_n\right)\frac{dZ_n}{d_n} \\ &= \frac{\left(\psi_n + f\left(H_n\right)\right)\left(f\left(Z_n\right)\psi_{n+1} + nf'\left(Z_n\right)\frac{d\Delta_n^f}{dn}\right) + nf'\left(Z_n\right) f\left(H_n\right)\frac{d\Delta_n^l}{dn}}{|A|}.\end{aligned}$$

which is positive whenever $\frac{d\Delta_n^l}{dn}, \frac{d\Delta_n^f}{dn} \geq 0$. Also, we can see that $\frac{d\Delta_n^l}{dn}, \frac{d\Delta_n^f}{dn} < 0$ are necessary but not sufficient for $\frac{d\lambda_n^2}{dn}$ to be negative.

When the leader performs R&D, i.e., the leader is one step ahead of the followers, the derivative of the pace of innovation is given by:

$$\begin{aligned}\frac{d\lambda_n^1}{dn} &= \frac{d\lambda_n^2}{dn} + f'\left(H_n\right)\frac{dH_n}{dn} \\ &= \frac{f\left(Z_n\right)\psi_n\psi_{n+1}}{|A|} + \frac{nf'\left(Z_n\right)\left(f\left(H_n\right) + f'\left(H_n\right) Z_n\right) + f'\left(H_n\right)\psi_{n+1}}{|A|}\frac{d\Delta_n^l}{dn} \\ &+ \frac{nf'\left(Z_n\right)\psi_n}{|A|}\frac{d\Delta_n^f}{dn} + \left(f\left(H_n\right) - f'\left(H_n\right) H_n\right)\frac{nf'\left(Z_n\right)\frac{d\Delta_n^f}{dn} + f\left(Z_n\right)\psi_{n+1}}{|A|}.\end{aligned}$$

By Lemma 1 the function $f(z)$ is concave and $f(0) = 0$; these two conditions imply $f(z) \geq f'(z)z$. Then the derivatives are positive whenever $\frac{d\Delta_n^l}{dn}, \frac{d\Delta_n^f}{dn} \geq 0$, and $\frac{d\Delta_n^l}{dn}, \frac{d\Delta_n^f}{dn} < 0$ are necessary but not sufficient for $\frac{d\lambda_n^1}{dn}$ to be negative. ■