Announcing High Prices to Deter Innovation*

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Abstract

Price announcements—similar to the ones made in media events by tech firms—are effective in deterring innovation. By announcing (and setting) a high price, a firm increases its rivals' short-run profits, reducing the rival firms' incentives to innovate by magnifying their Arrow's replacement effect. We show that the equilibrium prices are greater and the R&D investments are lower relative to when price announcements cannot be used strategically. We call this the R&D deterrence effect of price, and show that it induces equilibrium prices that may exceed the multiproduct monopoly prices and even dissipate the consumer benefits of innovation.

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1 Introduction

Steve Jobs unveiled the iPhone on January 9, 2007 at the Macworld conference, and announced the prices for the two versions of the phone: \$499 (4GB) and \$599 (8GB). A number of other recent examples suggest that price announcements at media events have become common practice in innovative industries (e.g., Microsoft unveiling its Surface Pro or Samsung launching the Galaxy S8 phone).[1] Price announcements are meaningful in that firms usually do not revise these prices until the introduction of their next generation of products (e.g., the price history of the various generations of the iPhone in Figure 1).

Price announcements have implications for firms' incentives to innovate. Consider a firm's unveiling of a new product and its price announcement decision. Announcing a higher price for the new product reduces the amount of business that the new product will steal from existing substitute products sold by rival firms. If the profit of rival firms remains high despite the new product, rivals will be less driven to innovate due to Arrow's replacement effect (Arrow, 1962). That is, firms can use the price announcements as a tool to manipulate the competitors' replacement effect. A higher price announcement has the cost of lower short-run profits, but the benefits of less R&D activity by rival firms. We call this latter effect the *R&D deterrence effect* of price announcements.

Arrow's replacement effect (or the profitability of a firm's existing products) has often been cited as a factor that kills innovation and even threaten the existence of firms in innovative industries (Christensen, 1993, 1997, Igami, 2017).[2] In this paper, we go one step further and analyze how firms can use prices to affect the replacement effect of rivals and, consequently, their incentives to innovate. Specifically, we study how price announcements impact innovation outcomes and welfare in equilibrium via their R&D deterrence effect. Are price announcements moving industries into a state of complacency, increasing the level of prices, and eroding the benefits of innovation?

We study price announcements in the context of a dynamic oligopoly model

[1] Recent examples include announcements by Apple (https://www.wsj.com/articles/apple-wwdc-event-watch-gets-upgrades-amazon-video-coming-to-apple-tv-1496684848), Microsoft (https://www.wsj.com/articles/microsofts-new-surface-pro-borrows-from-the-family-to-revive-sales-1495541700), Nintendo (https://www.wsj.com/articles/nintendo-to-launch-2ds-xl-handheld-game-device-in-july-1493388631, and Samsung (https://www.wsj.com/articles/samsung-launches-galaxy-s8-smartphone-1490799600).

[2] See, for instance, http://associationsnow.com/2014/09/innovation-complacency-dont-mix/ and http://www.nytimes.com/2010/09/27/technology/27nokia.html.

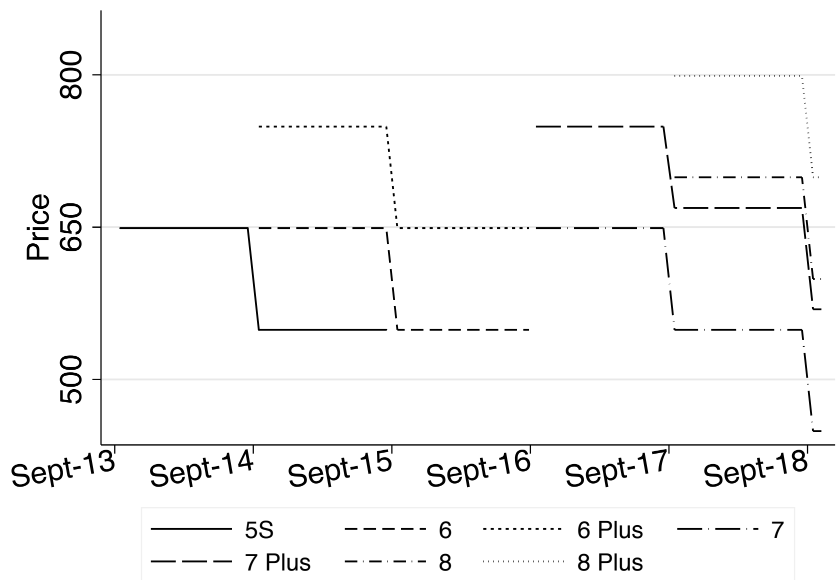

Figure 1: iPhone prices over time, by model

Source: Authors' calculations based on price announcements at Apple media events and past information posted on the Apple Store website. The prices are for the cheapest version of each model when purchasing it unlocked (i.e., carrier free).

where firms compete both in prices and in developing a new product that improves upon the existing products. Motivated by the examples of price announcements at media events, we assume that firms make public price announcements when they start selling their products and then, after observing the full profile of price announcements, they choose how much to invest in R&D. A greater R&D investment improves a firm's chance of inventing a *new* product. To isolate the role played by price announcements in deterring R&D, we compare the equilibrium outcomes under price announcements with the equilibrium outcomes when firms cannot use price announcements to strategically influence R&D investments.

We show that price announcements can be used by firms to manipulate their rivals' (Arrow's) replacement effect, thereby decreasing their incentives to innovate, and moving the industry into a state of complacency. Price announcements cause equilibrium prices to be higher and innovation rates to be lower relative to the equilibrium without announcements. The greater prices are at the expense of static profits, but increase the discounted value of the firms because of the lower innovation rates by rivals. These lower innovation rates decrease the probability of a rival firm introducing a new product, which would weaken the unsuccessful firm as a competitor.

The greater prices and the lower innovation rates in the equilibrium with price announcements cause a decrease in consumer welfare. We quantify the magnitude of the consumer welfare loss in two ways. First, the decrease in consumer welfare can lead the total surplus to decrease, despite an increase in firms' profits. Second, the higher prices caused by the R&D deterrence effect of price announcements may completely dissipate the consumers' benefits from new products. That is, we find instances where consumers would be better off if firms did not engage in R&D activity (i.e., cases with products that stay the same forever). Both results suggest that the loss in consumer welfare caused by the R&D deterrence effect of prices is of first order.

With respect to properties of the equilibrium prices with price announcements, we find that prices may surpass the multiproduct monopolist in some cases. As well, we show that the incentive to announce a high price varies non-monotonically with the degree of product differentiation: no surcharge exists when goods are perfect substitutes or completely differentiated, whereas the incentive to announce a high price peaks for intermediate levels of differentiation.

These results have several implications. First, they broaden our understanding on how prices can be used to soften competition along non-price dimensions. Second, they suggest that the R&D deterrence effect of price announcements has a first order effect on consumer welfare, implying that the measurement of the welfare gains of innovation will be biased unless the strategic role of price announcements are accounted for. Third, they provide an economic argument for why firms in innovative industries make use of price announcements (although we acknowledge that marketing and other factors may also be important in the decision of making price announcements).

Our paper contributes to several strands of the literature. First, it contributes to the literature on dynamic R&D competition. Our model resembles Loury (1979), Lee and Wilde (1980), and Reinganum (1982) in that innovation is uncertain and the arrival of the innovation follows a Poisson process with parameters that depend on the intensity of the firms' R&D investments. In contrast to these papers, we follow Marshall and Parra (2018) and explicitly model the product market game. Firms invest in R&D to gain a product market advantage, and choose their R&D investments according to the incremental value of the innovation (Arrow, 1962). The new mechanism in our model is that a firm's price announcement directly impacts the incremental value of innovation of rival firms, and can therefore be used to manipulate its rivals' incentives to conduct R&D.

Second, we contribute to the literature on strategic deterrence. Research has shown that firms may benefit from manipulating prices to signal cost efficiency or low market profitability (Milgrom and Roberts, 1982, Harrington, 1986), or to establish a reputation of being a "tough" competitor (Goolsbee and Syverson, 2008, Kreps and Wilson, 1982).[3] In all of these cases, firms sacrifice short-run profits to deter entry and increase the value of the firm in the long run. While most of the literature has focused on how predatory prices may affect the entry of new competitors, the analysis of how pricing may be used to soften competition along other dimensions (e.g., innovation or capital investment) has been less studied.

Two articles are closely related to ours. Gallini (1984) shows that incumbents may use licensing agreements to share profits with potential entrants to decrease their incentives to innovate. Relatedly, we show that announcing (and setting) high prices are an effective way to share rents with rivals, reducing the intensity of R&D competition. Besanko *et al.* (2014) study dynamic pricing decisions when firms learn by doing. Under learning, the dynamic pressure of future competition induces firms to decrease prices to expand quantity and therefore speed up the learning process. In contrast to their results, we find that the ability to use price announcements interacted with future competition generates an upward pressure on prices, where the greater prices are intended to decrease R&D activity by magnifying the Arrow's replacement effect of its rivals.

The rest of the paper is organized as follows. Section 2 introduces the model, and the equilibria with and without price announcements are discussed in Section 3 and Section 4, respectively. Additional properties of the R&D deterrence effect are discussed in Section 5, and Section 6 discusses model extensions. Lastly, Section 7 discusses managerial implications and concludes.

2 Model Setup

Consider a continuous-time infinitely lived oligopoly, where firms sell differentiated goods and compete in prices. At every instant of time, and for a given vector

[3] Economists have also analyzed how non-price strategic choices including advertising, licensing, R&D, capacity investments, patenting, and production-sharing agreements can be used to deter entry (e.g., Dixit 1980, Ellison and Ellison 2011, Gilbert and Newbery 1982, Spence 1977, Chen and Ross 2000).

of market prices $\mathbf{p}$, firm i earns a profit flow $\pi_i(\mathbf{p}) = (p_i - c_i)q_i(\mathbf{p})$ where q_i is the demand for firm i's product, and c_i is firm i's marginal cost of production. We assume $\partial q_i/\partial p_i < 0$ and $\partial q_i/\partial p_j > 0$ as well as some additional regularity conditions that guarantee a unique equilibrium in the static price game and a unique solution to the problem of a monopolist controlling the prices of all the goods (see Appendix C for details). For ease of exposition, we present our analysis for the case of a symmetric duopoly (i.e., $c_1 = c_2 = c$), and we later argue in Section 6 that our results generalize to the case of n firms and cost asymmetries.

Aside from competing in prices, firms compete in developing an innovation. We consider the case of a cost-saving innovation in our baseline model, and discuss quality-enhancing innovations in Section 6. The firm that successfully innovates, which we call the *leader*, obtains a patented innovation that decreases its marginal cost to βc with $\beta \in (0, 1)$. Firms invest in R&D by choosing a Poisson innovation rate x_i at a flow cost of $\kappa(x_i) = x_i^\gamma/\gamma$ with $\gamma \geq 2$. The flow cost $\kappa(x_i)$ is strictly increasing, strictly convex (i.e., $\kappa''(x) > 0$ for all $x \geq 0$), and satisfies $\kappa(0) = \kappa'(0) = 0$ and $\kappa'''(x) \geq 0$ for all $x \geq 0$.[4] The Poisson processes are independent among firms, generating a memoryless stochastic process.

We assume that after one firm successfully innovates, the industry reaches maturity.[5] Once the industry reaches maturity, firms no longer invest in R&D, and they play a static asymmetric price competition game at every instant of time. Define π^l and π^f to be the equilibrium profit flows earned by the leader and *follower* (i.e., the unsuccessful firm), respectively, after the new technology is invented. Define the equilibrium prices of the leader and follower to be p^l and p^f, respectively.

As reference points, let π^s be the profit flow in the unique symmetric equilibrium of the static price game, and let p^s be the equilibrium price of each firm. Similarly, let π^m be the per-product profit flow earned by a multiproduct monopolist controlling the prices of both goods, and let p^m be the monopoly price for each good.

Lemma 1 (Profit and Prices)**.** *In equilibrium, $\pi^f < \pi^s < \pi^l$ and $p^l < p^f < p^s$.*

As expected, the innovation gives the leader a competitive edge, increasing

[4] The functional form $\kappa(x) = x^\gamma/\gamma$ is only critical for the characterization of a sufficient condition that guarantees that the second order conditions of the firms' pricing problems hold. Equilibria can be constructed for alternative cost functions, $\kappa(x)$, satisfying the aforementioned assumptions.

[5] We discuss the case of a sequence of innovations in Section 6.

its profit flow and decreasing that of the follower. After a firm has successfully innovated, the values of the innovator (or leader) and follower are given by $L = \pi^l/r$ and $F = \pi^f/r$, respectively, where r is the discount rate.

Definition 1 (Post-innovation Values)**.** *When an innovation arrives, the market reaches maturity and the values of the technology leader and follower are given by* $L = \pi^l/r$ *and* $F = \pi^f/r$*, respectively.*

Depending on the initial level of the marginal costs c and the magnitude of the process innovation β, the profit flow of the leader may be higher or lower than the per-product monopoly profit flow, π^m. Because the distinction will be important in the analysis that follows, we call an innovation *incremental* when $\pi^l < \pi^m$, and *radical* when $\pi^l \geq \pi^m$.

Definition 2 (Incremental and Radical Innovations)**.** *An innovation is incremental when* $\pi^l < \pi^m$ *whereas it is radical when* $\pi^l \geq \pi^m$*.*

In what follows, we analyze the model under two different assumptions regarding the timing of play. First, in Section 3, we follow the dynamic oligopoly literature and study equilibrium outcomes when firms simultaneously decide on both prices and R&D investments (see, for instance, Ericson and Pakes 1995). In this case, prices cannot be used to strategically influence R&D investment decisions. The model with simultaneous decisions will serve as a benchmark and help us to isolate the strategic role played by price announcements.

In Section 4, we proceed to study the case where firms make simultaneous public price announcements at the beginning of the game, and credibly commit to those prices. Firms choose how much to invest in R&D in every period, and condition their choices on the full profile of price announcements.

We will focus on studying the Markov perfect equilibria of the game. At each instant of time, firms' strategies will be a function of the only state variable of the game—i.e., whether or not an innovation has arrived. When we study the model with price announcements, the firms' R&D investment strategies will also be a function of the announced prices.

3 No Price Announcements

We first analyze the game where firms choose both prices and R&D investments simultaneously at every instant of time.[6] Because of the timing of play, firms cannot use prices to affect their rivals' investment choices, making it a natural comparison for the model with price announcements.

Let r represent the discount rate. After a firm has successfully innovated, the values of the leader (or innovator) and follower are given by $L = \pi^l/r$ and $F = \pi^f/r$, respectively. Let V_i represent the value of firm i at time t before any firm has successfully innovated,

$$V_i = \max_{p_i, x_i} \int_t^{\infty} e^{-(r+x_i+x_j)(s-t)} \left(\pi_i(p_i, p_j) + x_i L + x_j F - \kappa(x_i)\right) ds. \tag{1}$$

To understand this value function, fix any instant of time $s > t$. With probability $\exp(-(x_i + x_j)(s-t))$, no innovation has arrived between t and s. At that instant, firm i receives the flow payoff $\pi_i(p_i, p_j)$; innovates at rate x_i; earns an expected payoff of successful innovation of $x_i L$; pays the flow cost of its R&D, $\kappa(x_i)$; faces innovation by its rival at rate x_j; and earns the expected payoff of losing the innovation race of $x_j F$. All of these payoffs are discounted by $\exp(-r(s-t))$. Due to the stationarity of the payoffs and the nature of Markov strategies, V_i can be rewritten as

$$V_i = \max_{p_i, x_i} \frac{\pi_i(p_i, p_j) + x_i L + x_j F - \kappa(x_i)}{r + x_i + x_j},$$

which does not depend on the time t.

Given the rival's strategy (x_j, p_j), the best response functions of firm i are implicitly defined by the first order conditions

$$\kappa'(x_i) = L - V_i, \qquad \frac{\partial \pi_i(p_i, p_j)}{\partial p_i} = 0. \tag{2}$$

The first condition in (2) shows that firms choose how much to invest in R&D by equating the incremental value of the innovation with the marginal cost of increasing the innovation rate, x_i. The incremental value of the innovation, $L - V_i$, depends on firm i's value under its *current* technology, V_i. The greater V_i, the lower firm i's incentives to invest in R&D (i.e., Arrow's replacement effect). The second condition in (2) shows that firms choose prices by maximizing their static

[6] A more general version of this model is discussed in Marshall and Parra (2018).

product market profits. Let V^{na}, p^{na}, and x^{na} represent the equilibrium values, prices, and investments when there are no announcements. Equilibrium existence and uniqueness is established in the following proposition.

Proposition 1 (Equilibrium with No Announcements)**.** *There is a unique symmetric Markov perfect equilibrium, (p^{na}, x^{na}, V^{na}), that solves equation* (1) *and conditions* (2)*. In equilibrium, $p^{na} = p^s$, $x^{na} > 0$, and $V^{na} \in (F, L)$.*

4 Price Announcements

We next consider the case with public price announcements. The timing of the game is as follows. At the beginning of the race, firms make simultaneous public price announcements, and they (credibly) commit to these prices until the next innovation arrives. Upon observing the announced prices, firms then choose how much to invest in R&D in every period.[7] As discussed in Section 4.4, our results hold even if firms frequently revise their price announcements, with arbitrary times between each announcement. That is, committing to a price until the next innovation arrives is not central to our results.[8]

We solve the game by backward induction. We first analyze the equilibrium investments and firm values for a given pair of prices $\mathbf{p} = (p_i, p_j)$. We then use the firm values as a function of prices to analyze the equilibrium of the pricing game. Multiple pricing equilibria may exist in this scenario. In every equilibria, market prices are higher and R&D investments are lower than those when pricing and investment decisions are determined simultaneously (i.e., the case where price choices cannot affect the rivals' R&D decisions). We show that a unique *low deterrence* equilibrium—an equilibrium with prices greater than p^s but less than the monopoly price p^m—exists under mild assumptions. Also, a *high deterrence* equilibrium—where prices exceed the monopoly price—may exist when innovations are incremental (see Definition 2).

[7] To simplify exposition, we assume that both firms invest in R&D. We emphasize, however, that a firm's incentive to announce a high price and deter a rival from introducing a new product exists even if the firm itself does not engage in R&D activities.

[8] In our baseline model, prices are not revised prior to the discovery of the next innovation because this period presents no changes to state variables that would incentivize a firm to revise its price. In Section 6, we show that our results extend to the case where we allow for demand shocks that lead to price announcement revisions.

4.1 R&D Investments

Let $V_i(\mathbf{p})$ represent the value of firm i at time t before any firm has successfully innovated as a function of price announcements $\mathbf{p} = (p_i, p_j)$, then

$$V_i(\mathbf{p}) = \max_{x_i} \int_t^\infty e^{-(r+x_i+x_j)(s-t)} \left(\pi_i(\mathbf{p}) + x_i L + x_j F - \kappa(x_i)\right) ds.$$

$V_i(\mathbf{p})$ has an interpretation that is similar to that of equation (1), with the difference that prices are now fixed at $\mathbf{p}$. As before, firms invest in R&D to earn the value of becoming the market leader, L (see Definition 1).

Using the principle of optimality, and conditioning on the opponents' strategy x_j, we can rewrite the problem above as

$$rV_i(\mathbf{p}) = \max_{x_i} \{\pi_i(\mathbf{p}) + x_i(L - V_i(\mathbf{p})) + x_j(F - V_i(\mathbf{p})) - \kappa(x_i)\}. \tag{3}$$

Given the price vector $\mathbf{p}$, each firm chooses how much to invest in R&D by solving the maximization problem in equation (3). Because equation (3) is independent of t, we can see that the stationarity of model makes the timing of the price announcement irrelevant; i.e., firms choose the same R&D investment regardless of when prices are announced.

Differentiating the right hand side of equation (3) gives us an implicit expression for firm i's R&D investment,

$$\kappa'(x_i) = L - V_i(\mathbf{p}). \tag{4}$$

As before, the investment rule equates the marginal cost of increasing the innovation rate, x_i, with the incremental value of an innovation, $L - V_i(\mathbf{p})$. Due to the convexity of κ, equation (4) has a unique solution and can be inverted and written as $x_i = f(L - V_i(\mathbf{p}))$, where f is a strictly increasing and concave function (see Lemma 6 in the Appendix).

Replacing the equilibrium investment decisions back into equation (3), and restricting attention to symmetric values among firms ($V_i = V_j = V$), we define the continuous function

$$\phi(V, \pi) \equiv \pi + f(L - V)(L + F - 2V) - \kappa(f(L - V)) - rV. \tag{5}$$

The function $\phi(V, \pi)$ will help us characterize how prices relate to the equilibrium

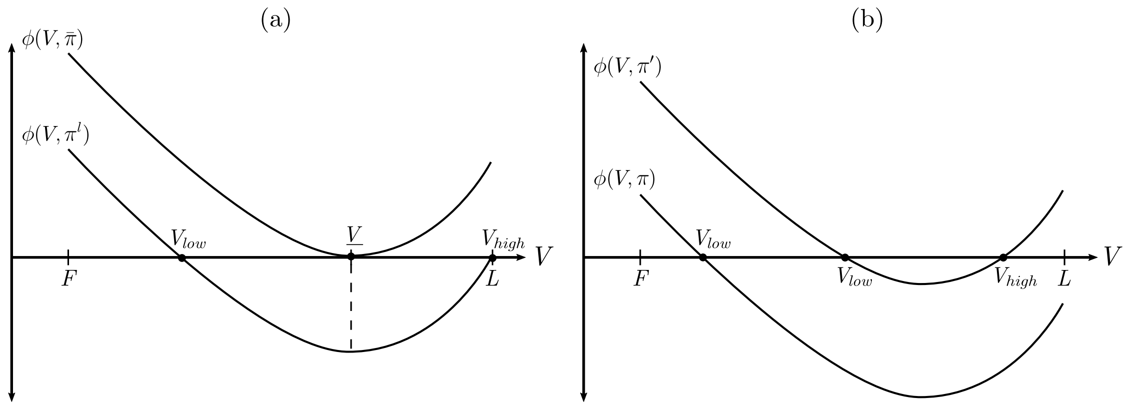

Figure 2: Equilibrium Multiplicity

value of V. Start by observing that, given a vector of (symmetric) prices $\mathbf{p}$, both firms earn the profit flow $\pi = \pi_i(\mathbf{p})$. For each π we can search for a candidate solution to the value function by looking at values of V satisfying $\phi(V,\pi) = 0$. Because deterrence is costly and firms cannot be deterred beyond the point where they choose not to invest in R&D (i.e., $x_j \geq 0$), no equilibrium with $V \geq L$ exists (we come back to this below). Also, it can be easily shown that values of $V < F$ are never candidate solutions for economically relevant values of π. For these reasons we restrict the analysis of $\phi(V,\pi)$ to values of $V \in [F, L]$ (see Definition 1 for the definition of L and F).

Lemma 2 (Shape of ϕ)**.** *(i) $\phi(F,\pi) > 0$ for any $\pi \geq \pi^f$. (ii) $\phi(L,\pi^l) = 0$ and $\phi(L,\pi) < 0$ for any $\pi < \pi^l$. (iii) For any given value of π, $\phi'(F,\pi) < 0$, $\phi'(L,\pi) > 0$, and $\phi''(V,\pi) > 0$ for all $V \in (F, L)$.*[9] *(iv) There exists a unique value $\underline{V} \in (F, L)$, independent of π, where $\phi(V,\pi)$ is minimized.*

The previous lemma characterizes ϕ as a function of V. It tells us that for any given value of π, $\phi(V,\pi)$ is U-shaped with $\phi'(V,\pi)$ being monotonically increasing (see Figure 2). Due to the linearity of ϕ in π, a change in π only results in a vertical shift of ϕ. These two facts imply that ϕ is uniquely minimized at $\underline{V}$, which is independent of π. Define $\bar{\pi}$ to be the unique value of π satisfying $\phi(\underline{V},\bar{\pi}) = 0$.

Lemma 3 (Equilibrium Candidates)**.** *For $\pi \in [\pi^f, \pi^l) \cup \{\bar{\pi}\}$ the equation $\phi(V,\pi) = 0$ has a unique solution. For $\pi \in [\pi^l, \bar{\pi})$, $\phi(V,\pi) = 0$ has two solutions. For $\pi > \bar{\pi}$, $\phi(V,\pi) = 0$ has no solution.*

[9]The prime denotes derivatives with respect to the V dimension.

The intuition of Lemma 3 can be seen graphically in Figure 2. Start by analyzing $\phi(V, \pi^l)$. Since $\phi(L, \pi^l) = 0$ and $\phi'(L, \pi^l) > 0$ we know that ϕ approaches zero at L from below. Also, since $\phi(F, \pi^l) > 0$ we know that $\phi(V, \pi^l)$ must cross zero at a value of $V \in (F, L)$. This value is represented by V_{low} in Figure 2 (Panel A). Moreover, since a lower π implies a downward shift of ϕ, and because $\phi(F, \pi) > 0$ for any $\pi \in [\pi^f, \pi^l)$, a solution to the left of $\underline{V}$ always exists for this profit range but no solution to the right of $\underline{V}$ exists (see Figure 2 (Panel B)). Similarly, when $\pi \in [\pi^l, \bar{\pi})$ the solution V_{low} still exists, but now a new solution to the right of $\underline{V}$, denoted by V_{high}, also exists (see $\phi(V, \pi')$ in Figure 2 (Panel B)). When $\pi = \bar{\pi}$, $\underline{V}$ is the only solution. Finally, when $\pi > \bar{\pi}$, no solution exists.

Since ϕ may never cross the horizontal axis for a sufficiently high profit level, we henceforth assume that $\pi^m \leq \bar{\pi}$ in order to guarantee the existence of $V_i(\mathbf{p})$. To see why this is sufficient, observe that π^m is the maximum profit flow that can be attained in a symmetric equilibrium. Therefore $\pi^m \leq \bar{\pi}$ guarantees that $V_i(\mathbf{p})$ is well defined for any relevant vector of symmetric prices.[10]

The two solutions of ϕ have different qualitative features. For instance, while V_{low} is increasing in π, the solution V_{high} is decreasing. By implicitly differentiating $\phi(V, \pi) = 0$ with respect to V_j, we can define the ratio

$$R(\mathbf{p}) \equiv \frac{dV_i}{dV_j} = \frac{f'(L - V(\mathbf{p}))(V(\mathbf{p}) - F)}{r + 2f(L - V(\mathbf{p})))} > 0. \tag{6}$$

$R(\mathbf{p})$ captures how a solution $V_i(\mathbf{p})$ changes with an increase in the opponents' value. As we can observe, firms' values are complements, since an increase in firm j's value increases firm i's value by $R(\mathbf{p})$. $R(\mathbf{p})$ will play an important role when we characterize the equilibrium of the pricing game, as it measures how increasing firm j's value benefits firm i (i.e., the payoff pass-through of increasing the rival's value).

Lemma 4 (Properties of High and Low Equilibria)**.** *The equilibrium value in every solution V_{low} is lower than the value in any solution V_{high}. In any solution V_{low}, $R(\mathbf{p}) \in (0, 1)$. In any solution V_{high}, $R(\mathbf{p}) > 1$. $V_{low}(\pi)$ is increasing in π, while $V_{high}(\pi)$ is decreasing.*

Because the equilibrium value in every solution V_{low} is lower than the value

[10]Note, however, that $\pi^m \leq \bar{\pi}$ is a sufficient condition. It is possible to find model parameters for which the unique equilibrium price vector under price announcements $\mathbf{p}^*$ satisfies $\pi(\mathbf{p}^*) < \bar{\pi}$—so that $V(\mathbf{p}^*)$ is well defined—but $\bar{\pi} < \pi^m$.

in any solution V_{high}, and because larger V's are associated with lower R&D investments levels (see equation (4)), we refer to a V_{low} solution as part of a *low deterrence* equilibrium. Similarly, we refer to V_{high} solutions as part of a *high deterrence* equilibrium. Recall from Lemma 3 that the solution V_{high} only exists when $\pi > \pi^l$. Since in equilibrium $\pi(\mathbf{p}) \leq \pi^m$—i.e., only profit flows less than the monopoly profits are attainable—we know that V_{high} solutions are not feasible when the innovation is radical ($\pi^m \leq \pi^l$). That is, a high deterrence equilibrium may only exist for incremental innovations ($\pi^m > \pi^l$).

Lemma 5 (Feasibility of Equilibria). *Under radical innovations (i.e., $\pi^l \geq \pi^m$) only the V_{low} solution is a candidate for equilibrium. Under incremental innovations (i.e., $\pi^m > \pi^l$), both solutions are candidates.*

4.2 Market Prices

After characterizing the possible value function solutions for a given price vector $\mathbf{p}$, we now proceed to look for equilibrium prices. Because the V_{low} solution is defined for all profit levels, we start by showing the existence of a low deterrence equilibrium. We leave the discussion of high deterrence equilibria to the end of this section.

Given beliefs about p_j, each firm i chooses its price announcement by solving $\max_{p_i} V_i(\mathbf{p})$.[11] Then, firm i's first order condition is equivalent to

$$\frac{dV_i(\mathbf{p})}{dp_i} = 0 \Leftrightarrow \frac{d\pi_i(\mathbf{p})}{dp_i} + \underbrace{\frac{dx_j}{dp_i}(F - V_i(\mathbf{p}))}_{\text{R\&D Deterrence Effect}>0} = \frac{d\pi_i(\mathbf{p})}{dp_i} + R(\mathbf{p})\frac{d\pi_j(\mathbf{p})}{dp_i} = 0 \quad (7)$$

(see equation (12) in the Appendix for the full expression of $dV_i(\mathbf{p})/dp_i$).

Condition (7) tells us that when a firm chooses its price it considers the impact of its price on its own profit, π_i, as well as on the expected loss that results when its rival successfully innovates, $x_j(F - V_i(\mathbf{p}))$. Recall that with the arrival of an innovation, the unsuccessful firm becomes less profitable, $F < V_i(\mathbf{p})$, which incentivizes each firm to reduce the likelihood of a successful innovation by its rival. Firms achieve this by using price announcements to deter their rivals' R&D investments, which we call this the *R&D deterrence effect of price*. Price announcements

[11]Since equation (3) does not depend on t, and as long as no firm has successfully innovated, the firms would make the same price announcement regardless of the timing of the announcement.

are effective deterrents because firms invest in R&D according to the incremental value of the innovation (i.e., $\kappa'(x_i) = L - V_i(\mathbf{p})$), and prices directly impact the incremental value of the innovation through the product market profit (see Lemma 4). In other words, price announcements are a tool to reduce a rival's incentives to replace its current technology (Arrow, 1962).

We show that the R&D deterrence effect is positive, which results in an upward pressure on prices solely driven by deterrence motives. The sign can be more easily seen when using implicit differentiation to rewrite the effect as $R(\mathbf{p})d\pi_j/dp_i$. The term $R(\mathbf{p}) = dV_i/dV_j > 0$ represents how an increase in V_j impacts V_i (see Lemma 4). $d\pi_j/dp_i > 0$ captures the price complementarities between the substitute goods; i.e., how much firm j benefits from an increase in p_i. The economics behind the positive R&D deterrence effect are that a higher price increases the rival's profit flow, which in turn increases the rival's pre-innovation values (V), and thus reduces the rival's incremental value of the innovation ($L - V$). That is, firms face incentives to set higher prices to increase the Arrow's replacement effect of their rival, making the rival complacent and less driven to innovate.[12]

To go further, evaluate the derivative of $V_i(\mathbf{p})$ with respect to p_i at the equilibrium price without price announcements, p^s. Since $\partial\pi_i(\mathbf{p}^s)/\partial p_i = 0$ in the equilibrium without price announcements and the R&D deterrence effect is positive, equation (7) is not satisfied at p^s, and $dV_i(\mathbf{p}^s)/dp_i > 0$.[13] Similarly, evaluating $dV_i(\mathbf{p})/dp_i$ at the monopoly price p^m we obtain

$$\frac{d\pi_i(\mathbf{p}^m)}{dp_i} + R(\mathbf{p}^m)\frac{d\pi_j(\mathbf{p}^m)}{dp_i} < \frac{d\pi_i(\mathbf{p}^m)}{dp_i} + \frac{d\pi_j(\mathbf{p}^m)}{dp_i} = 0,$$

where the inequality follows from $d\pi_j(\mathbf{p})/dp_i > 0$ and $R(\mathbf{p}) \in (0, 1)$ in any low deterrence equilibrium (see Lemma 4). In words, the first derivative of V_i with respect to p_i is negative at the monopoly price. Therefore, by the intermediate value theorem, there exists a price $p^a_{low} \in (p^s, p^m)$ such that equation (7) holds.

Existence and uniqueness of a low deterrence equilibrium is discussed in the following proposition. In the proposition, we provide a sufficient (although not

[12] It is important to highlight that the R&D deterrence effect is solely driven by the firms' incentives to influence the investments of their rivals and it is not driven by collusive behavior. For results on firms' investments under collusive behavior see Fershtman and Pakes (2000).

[13] For notational ease, we use $\mathbf{p}^s$ to denote the vector (p^s, p^s). Similarly for $\mathbf{p}^m$ below.

necessary) condition for equilibrium uniqueness based on the function,

$$\Psi(p) = \frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i^2} + \max\left\{0, \frac{\partial^2 \pi_j(\mathbf{p})}{\partial p_i^2}\right\} - \Lambda \frac{\partial \pi_i(\mathbf{p})}{\partial p_i}\frac{\partial \pi_j(\mathbf{p})}{\partial p_i}, \tag{8}$$

where $\mathbf{p} = (p, p)$,

$$\Lambda = \frac{1}{K^2}\left(\frac{f'(L-\underline{V})^2 - f''(L-\underline{V})K}{f'(L-\underline{V})}\right) > 0,$$

$K = r + 2f(L - \underline{V}) > 0$, $f(z) = z^{\frac{1}{\gamma-1}}$, γ is the coefficient on the cost function $\kappa(x)$, and $\underline{V}$ is defined in Lemma 2. Assuming that $\Psi(p) < 0$ for all $p \in (p^s, p^m)$ is sufficient to guarantee that there exists a unique low deterrence equilibrium. Below we provide examples with linear and logit demand functions where this condition is satisfied (see Table 1).

Proposition 2 (Low Deterrence Equilibrium)**.** *Assume $\Psi(p) < 0$ for all $p \in (p^s, p^m)$. There exists a unique* low deterrence *symmetric Markov perfect equilibrium, $(p^a_{low}, x^a_{low}, V^a_{low})$. In this equilibrium, firms deter their rivals' R&D ($x^a_{low} < x^{na}$) by announcing higher prices ($p^a_{low} \in (p^{na}, p^m)$) and they earn greater profits ($V^a_{low} > V^{na}$) relative to the case without price announcements.*

As in the rest of the literature on predation, firms are willing to sacrifice current market profits in order to deter their rivals' investments. We can see this in the first order condition in equation (7). Because the R&D deterrence effect is positive, we have that $\partial \pi_i(\mathbf{p}^a_{low})/\partial p_i < 0$. That is, given the rival's price, firm i would be better off by decreasing its price if it wanted to maximize its short-run profit. However, because of the dynamic benefits of deterring their rivals' R&D, firms are willing to make this sacrifice.

With respect to the magnitude of V^a_{low}, equation (7) implies that $V^a_{low} < L$ must hold in any equilibrium. To see this, recall that firms choose their R&D investments according to the incremental value of the innovation, $L - V$ (see equation (4)). If $V \geq L$, firms choose not to invest in R&D, as the innovation destroys (rather than creates) value. For this reason, at any vector of prices $\mathbf{p}$ such that $V \geq L$ we have that $dx_j/dp_i = 0$ (i.e., firm j's investment in R&D cannot be less than 0). When $dx_j/dp_i = 0$, equation (7) can only be satisfied at $\mathbf{p}^s = (p^s, p^s)$. However, at $\mathbf{p}^s$ we know that $V^{na} \equiv V(\mathbf{p}^s) < L$ (see Proposition 1), which contradicts the premise that $V \geq L$, and proves that $V \geq L$ cannot be part of an equilibrium with price announcements.

We next focus on the high deterrence equilibrium. Following similar arguments as above and because in a high deterrence solution we have that $R(\mathbf{p}) > 1$ (see Lemma 4), the derivative of $V_i(\mathbf{p})$ with respect p_i (see equation (7)) is non-zero for any price between p^s and p^m. Therefore, if an equilibrium exists, we must have that $p^a_{high} > p^m$, where p^a_{high} is the market price in a high deterrence equilibrium. Observe, however, that an equilibrium may not exist because the high deterrence solution $V_{high}(\mathbf{p})$ is defined for symmetric price vectors such that the corresponding profit flow is in the interval $[\pi^l, \pi^m]$ (see Lemma 3). Depending on the parameters of the model, the vector $\mathbf{p}$ that solves equation (7) may or may not satisfy this restriction. Properties of a high deterrence equilibrium are discussed in the following proposition.

Proposition 3 (High Deterrence Equilibrium)**.** *When innovations are incremental (i.e., $\pi^l < \pi^m$), there may exist a* high deterrence *equilibrium, $(p^a_{high}, x^a_{high}, V^a_{high})$. In this equilibrium, firms announce prices that are higher than the monopoly price ($p^a_{high} > p^m$), deter their rivals' R&D ($x^a_{high} < x^a_{low}$) by more and earn greater profits ($V^a_{high} > V^a_{low}$) relative to the low deterrence equilibrium.*

From the discussion above, we note that there is an interesting link between the problem of a firm choosing its price announcement and the problem of a multiproduct monopolist. Equation (7) shows that firms choosing their price announcements (at least partially) internalize the effect of their price on the profits of their rivals, although for reasons that differ from those of a multiproduct monopolist. In the case of a multiproduct monopolist, the firm fully internalizes the price externality because the monopolist wishes to maximize the joint profit. In the case of a firm choosing its price announcement, the firm does not capture the joint profit but it still internalizes the price externality because of the benefits of deterring its rival's R&D activity. With respect to the magnitude of the equilibrium price relative to the multiproduct monopoly price, we know that $R(\mathbf{p}) \in (0, 1)$ and $R(\mathbf{p}) > 1$ in any low and high deterrence equilibrium, respectively (see Lemma 4). That is, in all low (high) deterrence equilibria the firm puts more (less) weight on its own profit than on the profit of the rival. This leads to equilibrium prices that are less (greater) than the monopoly prices in all low (high) deterrence equilibria.

To illustrate our results and the existence of multiple equilibria with different levels of R&D deterrence, we present a series of examples in Table 1. In Panels A and B of Table 1, we present examples with linear and logit demand functions, respectively. In both panels we assume that the R&D cost function is $\kappa(x) = x^2/2$,

	Panel A: Linear Demand			Panel B: Logit Demand		
	I	II	III	I	II	III
Demand	$q_i = \dfrac{2-4p_i+2p_j}{3}$			$q_i = \dfrac{\exp\{-p_i\}}{1+\exp\{-p_i\}+\exp\{-p_j\}}$		
β	0.9	0.75	0.4	0.88	0.8	0.5
Other parameters	c=0.2, $r=0.05$, $\kappa(x)=x^2/2$			c=0.13, $r=0.03$, $\kappa(x)=x^2/2$		
Innovation type	Increm.	Increm.	Rad.	Increm.	Increm.	Rad.
Existence/Uniqueness						
Existence high eq.	Yes	No	-	Yes	No	-
Uniqueness low eq.	Yes	Yes	Yes	Yes	Yes	Yes
Prices						
p^s	0.4667	0.4667	0.4667	1.3379	1.3379	1.3379
p^m	0.6	0.6	0.6	1.5532	1.5532	1.5532
p^a_{low}	0.5015	0.4894	0.4897	1.4119	1.3896	1.3883
p^a_{high}	0.6641	-	-	1.5961	-	-
R&D						
$pace^{na}$	0.2391	0.5973	1.4803	0.1481	0.2430	0.6022
$pace^a_{low}$	0.1836	0.5824	1.4739	0.1123	0.2276	0.5957
$pace^a_{high}$	0.0613	-	-	0.0403	-	-
Consumer Surplus						
CS^{na}	3.8720	4.0184	4.3858	14.1265	14.1772	14.3726
CS^a_{low}	3.7654	3.9930	4.3752	13.9484	14.1084	14.3451
CS^a_{high}	2.8178	-	-	12.9471	-	-
Total Surplus						
TS^{na}	7.6957	7.9064	8.4582	28.0147	28.0906	28.3855
TS^a_{low}	7.6446	7.8959	8.4540	27.8724	28.0374	28.3646
TS^a_{high}	6.8192	-	-	26.9430	-	-

Table 1: R&D Deterrence Effect: Numerical Examples

Note: An innovation is incremental (Increm.) or radical (Rad.) when $\pi^m > \pi^l$ and $\pi^m \leq \pi^l$, respectively. Existence high eq. indicates whether a high deterrence equilibrium exists. Uniqueness low eq. indicates whether the condition for low deterrence equilibrium uniqueness in Proposition 2 is satisfied. $p^s, pace^{na}$ are the equilibrium outcomes under no price announcements, where *pace* is defined as $2x$. $p^a_{low}, pace^a_{low}$ and $p^a_{high}, pace^a_{high}$ are the equilibrium outcomes with price announcements in a low and high deterrence equilibrium, respectively. Consumer surplus (CS) is defined in equation (9), and total surplus is given by $CS + 2V$, where V is the value of a firm at the beginning of the game.

and we keep the demand function, the marginal cost (c), and the discount rate (r) fixed throughout the examples in each panel. The magnitude of the innovation (β) is the only parameter that varies across examples. Each panel has the same taxonomy. Columns I and II present examples with incremental innovations ($\pi^m > \pi^l$). From Lemma 5 we know that a high deterrence equilibrium may exist whenever the innovation is incremental, however, we find that a high deterrence equilibrium only exists in Column I of each panel (i.e., a high deterrence equilibrium is not guaranteed to exist when the innovation is incremental). Column III presents an example with a radical innovation ($\pi^m \leq \pi^l$), where we know from Lemma 5 that only a low deterrence equilibrium may exist. In all of the examples we have that $\Psi(p) < 0$ for all $p \in (p^s, p^m)$ (see Proposition 2), implying a unique low deterrence equilibrium.

4.3 Consumer Welfare

Propositions 2 and 3 imply that consumers face both higher prices in the pre-innovation period and a lower pace of innovation when firms make price announcements (i.e., $p^a_{low}, p^a_{high} > p^s$ and $x^a_{low}, x^a_{high} < x^{na}$). A lower pace of innovation implies that the innovation—and its negative impact on prices, $p^l < p^f < p^s < p^a$ (see Lemma 1)—will on average reach the market later in time. These observations combined imply that consumers face weakly greater prices throughout the industry lifetime, and thus earn less consumer surplus when firms make price announcements.

To see this more formally, define $cs(\mathbf{p})$ to be the consumer surplus flow at prices $\mathbf{p}$. We assume that $cs(\mathbf{p})$ is strictly decreasing in each dimension of $\mathbf{p}$, capturing that higher prices lead to a lower consumer surplus flow. Define the expected discounted consumer surplus in the market as a function of pre-innovation prices $\mathbf{p}$ and innovation pace λ by

$$CS(\mathbf{p}, \lambda) = \frac{1}{r}\left(\frac{rcs(\mathbf{p}) + \lambda cs(p^l, p^f)}{r + \lambda}\right), \tag{9}$$

where $cs(\mathbf{p})$ is the consumer surplus flow at the vector of pre-innovation prices $\mathbf{p}$, $cs(p^l, p^f)$ is the consumer surplus flow after the innovation reaches the market, and $\lambda = 2x$ is the pace of innovation in the pre-innovation period (see Appendix D for details).

It is not hard to verify that $CS(\mathbf{p}, \lambda)$ is decreasing in the pre-innovation prices $\mathbf{p}$.

Also, observe that $CS(\mathbf{p}, \lambda)$ equals the value of a perpetuity that pays consumers a convex combination of $cs(\mathbf{p})$ and $cs(p^l, p^f)$, where the weight on $cs(p^l, p^f)$ increases as a function of the pace of innovation λ. Therefore, for any vector $\mathbf{p}$ such that $cs(\mathbf{p}) < cs(p^l, p^f)$, $CS(\mathbf{p}, \lambda)$ is increasing in λ. These two properties imply that in any equilibrium with price announcements we have that $CS(\mathbf{p}^s, \lambda^{na}) > CS(\mathbf{p}^a, \lambda^a)$.

Proposition 4 (R&D Deterrence Harms Consumers)**.** *Consumer welfare with price announcements is less than consumer welfare without price announcements.*

The equilibrium welfare values in the examples in Table 1 are in line with Proposition 4. The table shows that the expected discounted consumer surplus in all equilibria with price announcements is smaller than its value in the equilibrium without price announcements. The table also shows that total surplus decreases with price announcements, suggesting a first order loss in consumer welfare.

4.4 Discussion: The Role of the Assumptions

In this section, we discuss two key assumptions needed for the R&D deterrence effect of price announcements to hold: price commitments and innovation uncertainty.

In our baseline model, firms announce and commit to a price until the next innovation arrives. Two observations are in order. First, rather than being an assumption, the commitment until the next innovation arrives is a result of the nonexistence of state variable changes prior to the arrival of the next innovation (e.g., demand shocks). That is, because the environment is not changing in the pre-innovation phase of the game, firms face no incentives to revise their price announcements. Second, we emphasize that committing to a price until the next innovation arrives is not central to our results. As long as a price announcement is a commitment lasting any positive measure of time, firms will affect the continuation value of their opponents, rendering price announcements effective in deterring the R&D of rivals. To address both of these points, Section 6.5 presents an extension of the model where we allow for demand shocks that lead to price announcement revisions in the pre-innovation phase of the game. All of our results carry through.

Innovation uncertainty also plays an important role for the firms' ability to deter R&D. To illustrate this point, consider the firms' incremental value of innovating when firms obtain a breakthrough at a known date. With arrival time certainty, the post-innovation values are the only relevant values for firms when choosing their

R&D investments. The incremental value of the innovation is then the benefit of inventing the innovation minus the value of not inventing the product (as opposed to the value of inventing the innovation minus the value of continuing competing in the R&D race). Since the pre-innovation prices have no impact on the post-innovation values, pre-innovation prices cannot affect the replacement effect of firms, and thus do not affect R&D decisions. That is, equilibrium prices with innovation certainty match the equilibrium prices of the static price game.

5 Properties of the R&D Deterrence Effect

In this section, we study two aspects of the R&D deterrence effect. First, how the R&D deterrence effect varies with the degree of product differentiation. Second, we study whether innovation benefits consumers despite the R&D deterrence effect of price announcements. Specifically, we compare consumer surplus under two scenarios: i) firms do not invest in R&D and products stay the same forever, and ii) firms invest in R&D and make price announcements. The tradeoff is that consumers enjoy the benefits of innovation in the latter, but face greater prices as well. We ask whether the value created by the new products surpasses the negative effect of the greater prices caused by price announcements. We henceforth focus on the low deterrence equilibrium and drop the *low* subindex that corresponds to low deterrence equilibrium objects.

To answer these questions we parameterize the demand system. Consumers derive utility from the products sold by each firm as well as from a numeraire good. Consumers maximize $U(q_1, q_2) + m$ subject to $I = m + p_1 q_1 + p_2 q_2$ where q_i and p_i are the quantity and price of good i, m is the numeraire good (with a price normalized to one), and $U(q_1, q_2) = q_1 + q_2 - (q_1^2 + \sigma q_1 q_2 + q_2^2)/2$.[14] The demand for good i is therefore given by

$$q_i = \frac{1}{1+\sigma} - \frac{1}{1-\sigma^2} p_i + \frac{\sigma}{1-\sigma^2} p_j, \tag{10}$$

where we restrict σ to be in the unit interval to capture the case of substitute goods.

As discussed in Singh and Vives (1984), this demand system can capture various degrees of product differentiation. The two extreme cases of this system are

[14] $U(q_1, q_2)$ is quadratic and strictly concave.

the case of independent goods when $\sigma = 0$ (full differentiation) and the case of homogeneous goods when $\sigma \to 1$ (no differentiation). Henceforth, we use σ as our measure of product differentiation.

Lastly, we use the indirect utility function to measure how price changes impact welfare. Because income does not affect the demand for goods 1 and 2, the compensating variation is equivalent to changes in consumer surplus.

5.1 Product Substitution

In this subsection we analyze how the degree of product differentiation impacts the magnitude of the R&D deterrence effect ($p^a - p^s$). Using equations (7) and (10), we can write the equilibrium markup as

$$p^a - c = \frac{(1-\sigma^2)q(\mathbf{p}^a)}{1-\sigma R(\mathbf{p}^a)}. \tag{11}$$

Similary, we can write the equilibrium markup under no price announcements as $p^s - c = (1-\sigma^2)q(\mathbf{p}^s)$.

Equation (11) captures how the degree of product differentiation affects the R&D deterrence effect. Under full product differentiation, represented by $\sigma = 0$, firms do not have incentives to deter their rivals' R&D investments, as rival prices do not impact their profits. Similarly, as σ approaches one, the degree of product differentiation vanishes and the goods become homogeneous. In this case, the incentives to undercut the rival in the product market outweigh the R&D deterrence effect and the price markup goes to zero. In both of these extreme cases, equation (11) equals the equilibrium markup under no price announcements, implying that the R&D deterrence effect vanishes. Lastly, equation (11) captures the upward pricing pressure caused by the R&D deterrence effect that exists for intermediate values of product differentiation $\sigma \in (0, 1)$. Since $R(\mathbf{p}) < 1$ holds in any low deterrence equilibrium, the denominator in (11) is strictly less than one, implying that $p^a > p^s$.

These observations combined lead us to conclude that the R&D deterrence effect is non-monotonic in the degree of product differentiation. The following proposition summarizes these results, and Figure 3 illustrates them with a numerical example.

Proposition 5 (The Magnitude of the R&D Deterrence Effect)**.** *The R&D deterrence effect is non-monotonic in the degree of product differentiation (σ). The*

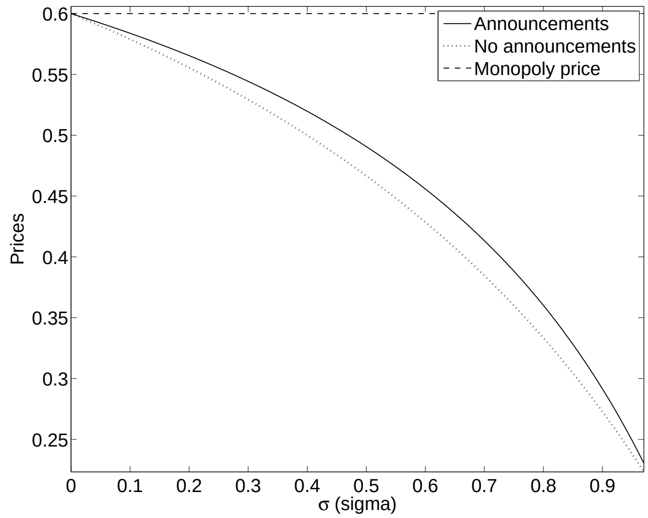

Figure 3: R&D Deterrence Effect and Product Differentiation

Note: The figure shows the equilibrium prices with price announcements (p^a), without price announcements (p^s), and the multiproduct monopoly price for the following parameters: $\kappa(x) = x^2/2$, $r = 0.02$, $\beta = 0.65$, and $c = 0.2$, $\sigma \in [0, 0.97]$. The sufficiency condition for equilibrium uniqueness, $\Psi(p) < 0$ for $p \in [p^s, p^m]$, is satisfied for every value of sigma.

effect is positive for $\sigma \in (0, 1)$, and it vanishes both when products are independent ($\sigma = 0$) and when products are homogeneous ($\sigma \to 1$).

5.2 Consumer Benefits of Innovation

In Proposition 4, we established that consumers are worse off in any equilibrium with price announcements relative to the equilibrium without price announcements. Table 1 also provides examples showing that total surplus decreases with price announcements, suggesting that the loss in consumer welfare caused by price announcements is of first order. A question that remains is if consumers would be better off if firms did not conduct R&D whatsoever (relative to the equilibrium with price announcements).

Why would consumers be better off if firms did not conduct R&D? From Proposition 2 and Proposition 3 we know that the prices faced by consumers in the period before a firm successfully innovates (p^a) are higher than the equilibrium prices when firms maximize static product market profits (p^s). We also established that consumers directly benefit when the innovation reaches the market, as prices fall: $p^l < p^f < p^s < p^a$ (see Lemma 1). These opposing welfare effects of the equilibrium with price announcements could lead to cases where the R&D deterrence effect of price announcements ($p^a > p^s$) outweighs the positive impact of innovation on consumer welfare ($p^l, p^f < p^s$). That is, to cases where innovation is welfare decreasing despite the consumer benefits of a new technology.

To answer the question of whether consumers benefit from innovation despite the R&D deterrence effect of price announcements, we compare the expected discounted consumer surplus under price announcements $CS(\mathbf{p}^a, \lambda^a)$ (see equation (9)) with the consumer surplus that exists with no R&D competition whatsoever; i.e., $CS^{\text{No Innov}} = cs(\mathbf{p}^s)/r$. The difference between $CS(\mathbf{p}^a, \lambda^a)$ and $CS^{\text{No Innov}}$ can be written as

$$\Delta CS = \frac{1}{r(r+\lambda)} \left(r\underbrace{(cs(\mathbf{p}^a) - cs(\mathbf{p}^s))}_{<0} + \lambda\underbrace{(cs(p^l, p^f) - cs(\mathbf{p}^s))}_{>0} \right).$$

The first term in the parentheses is negative because of the R&D deterrence effect ($p^a > p^{na}$), and captures the loss in welfare due to higher prices during the period when the firms are still performing R&D. The second term in the parentheses is positive because consumers face lower prices once the innovation reaches the market ($p^l, p^f < p^a$). Depending on the relative magnitude of each of these terms, consumers may be worse off when firms perform R&D.

Figure 4 shows that depending on the parameters of the model, consumers may benefit or lose with R&D competition. For some parameters, the R&D deterrence effect fully dissipates the utility gains of innovation (e.g., $\sigma = 0.2$). While for other parameters, consumers are still better off with innovation competition despite the R&D deterrence effect (e.g., $\sigma = 0.1$). Regardless, we conclude that the R&D deterrence effect is of first order in understanding the consumer benefits of innovation.

Result 1 (Welfare Dissipation)**.** *The R&D deterrence effect of price may dissipate the consumer benefits of innovation.That is, innovation may be welfare decreasing in presence of the R&D deterrence effect.*

6 Extensions

In this section, we extend the model to show that the R&D deterrence effect of price announcements exists more generally. With respect to market structure, we

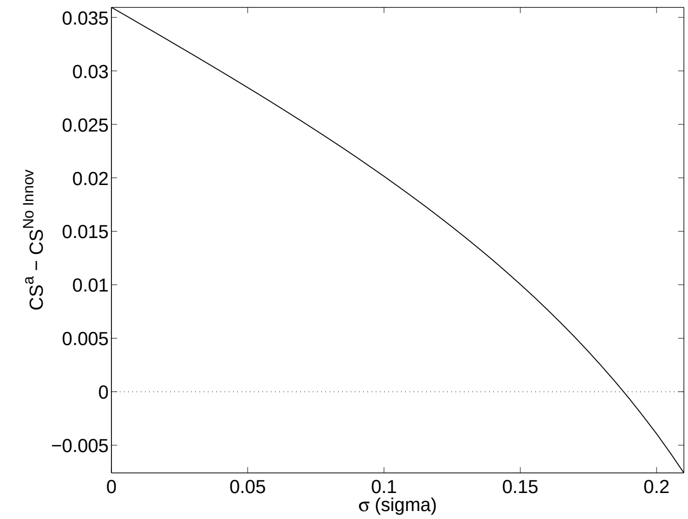

Figure 4: R&D Deterrence Effect and Consumer Benefits of Innovation

Note: The figure shows $\Delta CS = CS^a - CS^{\text{No Innov}}$ for the following parameters: $\kappa(x) = x^2/2$, $r = 0.04$, $\beta = 0.9$, and $c = 0.08$, $\sigma \in [0, 0.24]$. The sufficiency condition for equilibrium uniqueness, $\Psi(p) < 0$ for $p \in [p^s, p^m]$, is satisfied for every value of sigma.

consider the cases with n symmetric players and the duopoly case with asymmetric firms. With respect to the nature of innovations, we consider the case of sequential innovations and the case of quality-enhancing innovations (as opposed to the process innovations in our baseline model). Lastly, we discuss the case where firms face demand shocks that incentivize firms to revise their price announcements.

6.1 n Symmetric Competitors

Consider a industry served by n symmetric firms. As before, let π^l and π^f represent the profit flow obtained by the innovating firm and the non-successful followers after the innovation occurred. Likewise, let $L = \pi^l/r$ and $F = \pi^f/r$ represent the discounted values of being the technology leader and follower after the innovation arrives.

$V_i(\mathbf{p})$ represents the value of firm i at time t before any firm has successfully innovated as a function of the vector of Markov strategies $\mathbf{p}$,

$$V_i(\mathbf{p}) = \max_{x_i} \frac{\pi_i(\mathbf{p}) + x_i L + \sum_{j \neq i} x_j F - \kappa(x_i)}{r + \sum_k x_k},$$

where $V_i(\mathbf{p})$ now captures that an invention by any of the $n-1$ rivals makes firm i become a follower. Firm i's first order condition with respect to the R&D investment is given by equation (4).

Firm i chooses its price announcements by maximizing $V_i(\mathbf{p})$ given beliefs about its rivals' strategies. The first order condition with respect to price is equivalent to

$$\frac{dV_i}{dp_i} = 0 \Leftrightarrow \frac{d\pi_i(\mathbf{p})}{dp_i} \underbrace{- \sum_{j \neq i} \frac{dx_j}{dp_i}(V_i(\mathbf{p}) - F)}_{\text{R\&D Deterrence Effect}} = \frac{d\pi_i(\mathbf{p})}{dp_i} + \underbrace{\sum_{j \neq i} R(\mathbf{p}) \frac{d\pi_j}{dp_i}}_{>0} = 0,$$

where

$$R(\mathbf{p}) = \frac{f'(L - V(\mathbf{p}))(V(\mathbf{p}) - F)}{r + nf(L - V(\mathbf{p}))) - (n-2)f'(L - V(\mathbf{p}))(V(\mathbf{p}) - F)} > 0.$$

As before, firm i takes into consideration how its price will impact its product market profit as well as the R&D investment of each of its rivals. It is possible to show that $R(\mathbf{p}) \in (0, 1)$ in any low deterrence equilibrium. Thus, the existence of a low deterrence equilibrium with prices satisfying $p^a_{low} \in (p^s, p^m)$ follows from analogous arguments to those presented in Proposition 2.

6.2 Asymmetric Firms

Consider an asymmetric duopoly where firm 1 is investing in R&D to increase its cost advantage (i.e., lower its marginal cost from βc to $\beta^2 c$), and firm 2 is investing to match firm 1's marginal cost (i.e., lower its marginal cost from c to βc). Once one of the two firms succeeds, the industry reaches maturity and the firms no longer invest in R&D. The firms' post-innovation values differ depending on which firm successfully innovates. If firm i innovates, the values are given by L_i and F_i, with $L_1 > L_2 = F_1 > F_2$.[15]

Let $V_i(\mathbf{p})$ represent the value of firm i at time t before any firm has successfully innovated as a function of the firms' Markov strategies,

$$V_i(\mathbf{p}) = \max_{x_i} \frac{\pi_i(\mathbf{p}) + x_i L_i + x_j F_j - \kappa(x_i)}{r + x_i + x_j},$$

where $\pi_1(\mathbf{p}) = (p_1 - \beta c)q_1(\mathbf{p})$ and $\pi_2(\mathbf{p}) = (p_2 - c)q_2(\mathbf{p})$. Firm i's first order condition with respect to the R&D investment is given by $\kappa'(x_i) = L_i - V_i(\mathbf{p})$.

[15]These inequalities follow from arguments analogous to those in Lemma 1.

The price announcements are chosen by maximizing $V_i(\mathbf{p})$ given beliefs about p_j, and must satisfy

$$\frac{dV_i}{dp_i} = 0 \Leftrightarrow \frac{d\pi_i(\mathbf{p})}{dp_i} \underbrace{- \frac{dx_j}{dp_i}(V_i(\mathbf{p}) - F_i)}_{\text{R\&D Deterrence Effect}} = \frac{d\pi_i(\mathbf{p})}{dp_i} + \underbrace{R_i(\mathbf{p})\frac{d\pi_j}{dp_i}}_{>0} = 0,$$

with

$$R_i(\mathbf{p}) = \frac{f'(L - V_j(\mathbf{p}))(V_i(\mathbf{p}) - F)}{r + f(L - V_i(\mathbf{p}))) + f(L - V_j(\mathbf{p})))} > 0.$$

Arguments analogous to those in Proposition 2 establish existence of a low deterrence equilibrium with the properties discussed in Section 4.

6.3 Sequential Innovations

We next use the analysis above to argue that the existence of the R&D deterrence effect extends to scenarios where firms compete to develop a sequence of new technologies. Consider the case where two symmetric firms compete to develop a sequence of two technologies.[16] Each new technology reduces a firm's marginal cost by a factor of β. Once the two technologies have been invented, the industry reaches maturity, and the firms no longer invest in R&D.

The analysis of the game after the first technology has been invented is equivalent to the analysis in Section 6.2, where we studied the asymmetric case where one of the firms has a cost advantage of magnitude β. The analysis of the game before the first technology has been invented is equivalent to the symmetric analysis in Section 2 when replacing L and F for V_1 and V_2, respectively, and where V_1 and V_2 are the pre-innovation equilibrium values in Section 6.2. Because the R&D deterrence effect exists in both of these versions of the model, it follows that the R&D deterrence effect and its properties extend to the case of sequential innovations.

6.4 Quality-Enhancing Innovations

In the previous sections we focused on cost-saving innovations. Our results, however, also apply to markets in which firms compete in developing quality-enhancing innovations.

[16]The argument provided here extends to the case of k innovations. We consider the case of two innovations for ease of exposition.

To illustrate this, let θ be a vector describing the (single-dimensional) quality of each product, and let $q_i(\mathbf{p}, \theta)$ be the demand of firm i given the price vector $\mathbf{p}$ and the vector of product qualities θ. In addition to the assumptions discussed in Section 2, we assume that products are *substitutes* in quality, that is, $q_i(\mathbf{p}, \theta)$ strictly increases in θ_i, but strictly decreases in θ_j. This assumption is quite general and includes a wide class of discrete-choice demand models (e.g., logit model).[17]

In this environment with substitutes goods, it is not hard to verify that an increase in the quality of product i benefits firm i and makes every rival of firm i worse off. Consequently, pre and post-innovation profits satisfy Lemma 1 ($\pi^f < \pi^s < \pi^l$). This post-innovation asymmetry between market leader and market follower generates the same incentives to deter R&D as in the case of cost-saving innovations. Because, the analysis in Section 4 only relied on the profit ordering in Lemma 1, the innovation deterrence results extend to this environment.

6.5 Demand Shifters and Revised Price Announcements

In the baseline model, we assume that firms make simultaneous public price announcements at the beginning of the game, and they (credibly) commit to these prices until the next innovation arrives. In our baseline model, price announcements are not revised prior to the discovery of the next innovation because this period presents no changes to state variables that that would incentivize a firm to revise its price. In this extension, we allow for changes to state variables prior to the arrival of the next innovation (e.g., demand shocks), which lead to revised price announcements. This extension is empirically relevant—e.g., Amazon revised its price for the Amazon Fire smartphone because of disappointing sales—and illustrates that our results do not depend on a price commitment that lasts until the next innovation arrives.[18] That is, the same results obtain if firms make a sequence of short-term price announcements over time.

To make this argument, we consider a version of our baseline model where the industry may face a permanent demand shock. We assume that the demand shock arrives at Poisson rate μ, and it may arrive before or after the arrival of an innovation. Because the demand shock impacts profits, firms have incentives to revise their price announcements when faced with the demand shock.

[17] This assumption, however, does exclude cases where an innovation increases the desirability of every product in the market (e.g., improved industry standard).

[18] See, for instance, https://www.businessinsider.com/amazon-fire-phone-price-drop-2014-11.

Let L and F represent the discounted values of being the technology leader and follower after the innovation arrives, and L^{shock} and F^{shock} the discounted values once the demand shock impacts the industry,

$$
\begin{aligned}
L = \frac{\pi^l + \mu L^{shock}}{r+\mu}, \qquad F = \frac{\pi^f + \mu F^{shock}}{r+\mu}, \\
L^{shock} = \frac{\pi^{l,shock}}{r}, \qquad F^{shock} = \frac{\pi^{f,shock}}{r},
\end{aligned}
$$

where π^j and $\pi^{j,shock}$ ($j \in \{f, l\}$) are the profit flows with and without the demand shock.

Let $V_i(\mathbf{p})$ and $V_i^{shock}(\mathbf{p})$ represent the value of firm i at time t before any firm has successfully innovated as a function of both the firms' Markov strategies and whether the demand shock has arrived,

$$
\begin{aligned}
V_i(\mathbf{p}) &= \max_{x_i} \frac{\pi_i(\mathbf{p}) + x_i L_i + x_j F_j + \mu V^{shock} - \kappa(x_i)}{r + x_i + x_j + \mu}, \\
V_i^{shock}(\mathbf{p}) &= \max_{x_i^{shock}} \frac{\pi_i^{shock}(\mathbf{p}) + x_i^{shock} L_i^{shock} + x_j^{shock} F_j^{shock} - \kappa(x_i^{shock})}{r + x_i^{shock} + x_j^{shock}},
\end{aligned}
$$

where $V^{shock} = \max_{p_i} V_i^{shock}(\mathbf{p})$.

The price announcements are chosen by maximizing $V_i(\mathbf{p})$ and $V_i^{shock}(\mathbf{p})$ given beliefs about p_j, and must satisfy

$$
\frac{dV_i}{dp_i} = 0 \Leftrightarrow \frac{d\pi_i(\mathbf{p})}{dp_i} \underbrace{- \frac{dx_j}{dp_i}(V_i(\mathbf{p}) - F)}_{\text{R\&D Deterrence Effect}} = \frac{d\pi_i(\mathbf{p})}{dp_i} + \underbrace{R_i(\mathbf{p})\frac{d\pi_j}{dp_i}}_{>0} = 0,
$$

and $dV_i^{shock}/dp_i = 0 \Leftrightarrow$

$$
\frac{d\pi_i^{shock}(\mathbf{p})}{dp_i} \underbrace{- \frac{dx_j^{shock}}{dp_i}(V_i^{shock}(\mathbf{p}) - F^{shock})}_{\text{R\&D Deterrence Effect}} = \frac{d\pi_i^{shock}(\mathbf{p})}{dp_i} + \underbrace{R_i^{shock}(\mathbf{p})\frac{d\pi_j^{shock}}{dp_i}}_{>0} = 0,
$$

with

$$
R(\mathbf{p}) = \frac{f'(L - V(\mathbf{p}))(V(\mathbf{p}) - F)}{r + 2f(L - V(\mathbf{p})) + \mu} > 0
$$

and

$$
R^{shock}(\mathbf{p}) = \frac{f'(L^{shock} - V^{shock}(\mathbf{p}))(V^{shock}(\mathbf{p}) - F^{shock})}{r + 2f(L^{shock} - V^{shock}(\mathbf{p}))} > 0.
$$

The existence of a low deterrence equilibrium, with the properties discussed in

Section 4, follows from arguments analogous to those in Proposition 2.

7 Concluding Remarks

We study how price announcements affect equilibrium market prices, firms' innovation rates, and welfare outcomes in a dynamic oligopoly model. We find that under price announcements, prices are always greater than the static oligopoly prices and may even exceed the multiproduct monopoly prices. Although price announcements are profitable for firms, price announcements decrease consumer surplus and may even decrease total surplus. The decrease in consumer surplus is caused by both higher prices and lower R&D investments. We show that the higher prices in the equilibrium with price announcements can even dissipate the consumer benefits of innovation. That is, we show instances where consumers would be better off if the industry exhibited no R&D activity whatsoever (i.e., when products stay the same forever). These results combined suggest that the effect of price announcements on consumer welfare is of first order and that the measurement of the welfare benefits of innovation will be biased unless the strategic price effects associated to R&D deterrence are considered.

These results provide an economic rationale for the use of price announcements in media events by tech firms. Our findings have strong managerial implications on the trade-offs that firms face when laying out their pricing strategies. Although pricing new products aggressively may attract new consumers—increasing the market share and short-run profits of the new products—competitors may react with an aggressive R&D plan. Depending on the firms' capabilities relative to competitors to come up with new products, the manager may prefer to avoid an intensification of R&D competition. This is particularly true in mature industries where product improvements are less clear and require large investments.

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Appendix

A. Omitted Proofs

Proof of Lemma 1. Using the envelope theorem and the demand regularity conditions (A), (B), and (D) (see section C of the Appendix) we find:

$$\frac{dp_i}{dc_i} = -\frac{-\frac{dq_i}{dp_i}}{2\frac{dq_i}{dp_i} + \frac{d^2q_i}{dp_i^2}(p_i - c)} \in (0,1), \qquad \frac{dp_j}{dp_i} = -\frac{\frac{dq_j}{dp_i} + \frac{d^2q_j}{dp_i dp_j}(p_j - c)}{2\frac{dq_j}{dp_j} + \frac{d^2q_j}{dp_j^2}(p_j - c)} \in (0,1)$$

Since $\frac{dp_j}{dc_i} = \frac{dp_j}{dp_i}\frac{dp_i}{dc_i} < \frac{dp_i}{dc_i}$, prices are lower when the cost-saving innovation arrives. In particular, $p^l < p^f < p^s$. Similarly for the profits we find:

$$\frac{d\pi_j}{dc_i} = \frac{dq_j}{dp_i}\frac{dp_i}{dc_i}(p_j - c) > 0, \qquad \frac{d\pi_i}{dc_i} = -q_i + \frac{dq_i}{dp_j}\frac{dp_j}{dp_i}\frac{dp_i}{dc_i}(p_i - c)$$

Given the results above, it is easy to see that $d\pi_j/dc_i$ is positive. For $d\pi_i/dc_i$, observe that at prices satisfying the first order condition can be rewritten as:

$$\frac{d\pi_i}{dc_i} = \left(\frac{dq_i}{dp_i} + \frac{dq_i}{dp_j}\frac{dp_j}{dp_i}\frac{dp_i}{dc_i}\right)(p_i - c) < \left(\frac{dq_i}{dp_i} + \frac{dq_i}{dp_j}\right)(p_i - c) < 0,$$

where we use dp_j/dp_i and $dp_i/dc_i \in (0,1)$ for the first inequality, and demand regularity condition (C) for the second. We conclude that $\pi^f < \pi^s < \pi^l$. ■

Proof of Lemma 2. (i) and (ii): Observe that $\phi(F,\pi) = \pi - \pi^f + f(L-F)(L-F) - \kappa(f(L-F))$. Due to the convexity of κ and $\kappa(0) = 0$, we have that $x\kappa'(x) > \kappa(x)$ for all $x > 0$. Equation (4) implies $\kappa'(f(L-F)) = (L-F)$. Replacing back, we obtain $\phi(F,\pi) = \pi - \pi^f + x\kappa'(x) - \kappa(x)$. Thus, $\pi > \pi^f$ is sufficient for $\phi(F,\pi) > 0$. Similarly, $\phi(L,\pi) = \pi - \pi^l$. Thus $\phi(L,\pi) < 0$ whenever $\pi < \pi^l$ and $\phi(L,\pi) = 0$ when $\pi = \pi^l$.

(iii) Differentiating ϕ with respect to V we obtain: $\phi'(V,\pi) = -r - 2f(L-V) + f'(L-V)(V-F)$; and $\phi''(V,\pi) = 3f'(L-V) - f''(L-V)(V-F)$, which is positive for $V \in (F,L)$ as $f'(x) > 0$ and $f''(x) \le 0$ (see Lemma 6). Then, $\phi'(F,\pi) = -r - 2f(L-F) < 0$. On the other hand, $\lim_{V\to L}\phi'(V,\pi) = -r + (L-F)\lim_{V\to L} f'(L-V)$. Here we have two cases (see Lemma 6), if $\gamma > 2$ then $\lim_{V\to L} f'(L-V) = \infty$ and $\phi'(L,\pi) > 0$. If $\gamma = 2$, then $\phi'(L,\pi) = -r + (L-F)$ and $\phi'(L,\pi) > 0$ whenever $L - F > r$.[19]

(iv) Observe that $\phi'(V,\pi)$ is independent of π. Also, since $\phi'(F,\pi) < 0$ and $\phi'(L,\pi) > 0$, the intermediate value theorem implies that there exists a $\underline{V} \in (F,L)$ such that $\phi'(\underline{V},\pi) = 0$. Finally, because $\phi'' > 0$, $\underline{V}$ is unique and corresponds to a minimum.[20] ■

Proof of Lemma 3. Since $\phi(\underline{V},\bar{\pi}) = 0$ and $\phi''(V,\pi) > 0$, there is a unique solution when $\pi = \bar{\pi}$. Because $\phi(\underline{V},\pi) < 0$ for $\pi < \bar{\pi}$, and $\phi(F,\pi) > 0$ for $\pi \ge \pi^f$, the

[19] This implies that $\phi'(L,\pi) < 0$ (and consequently $\phi'(V,\pi) < 0$ for all $V \in [F,L]$) when $L - F < r$. Because our results still apply in this scenario, and since in all relevant economic applications we normally have $L - F > r$, we chose not to overwhelm the reader with technical details in the main text and present the arguments for this case in footnotes 20 and 21.

[20] In the $L - F < r$ and $\gamma = 2$ scenario, $\underline{V} = L$ and $\bar{\pi} = \pi^l$.

intermediate value theorem implies that there exists a solution to the left of $\underline{V}$ when $\pi \in [\pi^f, \bar{\pi})$. Because ϕ is monotone in $V \in [F, \underline{V}]$, the solution to the left is unique. Similarly, because of the monotonicity of ϕ to the right of $\underline{V}$, there is no solution larger than $\underline{V}$ when $\phi(L, \pi) < 0$ (i.e., $\pi < \pi^l$) and a unique solution when $\phi(L, \pi) \geq 0$ (i.e., $\pi \geq \pi^l$).[21] ■

Proof of Lemma 4. From the proof of Lemma 3 we know that for any $\pi < \bar{\pi}$, $V_{low} < \underline{V} < V_{high}$, which proves the first claim. For the second claim simply observe that $\phi'(V, \pi) < 0$ (or $\phi'(V, \pi) = 0$ or $\phi'(V, \pi) > 0$) is equivalent to $R(\mathbf{p}) < 1$ (or $R(\mathbf{p}) = 1$ or $R(\mathbf{p}) > 1$, respectively) after manipulating the inequality and using the definition of $R(\mathbf{p})$. Since $R(\mathbf{p}) > 0$, $\phi'(\underline{V}, \pi) = 0$ and $\phi''(V, \pi) > 0$, it follows that $R(\mathbf{p}) \in (0, 1)$ at V_{low} and $R(\mathbf{p}) > 1$ at V_{high}. For the last statement, at any solution $V^a \in \{V_{low}, V_{high}\}$ of $\phi(V, \pi)$, observe that $dV^a/d\pi = -\phi'(V^a, \pi)^{-1}$. The result follows. ■

Proof of Lemma 5. See argument in the text. ■

Proof of Proposition 1. The first order condition for price in (2) and the demand regularity conditions imply that $\mathbf{p}^s = (p^s, p^s)$ is the unique equilibrium price vector and $\pi^s = \pi_i(\mathbf{p}^s)$ is the equilibrium profit earned by the firms. Using the condition (2) for R&D investments, we can construct $\phi(V, \pi^s)$ which, by Lemma 3 single crosses zero at a value $V^{na} \in (F, L)$ since $\pi^s \in (\pi^f, \pi^l)$ (see Lemma 1). Lastly, strict convexity of $\kappa(x)$ guarantees that x^{na} is the unique solution to equation (2) given V^{na} and L. ■

Proof of Proposition 2. The existence of the function $V_{low}(\mathbf{p})$ follows from Lemmas 2 and 3. The existence of $p^a_{low} \in (p^s, p^m)$ follows from Lemma 4 and the application of the intermediate value theorem, as discussed in the text. For completeness and to better show the differences between a *high* and *low* deterrence equilibrium, we present a complete derivation of the first order conditions. Then, we prove that p^a_{low} is indeed an equilibrium by showing that the second order condition holds at all solutions of the first order condition (hence, the equilibrium is unique). Then, we show that $\pi(\mathbf{p}^a_{low}) > \pi^s$ which, by Lemma 4, immediately implies $V^a_{low} > V^{na}$ and $x^a_{low} < x^{na}$.
First order condition: Using implicit differentiation, the derivative of V_i with respect to p_i is

$$\frac{dV_i}{dp_i} = \frac{\frac{d\pi_i}{dp_i} + f'(L - V_j)\frac{dV_j}{dp_i}(V_i - F)}{r + f(L - V_j) + f(L - V_i)}, \quad \text{where} \quad \frac{dV_j}{dp_i} = \frac{\frac{d\pi_j}{dp_i} + f'(L - V_i)\frac{dV_i}{dp_i}(V_j - F)}{r + f(L - V_j) + f(L - V_i)}.$$

Observe that at the optimum $dV_i/dp_i = 0$. Using this condition in dV_j/dp_i and replacing back into dV_i/dp_i delivers equation (7). More generally (i.e., not necessarily at the optimum), we obtain

$$\frac{dV_i}{dp_i} = \frac{1}{K}\frac{\frac{d\pi_i}{dp_i} + R_i\frac{d\pi_j}{dp_i}}{(1 - R_iR_j)} \tag{12}$$

where $K = r + f(L - V_i) + f(L - V_j) > 0$ and $R_i = f'(L - V_j)(V_i - F)/K > 0$. The expression above captures how in a *high* deterrence equilibrium the derivative of V_i with

[21] In the $L - F < r$ and $\gamma = 2$ scenario, $\phi(V, \pi)$ is strictly decreasing for all relevant V. Since $\phi(L, \pi^l) = 0$, this means that $\phi(V, \pi)$ single-crosses zero to the left of $\underline{V} = L$ whenever $\pi < \pi^l$, and it never crosses zero when $\pi > \pi^l$. That is, only low deterrence equilibria exist.

respect to p_i has the opposite sign of the same derivative in a *low* deterrence equilibrium, since $R_i > 1$ for $i \in \{1, 2\}$ in any high deterrence equilibrium.
Second order condition: Differentiating (12) with respect to p_i and using that in equilibrium $-d\pi_i/dp_i = R_i d\pi_j/dp_i$, we obtain:

$$\frac{d^2V_i}{dp_i^2} = \frac{\frac{d^2\pi_i}{dp_i^2} + R_i\frac{d^2\pi_j}{dp_i^2} - \frac{1}{K^2}\left(\frac{f'(L-V_j)^2 - f''(L-V_j)K}{f'(L-V_j)}\right)\frac{d\pi_i}{dp_i}\frac{d\pi_j}{dp_i}}{K\left(1 - R_iR_j\right)}. \tag{13}$$

The denominator is positive in a low deterrence equilibrium whereas it is negative in a high deterrence equilibrium (Lemma 4). We show that $\Psi(p) < 0$ for every $p \in (p^s, p^m)$ implies that at every price where (7) is satisfied, an upper-bound of (13) is negative.

Because the denominator is positive, we ignore it to determine the sign of d^2V_i/dp_i^2. We know that $d^2\pi_i/dp_i^2 < 0$ because of demand regularity condition (A), which guarantees uniqueness of the static oligopoly game. To bound the second term from above, we use $\max\{0, d^2\pi_j/dp_i^2\}$ (if $d^2\pi_j/dp_i^2 > 0$ take $R_i = 1$, if $d^2\pi_j/dp_i^2 < 0$ take $R_i = 0$). Before bounding the third (and last) term, we show that it is positive (i.e., the negative 1 multiplies a negative term). $d\pi_i/dp_i < 0$ because every price that satisfies (7) is greater than p^s, and we have that $d\pi_i(\mathbf{p}^s)/dp_i = 0$ and $d^2\pi_i/dp_i^2 < 0$. $d\pi_j/dp_i > 0$ because $dq_j/dp_i > 0$. Finally the term in parenthesis is positive by Lemma 6. We bound the term in parenthesis using the parametric specification of the cost function, $\kappa(x) = x^\gamma/\gamma$, which implies $f(z) = z^{1/(\gamma-1)}$, and

$$\frac{1}{K^2}\left(\frac{f'\left(L-V\right)^2 - f''\left(L-V\right)K}{f'\left(L-V\right)}\right) = \frac{(\gamma-1)^{-1}}{(r+2f(L-V))^2}\left(\frac{1}{(L-V)^{\frac{\gamma-2}{\gamma-1}}} + \frac{\gamma-2}{L-V}\right).$$

The expression above is increasing in V. Because every low deterrence equilibrium satisfies $V_{low} \le \underline{\mathrm{V}}$, we use $\underline{\mathrm{V}}$ to bound the expression above. Thus, $d^2V_i/dp_i^2 < \Psi(p^a_{low})/(K(1-R_iR_j)) < 0$, where the second inequality follows from the assumption that $\Psi(p) < 0$ for all $p \in (p^s, p^m)$. $d^2V_i/dp_i^2 < 0$ implies that p^a_{low} is a local maximum, and because this is true for every p satisfying (7), there is no local minimum satisfying the first order condition. Hence, the equilibrium is unique.
Proof of $\pi(\mathbf{p}^a_{low}) > \pi^s$*:* The solution to the problem of a firm that controls the price of all products in the case of symmetric demand functions (i.e., $\max_p \pi(p,p)$) is given by the multiproduct monopoly price, p^m. This implies that $\pi(p,p)$ is increasing in p until the price reaches the monopoly price, p^m. Because $p^s < p^a_{low} < p^m$, the result follows. ■

Proof of Proposition 3. High deterrence equilibria are shown in Table 1. For $p^a_{high} > p^m$ (and therefore $p^a_{high} > p^a_{low}$) see the discussion in the text. By Lemma 1, we know V^a_{high} (if it exists) is larger than V^a_{low}. This, in conjunction with equation (4), implies $x^a_{high} < x^a_{low}$. Finally, to check that a price satisfying equation (7) is indeed a high deterrence equilibrium, we need to check the second order condition (13) (we do so numerically in the examples in the table). Since $R_i > 1$, the denominator is negative, thus the numerator has to be positive. Examples suggest show that it is possible to simultaneously satisfy $\Psi(p) < 0$ for $p \in (p^s, p^m)$ and have the numerator of (13) be positive. ■

Proof of Proposition 4. See argument in the text. ■

Proof of Proposition 5. See argument following equation (11) in the main text. ■

B. Auxiliary Results

Lemma 6. *The function $f(z)$ implicitly defined by $\kappa'(f(z)) = z$ satisfies $f(0) = 0$ and is increasing ($f'(z) > 0$ for all $z > 0$). When $\gamma > 2$, $\lim_{z\to 0} f'(z) = \infty$. When $\gamma = 2$, $f(z) = 1$ for all $z \geq 0$. When $\kappa'''(x) > 0$, $f(z)$ is concave $f''(z) < 0$ for all $z > 0$, and when $\kappa'''(x) = 0$, $f''(z) = 0$ for all $z > 0$.*

Proof. The first statement follows from $\kappa'(0) = 0$. The second follows from the derivative of $f(z)$ being equal to $f'(z) = 1/\kappa''(f(z))$ and the fact that $\kappa(x)$ is strictly convex (i.e., $\kappa''(x) > 0$). The limiting result follows from $\kappa''(0) = 0$ when $\gamma > 2$ and $\kappa''(0)$ is a positive constant when $\gamma = 2$. Similarly, $f''(z) = -\kappa'''(f(z))/(\kappa''(f(z))^3)$ and the results follows from the value of $\kappa'''(x)$. ■

C. Demand Regularity Conditions

We assume that the following conditions hold for every symmetric price vector such that the first order condition $q_i + dq_i/dp_i\ (p_i - c) = 0$ is satisfied.

$$-\frac{dq_i}{dp_i} > \frac{d^2q_i}{dp_i^2}(p_i - c), \quad \text{(A)} \qquad \frac{dq_i}{dp_j} > -\frac{d^2q_i}{dp_j dp_i}(p_i - c), \quad \text{(B)} \qquad -\frac{dq_i}{dp_i} > \frac{dq_i}{dp_j}, \quad \text{(C)}$$

$$-\frac{dq_i}{dp_i} > \frac{dq_i}{dp_j} + \left(\frac{d^2q_i}{dp_i^2} + \frac{d^2q_i}{dp_j dp_i}\right)(p_i - c). \qquad \text{(D)}$$

(C) requires that own price effects are larger than those of the opponent. (A) guarantees both the second order condition for the oligopolist firm problem, and that $dp_i/dc_i < 1$. Finally, (B) and (D) are required for j to not over react to an i's price change; i.e., $dp_j/dp_i \in (0, 1)$.

To determine the conditions for when the multiproduct monopolist's problem has a unique solution, define the function: $F(p_1, p_2) \equiv \pi_1(p_1, p_2) + \pi_2(p_1, p_2)$. Let F_i represent the derivative of F with respect to p_i. Uniqueness is guaranteed if $F_{i,i} < 0$ and $F_{1,1}F_{2,2} - (F_1F_2)^2 > 0$ at any price vector such that $F_i = 0$.

D. Discounted Expected Consumer Surplus

The discounted expected consumer surplus at time t, before the innovation has arrived, and given a vector of price announcements, $\mathbf{p}$, is given by

$$CS(\mathbf{p}) = \int_t^\infty e^{-(r+\lambda)(s-t)} \left(cs(\mathbf{p}) + \lambda cs(p^l, p^f)/r\right) ds = \frac{rcs(\mathbf{p}) + \lambda cs(p^l, p^f)}{r(r+\lambda)},$$

where $cs(\mathbf{p})$ is the consumer surplus flow at prices $\mathbf{p}$, $cs(p^l, p^f)/r$ is the discounted consumer surplus after the innovation arrives, λ is the pace of innovation, and r is the discount rate. The interpretation of $CS(\mathbf{p})$ is similar to that of equation (1).