

# Announcing High Prices to Deter Innovation\*

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## Abstract

Price announcements—similar to the ones made in media events by tech firms—are effective in deterring innovation. By announcing (and setting) a high price, a firm increases its rivals' short-run profits, which generates a complacency effect that reduces the rival firms' incentives to innovate. We show that the equilibrium prices are greater and the R&D investments are lower relative to when price announcements cannot be used strategically. We call this the R&D deterrence effect of price, and show that it induces equilibrium prices that may exceed the multiproduct monopoly prices and even dissipate the consumer benefits of innovation.

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# 1 Introduction

Whether firms use prices to deter entry is a question that is central to industrial organization and antitrust policy. Research has shown that firms may benefit from manipulating prices to signal cost efficiency or low market profitability (Milgrom and Roberts, 1982, Harrington, 1986), or to establish a reputation of being a “tough” competitor (Goolsbee and Syverson, 2008, Kreps and Wilson, 1982).<sup>1</sup> In all of these cases, firms sacrifice short-run profits to deter entry and increase the value of the firm in the long run. While most of the literature has focused on how predatory prices may affect the entry of new competitors, the analysis of how pricing may be used to soften competition along other dimensions (e.g., innovation or capital investment) has been less studied.

In this paper, we address the question of how firms may use prices to deter innovation. We tackle this question in the context of a dynamic oligopoly model where firms compete both in prices and in developing an innovation that improves upon the existing products. Firms make a simultaneous public price announcement before they start selling their products and then, after observing the full profile of price announcements, they choose how much to invest in R&D. Public price announcements are common in innovative industries, with prices being announced in media events even before the products reach the market.<sup>2</sup> To isolate the role played by price announcements in deterring R&D, we compare the equilibrium under price announcements with the equilibrium in the case where firms cannot use price announcements to strategically influence R&D investments (i.e., the case where firms choose both prices and R&D investments simultaneously).

Our model resembles Loury (1979), Lee and Wilde (1980), and Reinganum (1982) in that innovation is uncertain and the arrival of the innovation follows a Poisson process with parameters that depend on the intensity of the firms’ R&D investments. In contrast to these papers, we follow Marshall and Parra (2018) and explicitly model the product market game. Firms invest in R&D to gain a product

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<sup>1</sup>Economists have also analyzed how non-price strategic choices including advertising, licensing, R&D, capacity investments, patenting, and production-sharing agreements can be used to deter entry (e.g., Dixit 1980, Ellison and Ellison 2011, Gilbert and Newbery 1982, Spence 1977, Chen and Ross 2000).

<sup>2</sup>Recent examples include announcements by Apple (<https://www.wsj.com/articles/apple-wwdc-event-watch-gets-upgrades-amazon-video-coming-to-apple-tv-1496684848>), Microsoft (<https://www.wsj.com/articles/microsofts-new-surface-pro-borrows-from-the-family-to-revive-sales-1495541700>), Nintendo (<https://www.wsj.com/articles/nintendo-to-launch-2ds-xl-handheld-game-device-in-july-1493388631>), and Samsung (<https://www.wsj.com/articles/samsung-launches-galaxy-s8-smartphone-1490799600>).

market advantage, and choose their R&D investments according to the incremental value of the innovation (Arrow, 1962). The price announcements directly impact the incremental value of the innovation, and can therefore be used to manipulate a rivals' incentives to conduct R&D.

Announcing a higher price is effective in deterring a rival's innovation because it induces complacency by increasing the rival's short-run product market profit. A greater short-run product market profit decreases the incremental value of the innovation, which in turn decreases the rival's incentives to invest in R&D. We call this the R&D deterrence effect of price. We show that there exists an equilibrium with R&D deterrence, with prices that are greater and R&D investments that are lower relative to when firms cannot use price announcements to strategically influence innovation incentives. As in the rest of the literature on predation, firms choose to sacrifice short-run profits by announcing a higher price to lower their rivals' incentives to innovate. The short-run sacrifices pay off and firms earn greater discounted profits by deterring their rivals' R&D investments.

With respect to the properties of the R&D deterrence effect, we show that the equilibrium prices with price announcements are always greater than the static oligopoly prices (i.e., the prices when firms cannot use price announcements to strategically influence R&D investments) and may even exceed the multiproduct monopolist prices. The R&D deterrence effect is also found to be non-monotonic in the degree of product differentiation, with the effect vanishing in the extremes cases of independent goods (full differentiation) and homogeneous goods (no differentiation). Lastly, we show that the higher prices caused by the R&D deterrence effect can dissipate the consumer benefits of innovation, in the sense that consumers would be better off if the industry exhibited no R&D activity.

These results have several implications. First, they broaden our understanding on how prices can be used to soften competition along non-price dimensions. Second, they suggest that the R&D deterrence effect of price has a first order effect on consumer welfare, which implies that measurements of the welfare gains of innovation will be biased unless the strategic role of price announcements are accounted for. Third, they provide an economic argument for why firms in innovative industries make use of price announcements (although we acknowledge that marketing and other factors may also be important in the decision of making price announcements).

Lastly, our results suggest that firm complacency can be manipulated by rivals

in equilibrium. Complacency has often been cited as a factor that kills innovation and may even threaten the existence of firms in innovative industries (Christensen, 2013). Examples with popular press coverage include Blackberry, Nokia, and Motorola.<sup>3</sup> While the literature has recognized that the incentives to innovate are impacted by the profitability of existing products (Arrow, 1962, Igami, 2017), our analysis goes beyond these results and shows that firms may choose prices so as to magnify their rivals' complacency and deter innovation.

Two articles are closely related to ours. Gallini (1984) shows that incumbents may use licensing agreements to share profits with potential entrants to decrease their incentives to innovate. Relatedly, we show that price announcements are effective in inducing complacency and reducing the intensity of R&D competition. Besanko *et al.* (2014) study dynamic pricing decisions when firms learn by doing. Under learning, the dynamic pressure of future competition induces firms to decrease prices to expand quantity and therefore speed up the learning process. In contrast to their results, we find that the ability to use price announcements interacted with future competition generates an upward pressure on prices, where the greater prices are intended to decrease R&D activity by inducing complacency.

The rest of the paper is organized as follows. Section 2 introduces the model, and the equilibria without price announcements and with price announcements are discussed in Section 3 and Section 4, respectively. Additional properties of the R&D deterrence effect are discussed in Section 5, and Section 6 discusses model extensions. Lastly, Section 7 concludes.

## 2 Model Setup

Consider a continuous-time infinitely lived oligopoly, where firms sell differentiated goods and compete in prices. At every instant of time, and for a given vector of market prices  $\mathbf{p}$ , firm  $i$  earns a profit flow  $\pi_i(\mathbf{p}) = (p_i - c_i)q_i(\mathbf{p})$  where  $q_i$  is the demand for firm  $i$ 's product, and  $c_i$  is firm  $i$ 's marginal cost of production. We assume  $\partial q_i/\partial p_i < 0$  and  $\partial q_i/\partial p_j > 0$  as well as some additional regularity conditions that guarantee a unique equilibrium in the static price game and a unique solution to the problem of a monopolist controlling the prices of all the goods (see Appendix C for details). For ease of exposition, we present our analysis

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<sup>3</sup>See, for instance, <http://associationsnow.com/2014/09/innovation-complacency-dont-mix/> and <http://www.nytimes.com/2010/09/27/technology/27nokia.html>.

for the case of a symmetric duopoly (i.e.,  $c_1 = c_2 = c$ ), and we later argue in Section 6 that our results generalize to the case of  $n$  firms and cost asymmetries.

Aside from competing in prices, firms compete in developing a cost-saving innovation.<sup>4</sup> The firm that successfully innovates, which we call the *leader*, obtains a patented innovation that decreases its marginal costs to  $\beta c$  with  $\beta \in (0, 1)$ . Firms invest in R&D by choosing a Poisson innovation rate  $x_i$  at a flow cost of  $\kappa(x_i) = x_i^\gamma/\gamma$  with  $\gamma \geq 2$ . The flow cost  $\kappa(x_i)$  is strictly increasing, strictly convex (i.e.,  $\kappa''(x) > 0$  for all  $x \geq 0$ ), and satisfies  $\kappa(0) = \kappa'(0) = 0$  and  $\kappa'''(x) \geq 0$  for all  $x \geq 0$ .<sup>5</sup> The Poisson processes are independent among firms, generating a memoryless stochastic process.

We assume that after one firm successfully innovates, the industry reaches maturity.<sup>6</sup> Once the industry reaches maturity, firms no longer invest in R&D, and they play a static asymmetric price competition game at every instant of time. Define  $\pi^l$  and  $\pi^f$  to be the equilibrium profit flows earned by the leader and *follower* (i.e., the unsuccessful firm), respectively, after the innovation is invented. Define the equilibrium prices of the leader and follower to be  $p^l$  and  $p^f$ , respectively.

As reference points, let  $\pi^s$  be the profit flow in the unique symmetric equilibrium of the static price game, and let  $p^s$  be the equilibrium price of each firm. Similarly, let  $\pi^m$  be the per-product profit flow earned by a multiproduct monopolist controlling the prices of both goods, and let  $p^m$  be the monopoly price for each good.

**Lemma 1.** *In equilibrium,  $\pi^f < \pi^s < \pi^l$  and  $p^l < p^f < p^s$ .*

As expected, the innovation gives the leader a competitive edge, increasing its profit flow and decreasing that of the follower. Depending on the initial level of the marginal costs  $c$  and the magnitude of the process innovation  $\beta$ , the profit flow of the leader may be higher or lower than the per-product monopoly profit flow,  $\pi^m$ . Because the distinction will be important in the analysis that follows, we call an innovation *incremental* when  $\pi^l < \pi^m$ , and *radical* when  $\pi^l \geq \pi^m$ .

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<sup>4</sup>For the sake of exposition, we present the model as firms competing in developing a process innovation that lowers the innovator's marginal cost. The model, however, can accommodate other forms of innovation. For instance, we could alternatively assume that the innovation increases product quality.

<sup>5</sup>The functional form  $\kappa(x) = x^\gamma/\gamma$  is only critical for the characterization of a sufficient condition that guarantees that the second order conditions of the firms' pricing problems hold. Equilibria can be constructed for alternative cost functions,  $\kappa(x)$ , satisfying the aforementioned assumptions.

<sup>6</sup>We discuss the case of sequential innovations in Section 6.

**Definition 1.** *An innovation is incremental when  $\pi^l < \pi^m$  whereas it is radical when  $\pi^l \geq \pi^m$ .*

In what follows, we analyze the model under two different assumptions regarding the timing of play. First, in Section 3, we study the equilibrium outcomes when firms choose both prices and R&D investments simultaneously. In this case, prices cannot be used to strategically influence R&D investment decisions. The model with simultaneous decisions will serve as a benchmark and help us to isolate the strategic role played by price announcements.

In Section 4 we proceed to study the case where firms make simultaneous public price announcements at the beginning of the game, and credibly commit to those prices until the innovation arrives. Firms then choose how much to invest in R&D in every period, and condition their choices on the full profile of price announcements. Alternatively, we could assume that firms play a two-stage game at every instant of time, with firms first making simultaneous public price announcements, and then choosing how much to invest in R&D after observing all of the price announcements. Due to the properties of the Poisson arrival process and the stationarity of the model, both formulations are equivalent.<sup>7</sup>

In what follows, we focus on studying the Markov perfect equilibria of the game. At each instant of time, firms' strategies will be a function of the only state variable of the game—i.e., whether or not an innovation has arrived. When we study the model with price announcements, the firms' R&D investment strategies will also be a function of the announced prices.

### 3 No Price Announcements

We first analyze the game where firms choose both prices and R&D investments simultaneously at every instant of time.<sup>8</sup> Because of the timing of play, firms cannot use prices to affect their rivals' investment choices, making it a natural comparison for the model with price announcements.

Let  $r$  represent the discount rate. After a firm has successfully innovated, the values of the innovator (or leader) and follower are given by  $L = \pi^l/r$  and  $F = \pi^f/r$ , respectively. Let  $V_i$  represent the value of firm  $i$  at time  $t$  before any

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<sup>7</sup>As shown in Reinganum (1982), open and closed-loop decisions coincide under a Poisson innovation arrival process.

<sup>8</sup>A more general version of this model is discussed in Marshall and Parra (2018).

firm has successfully innovated,

$$V_i = \max_{p_i, x_i} \int_t^\infty e^{-(r+x_i+x_j)(s-t)} (\pi_i(p_i, p_j) + x_i L + x_j F - \kappa(x_i)) ds. \quad (1)$$

To understand this value function, fix any instant of time  $s > t$ . With probability  $\exp(-(x_i + x_j)(s - t))$ , no innovation has arrived between  $t$  and  $s$ . At that instant, firm  $i$  receives the flow payoff  $\pi_i(p_i, p_j)$ ; innovates at rate  $x_i$ ; earns an expected payoff of successful innovation of  $x_i L$ ; pays the flow cost of its R&D,  $\kappa(x_i)$ ; faces innovation by its rival at rate  $x_j$ ; and earns the expected payoff of losing the innovation race of  $x_j F$ . All of these payoffs are discounted by  $\exp(-r(s - t))$ . Due to the stationarity of the payoffs and the nature of Markov strategies,  $V_i$  can be rewritten as

$$V_i = \max_{p_i, x_i} \frac{\pi_i(p_i, p_j) + x_i L + x_j F - \kappa(x_i)}{r + x_i + x_j},$$

which does not depend on the time  $t$ .

Given the rival's strategy  $(x_j, p_j)$ , the best response functions of firm  $i$  are implicitly defined by the first order conditions

$$\kappa'(x_i) = L - V_i, \quad \frac{\partial \pi_i(p_i, p_j)}{\partial p_i} = 0. \quad (2)$$

The first condition in (2) shows that firms choose how much to invest in R&D by equating the incremental value of the innovation with the marginal cost of increasing the innovation rate,  $x_i$ . The second condition in (2) shows that firms choose prices by maximizing their static product market profits. Let  $V^{na}$ ,  $p^{na}$ , and  $x^{na}$  represent the equilibrium values, prices, and investments when there is no announcements. Equilibrium existence and uniqueness is established in the following proposition.

**Proposition 1.** *There is a unique symmetric Markov perfect equilibrium,  $(p^{na}, x^{na}, V^{na})$ , that solves equation (1) and conditions (2). In equilibrium,  $p^{na} = p^s$ ,  $x^{na} > 0$ , and  $V^{na} \in (F, L)$ .*

## 4 Price Announcements

We next consider the case with public price announcements. The timing of the game is as follows. At the beginning of the race, firms make simultaneous public price announcements, and they (credibly) commit to these prices until the next

innovation arrives. Upon observing the announced prices, firms then choose how much to invest in R&D in every period. We solve the game by backward induction. We first analyze the equilibrium investments and firm values for a given pair of prices  $\mathbf{p} = (p_i, p_j)$ . We then use the firm values as a function of prices to analyze the equilibrium of the pricing game.

Multiple pricing equilibria may exist in this scenario. In every equilibria, market prices are higher and R&D investments are lower than those when pricing and investment decisions are determined simultaneously (i.e., the case where price choices cannot affect the rivals' R&D decisions). We show that a unique *low deterrence* equilibrium—an equilibrium with prices greater than  $p^s$  but less than the monopoly price  $p^m$ —exists under mild assumptions. Also, a *high deterrence* equilibrium—where prices exceed the monopoly price—may exist when innovations are incremental (see Definition 1).

## 4.1 R&D Investments

Let  $V_i(\mathbf{p})$  represent the value of firm  $i$  at time  $t$  before any firm has successfully innovated as a function of price announcements  $\mathbf{p} = (p_i, p_j)$ ,

$$V_i(\mathbf{p}) = \max_{x_i} \int_t^\infty e^{-(r+x_i+x_j)(s-t)} (\pi_i(\mathbf{p}) + x_i L + x_j F - \kappa(x_i)) ds.$$

$V_i(\mathbf{p})$  has an interpretation that is similar to that of equation (1), with the difference that now prices are fixed at  $\mathbf{p}$ . Using the principle of optimality, and conditioning on the opponents' strategy  $x_j$ , we can rewrite the problem above as

$$rV_i(\mathbf{p}) = \max_{x_i} \{\pi_i(\mathbf{p}) + x_i(L - V_i(\mathbf{p})) + x_j(F - V_i(\mathbf{p})) - \kappa(x_i)\}. \quad (3)$$

Given the price vector  $\mathbf{p}$ , each firm chooses how much to invest in R&D by solving the maximization problem in equation (3). Because equation (3) is independent of  $t$ , we can see that the stationarity of model makes the timing of the price announcement irrelevant; i.e., firms choose the same R&D investment regardless of when prices are announced.

Differentiating the right hand side of equation (3) gives us an implicit expression for firm  $i$ 's R&D investment,

$$\kappa'(x_i) = L - V_i(\mathbf{p}). \quad (4)$$



As before, the investment rule equates the marginal cost of increasing the innovation rate,  $x_i$ , with the incremental value of an innovation,  $L - V_i(\mathbf{p})$ . Due to the convexity of  $\kappa$ , equation (4) has a unique solution and can be inverted and written as  $x_i = f(L - V_i(\mathbf{p}))$ , where  $f$  is a strictly increasing and concave function (see Lemma 6 in the Appendix).

Replacing the equilibrium investment decisions back into equation (3), and restricting attention to symmetric values among firms ( $V_i = V_j = V$ ), we define the continuous function

$$\phi(V, \pi) \equiv \pi + f(L - V)(L + F - 2V) - \kappa(f(L - V)) - rV. \quad (5)$$

The function  $\phi(V, \pi)$  will help us characterize how prices relate to the equilibrium value of  $V$ . Start by observing that, given a vector of (symmetric) prices  $\mathbf{p}$ , both firms earn the profit flow  $\pi = \pi_i(\mathbf{p})$ . For each  $\pi$  we can search for a candidate solution to the value function by looking at values of  $V$  satisfying  $\phi(V, \pi) = 0$ . Because deterrence is costly and firms cannot be deterred beyond the point where they choose not to invest in R&D (i.e.,  $x_j \geq 0$ ), no equilibrium with  $V \geq L$  exists (we come back to this below). Also, it can be easily shown that values of  $V < F$  are never candidate solutions for economically relevant values of  $\pi$ . For these reasons we restrict the analysis of  $\phi(V, \pi)$  to values of  $V \in [F, L]$ .

**Lemma 2.** *(i)  $\phi(F, \pi) > 0$  for any  $\pi \geq \pi^f$ . (ii)  $\phi(L, \pi^l) = 0$  and  $\phi(L, \pi) < 0$  for any  $\pi < \pi^l$ . (iii) For any given value of  $\pi$ ,  $\phi'(F, \pi) < 0$ ,  $\phi'(L, \pi) > 0$ , and  $\phi''(V, \pi) > 0$  for all  $V \in (F, L)$ .<sup>9</sup> (iv) There exists a unique value  $\underline{V} \in (F, L)$ , independent of  $\pi$ , where  $\phi(V, \pi)$  is minimized.*

The previous lemma characterizes  $\phi$  as a function of  $V$ . It tells us that for any given value of  $\pi$ ,  $\phi(V, \pi)$  is U-shaped with  $\phi'(V, \pi)$  being monotonically increasing. Due to the linearity of  $\phi$  in  $\pi$ , a change in  $\pi$  only results in a vertical shift of  $\phi$ . These two facts imply that  $\phi$  is uniquely minimized at  $\underline{V}$ , which is independent of  $\pi$ . Define  $\bar{\pi}$  to be the unique value of  $\pi$  satisfying  $\phi(\underline{V}, \bar{\pi}) = 0$ .

**Lemma 3.** *For  $\pi \in [\pi^f, \pi^l] \cup \{\bar{\pi}\}$  the equation  $\phi(V, \pi) = 0$  has a unique solution. For  $\pi \in [\pi^l, \bar{\pi})$ ,  $\phi(V, \pi) = 0$  has two solutions. For  $\pi > \bar{\pi}$ ,  $\phi(V, \pi) = 0$  has no solution.*

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<sup>9</sup>The prime denotes derivatives with respect to the  $V$  dimension.

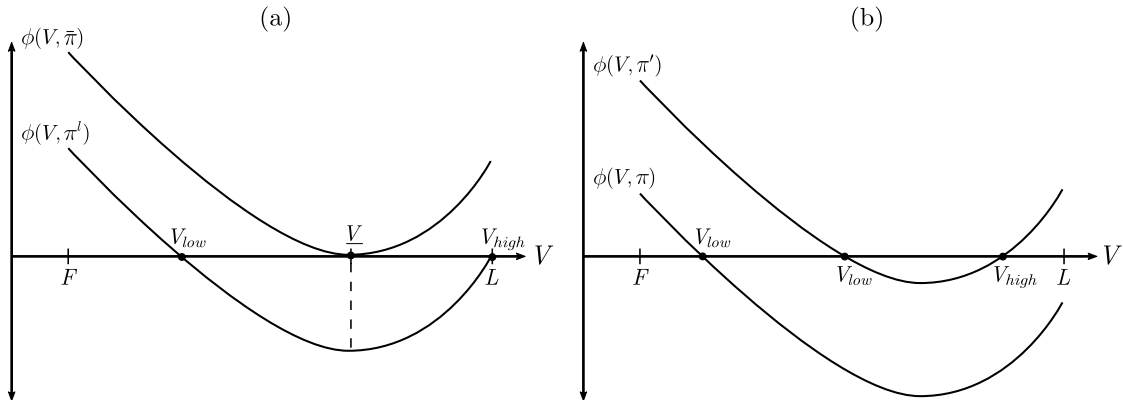


Figure 1: Equilibrium Multiplicity

The intuition of Lemma 3 can be seen graphically in Figure 1. Start by analyzing  $\phi(V, \pi^l)$ . Since  $\phi(L, \pi^l) = 0$  and  $\phi'(L, \pi^l) > 0$  we know that  $\phi$  approaches zero at  $L$  from below. Also, since  $\phi(F, \pi^l) > 0$  we know that  $\phi(V, \pi^l)$  must cross zero at a value of  $V \in (F, L)$ . This value is represented by  $V_{low}$  in Figure 1 (Panel A). Moreover, since a lower  $\pi$  implies a downward shift of  $\phi$ , and because  $\phi(F, \pi) > 0$  for any  $\pi \in [\pi^f, \pi^l]$ , a solution to the left of  $\underline{V}$  always exists for this profit range but no solution to the right of  $\underline{V}$  exists (see Figure 1 (Panel B)). Similarly, when  $\pi \in [\pi^l, \bar{\pi})$  the solution  $V_{low}$  still exists, but now a new solution to the right of  $\underline{V}$ , denoted by  $V_{high}$ , also exists (see  $\phi(V, \pi')$  in Figure 1 (Panel B)). When  $\pi = \bar{\pi}$ ,  $\underline{V}$  is the only solution. Finally, when  $\pi > \bar{\pi}$ , no solution exists.

Since  $\phi$  may never cross the horizontal axis for a sufficiently high profit level, we henceforth assume that  $\pi^m \leq \bar{\pi}$  in order to guarantee the existence of  $V_i(\mathbf{p})$ . To see why this is sufficient, observe that  $\pi^m$  is the maximum profit flow that can be attained in a symmetric equilibrium. Therefore  $\pi^m \leq \bar{\pi}$  guarantees that  $V_i(\mathbf{p})$  is well defined for any relevant vector of symmetric prices.<sup>10</sup>

The two solutions of  $\phi$  have different qualitative features. For instance, while  $V_{low}$  is increasing in  $\pi$ , the solution  $V_{high}$  is decreasing. By implicitly differentiating  $\phi(V, \pi) = 0$  with respect  $V_j$  we can define the ratio

$$R(\mathbf{p}) \equiv \frac{dV_i}{dV_j} = \frac{f'(L - V(\mathbf{p}))(V(\mathbf{p}) - F)}{r + 2f(L - V(\mathbf{p}))} > 0. \quad (6)$$

$R(\mathbf{p})$  captures how a solution  $V_i(\mathbf{p})$  changes with an increase in the opponents'

<sup>10</sup>Note, however, that  $\pi^m \leq \bar{\pi}$  is a sufficient condition. It is possible to find model parameters for which the unique equilibrium price vector under price announcements  $\mathbf{p}^*$  satisfies  $\pi(\mathbf{p}^*) < \bar{\pi}$ —so that  $V(\mathbf{p}^*)$  is well defined—but  $\bar{\pi} < \pi^m$ .

value. As we can observe, firms' values are complements, since an increase in firm  $j$ 's value increases firm  $i$ 's value by  $R(\mathbf{p})$ .  $R(\mathbf{p})$  will play an important role when we characterize the equilibrium of the price game, as it measures how increasing firm  $j$ 's value benefits firm  $i$  (i.e., the payoff pass-through of increasing the rival's value).

**Lemma 4.** *Every solution  $V_{low}$  is lower than any solution  $V_{high}$ . In any solution  $V_{low}$ ,  $R(\mathbf{p}) \in (0, 1)$ . In any solution  $V_{high}$ ,  $R(\mathbf{p}) > 1$ .  $V_{low}(\pi)$  is increasing in  $\pi$ , while  $V_{high}(\pi)$  is decreasing.*

Because every solution  $V_{low}$  is lower than any solution  $V_{high}$ , and because larger  $V$ 's are associated with lower R&D investments levels (see equation (4)), we refer to a  $V_{low}$  solution as part of a *low deterrence* equilibrium. Similarly, we refer to  $V_{high}$  solutions as part of a *high deterrence* equilibrium. Recall from Lemma 3 that the solution  $V_{high}$  only exists when  $\pi > \pi^l$ . Since in equilibrium  $\pi(\mathbf{p}) \leq \pi^m$ —i.e., only profit flows less than the monopoly profits are attainable—we know that  $V_{high}$  solutions are not feasible when the innovation is radical ( $\pi^m \leq \pi^l$ ). That is, a high deterrence equilibrium may only exist for incremental innovations ( $\pi^m > \pi^l$ ).

**Lemma 5.** *Under radical innovations (i.e.,  $\pi^l \geq \pi^m$ ) only the  $V_{low}$  solution is a candidate for equilibrium. Under incremental innovations (i.e.,  $\pi^m > \pi^l$ ), both solutions are candidates.*

## 4.2 Market Prices

After characterizing the possible value function solutions for a given price vector  $\mathbf{p}$ , we now proceed to look for equilibrium prices. Because the  $V_{low}$  solution is defined for all profit levels, we start by showing the existence of a low deterrence equilibrium. We leave the discussion of high deterrence equilibria to the end of this section.

Given beliefs about  $p_j$ , each firm  $i$  chooses its prices in the first stage of the period by solving  $\max_{p_i} V_i(\mathbf{p})$ .<sup>11</sup> Then, firm  $i$ 's first order condition is equivalent to

$$\frac{dV_i(\mathbf{p})}{dp_i} = 0 \Leftrightarrow \frac{d\pi_i(\mathbf{p})}{dp_i} + \underbrace{\frac{dx_j}{dp_i}(F - V_i(\mathbf{p}))}_{\text{R\&D Deterrence Effect} > 0} = \frac{d\pi_i(\mathbf{p})}{dp_i} + R(\mathbf{p}) \frac{d\pi_j(\mathbf{p})}{dp_i} = 0 \quad (7)$$

<sup>11</sup>Since equation (3) does not depend on  $t$ , and as long as no firm has successfully innovated, the firms would make the same price announcement regardless of the timing of the announcement.

(see equation (14) in the Appendix for the full expression of  $dV_i(\mathbf{p})/dp_i$ ).

Condition (7) tells us that when a firm chooses its price it considers the impact of its price on its own profit,  $\pi_i$ , as well as on the expected loss that results when its rival successfully innovates,  $x_j(F - V_i(\mathbf{p}))$ . Recall that with the arrival of an innovation, the unsuccessful firm becomes less profitable,  $F < V_i(\mathbf{p})$ , which incentivizes each firm to reduce the likelihood of a successful innovation by its rival. Firms achieve this by using price announcements to deter their rivals' R&D investments, which we call this the *R&D deterrence effect of price*. Price announcements are effective deterrents because firms invest in R&D according to the incremental value of the innovation (i.e.,  $\kappa'(x_i) = L - V_i(\mathbf{p})$ ), and prices directly impact the incremental value of the innovation through the product market profit (see Lemma 4).

We show that the R&D deterrence effect is positive, which results in an upward pressure on prices solely driven by deterrence motives. The sign can be more easily seen when using implicit differentiation to rewrite the effect as  $R(\mathbf{p})d\pi_j/dp_i$ . The term  $R(\mathbf{p}) = dV_i/dV_j > 0$  represents how an increase in  $V_j$  impacts  $V_i$  (see Lemma 4).  $d\pi_j/dp_i > 0$  captures the price complementarities between the substitute goods; i.e., how much firm  $j$  benefits from an increase in  $p_i$ . The economics behind the positive R&D deterrence effect are that a higher price increases the rival's profit flow, which in turn increases the rival's pre-innovation values ( $V$ ), and thus reduces the rival's incremental value of the innovation ( $L - V$ ). That is, firms face incentives to set higher prices to make their rivals complacent and less driven to innovate.

To go further, evaluate the derivative of  $V_i(\mathbf{p})$  with respect to  $p_i$  at the equilibrium price without price announcements,  $p^s$ . Since  $\partial\pi_i(\mathbf{p}^s)/\partial p_i = 0$  in the equilibrium without price announcements and the R&D deterrence effect is positive, equation (7) is not satisfied at  $p^s$ , and  $dV_i(\mathbf{p}^s)/dp_i > 0$ .<sup>12</sup> Similarly, evaluating  $dV_i(\mathbf{p})/dp_i$  at the monopoly price  $p^m$  we obtain

$$\frac{d\pi_i(\mathbf{p}^m)}{dp_i} + R(\mathbf{p}^m)\frac{d\pi_j(\mathbf{p}^m)}{dp_i} < \frac{d\pi_i(\mathbf{p}^m)}{dp_i} + \frac{d\pi_j(\mathbf{p}^m)}{dp_i} = 0.$$

where the inequality follows from  $d\pi_j(\mathbf{p})/dp_i > 0$  and  $R(\mathbf{p}) \in (0, 1)$  in any low deterrence equilibrium (see Lemma 4). In words, the first derivative of  $V_i$  with respect to  $p_i$  is negative at the monopoly price. Therefore, by the intermediate

<sup>12</sup>For notational ease, we use  $\mathbf{p}^s$  to denote the vector  $(p^s, p^s)$ . Similarly for  $\mathbf{p}^m$  below.

value theorem, there exists a price  $p_{low}^a \in (p^s, p^m)$  such that equation (7) holds.

Existence and uniqueness of a low deterrence equilibrium is discussed in the following proposition. In the proposition, we provide a sufficient (although not necessary) condition for equilibrium uniqueness based on the function,

$$\Psi(p) = \frac{\partial^2 \pi_i(\mathbf{p})}{\partial p_i^2} + \max \left\{ 0, \frac{\partial^2 \pi_j(\mathbf{p})}{\partial p_i^2} \right\} - \Lambda \frac{\partial \pi_i(\mathbf{p})}{\partial p_i} \frac{\partial \pi_j(\mathbf{p})}{\partial p_i}, \quad (8)$$

where  $\mathbf{p} = (p, p)$ ,

$$\Lambda = \frac{1}{K^2} \left( \frac{f'(L - \underline{V})^2 - f''(L - \underline{V})K}{f'(L - \underline{V})} \right) > 0,$$

$K = r + 2f(L - \underline{V}) > 0$ , and  $f(z) = z^{\frac{1}{\gamma-1}}$ ,  $\gamma$  is the coefficient on the cost function  $\kappa(x)$ , and  $\underline{V}$  is defined in Lemma 2. Assuming that  $\Psi(p) < 0$  for all  $p \in (p^s, p^m)$  is sufficient to guarantee that there exists a unique low deterrence equilibrium. Below we provide examples with linear and logit demand functions where this condition is satisfied (see Table 1).

**Proposition 2** (Low Deterrence Equilibrium). *Assume  $\Psi(p) < 0$  for all  $p \in (p^s, p^m)$ . There exists a unique low deterrence symmetric Markov perfect equilibrium,  $(p_{low}^a, x_{low}^a, V_{low}^a)$ . In this equilibrium, firms deter their rivals' R&D ( $x_{low}^a < x^{na}$ ) by announcing higher prices ( $p_{low}^a \in (p^{na}, p^m)$ ) and they earn greater profits ( $V_{low}^a > V^{na}$ ) relative to the case without price announcements.*

As in the rest of the literature on predation, firms are willing to sacrifice current market profits in order to deter the rival's investments. We can see this in the first order condition in equation (7). Because the R&D deterrence effect is positive, we have that  $\partial \pi_i(\mathbf{p}_{low}^a)/\partial p_i < 0$ . That is, given the rival's price, firm  $i$  would be better off by decreasing its price if it wanted to maximize its short-run profit. However, because of the dynamic benefits of deterring its rival's R&D, firms are willing to make this sacrifice.

With respect to the magnitude of  $V_{low}^a$ , equation (7) implies that  $V_{low}^a < L$  must hold in any equilibrium. To see this, recall that firms choose their R&D investments according to the incremental value of the innovation,  $L - V$  (see equation (4)). If  $V \geq L$ , firms choose not to invest in R&D, as the innovation destroys (rather than creates) value. For this reason, at any vector of prices  $\mathbf{p}$  such that  $V \geq L$  we have that  $dx_j/dp_i = 0$  (i.e., firm  $j$ 's investment in R&D cannot be less than 0). When

$dx_j/dp_i = 0$ , equation (7) can only be satisfied at  $\mathbf{p}^s = (p^s, p^s)$ . However, at  $\mathbf{p}^s$  we know that  $V^{na} \equiv V(\mathbf{p}^s) < L$  (see Proposition 1), which contradicts the premise that  $V \geq L$ , and proves that  $V \geq L$  cannot be part of an equilibrium with price announcements.

We next focus on the high deterrence equilibrium. Following similar arguments as above and because in a high deterrence solution we have that  $R(\mathbf{p}) > 1$  (see Lemma 4), the derivative of  $V_i(\mathbf{p})$  with respect  $p_i$  (see equation (7)) is non-zero for any price between  $p^s$  and  $p^m$ . Therefore, if an equilibrium exists, we must have that  $p_{high}^a > p^m$ , where  $p_{high}^a$  is the market price in a high deterrence equilibrium. Observe, however, that an equilibrium may not exist because the high deterrence solution  $V_{high}(\mathbf{p})$  is defined for symmetric price vectors such that the corresponding profit flow is in the interval  $[\pi^l, \pi^m]$  (see Lemma 3). Depending on the parameters of the model, the vector  $\mathbf{p}$  that solves equation (7) may or may not satisfy this restriction. Properties of a high deterrence equilibrium are discussed in the following proposition.

**Proposition 3** (High Deterrence Equilibrium). *When innovations are incremental (i.e.,  $\pi^l < \pi^m$ ), there may exist a high deterrence equilibrium,  $(p_{high}^a, x_{high}^a, V_{high}^a)$ . In this equilibrium, firms announce higher prices than the monopoly price ( $p_{high}^a > p^m$ ), deter their rivals' R&D ( $x_{high}^a < x_{low}^a$ ) by more and earn greater profits ( $V_{high}^a > V_{low}^a$ ) relative to the low deterrence equilibrium.*

From the discussion above, we note that there is an interesting link between the problem of a firm choosing its price announcement and the problem of a multiproduct monopolist. Equation (7) shows that firms choosing their price announcements (at least partially) internalize the effect of their price on the profits of their rivals, although for reasons that differ from those of a multiproduct monopolist. In the case of a multiproduct monopolist, the firm fully internalizes the price externality because the monopolist wishes to maximize the joint profit. In the case of a firm choosing its price announcement, the firm does not capture the joint profit but it still internalizes the price externality because of the benefits of deterring its rival's R&D activity. With respect to the magnitude of the equilibrium price relative to the multiproduct monopoly price, we know that  $R(\mathbf{p}) \in (0, 1)$  and  $R(\mathbf{p}) > 1$  in any low and high deterrence equilibrium, respectively (see Lemma 4). That is, in all low (high) deterrence equilibria the firm puts more (less) weight on its own profit than on the profit of the rival. This leads to equilibrium prices that are less (greater) than the monopoly prices in all low (high) deterrence equilibria.

	Panel A: Linear Demand			Panel B: Logit Demand		
	I	II	III	I	II	III
Demand	$q_i = \frac{1 - 2p_i + p_j}{3}$			$q_i = \frac{\exp\{-p_i\}}{1 + \exp\{-p_i\} + \exp\{-p_j\}}$		
$\beta$	0.9	0.75	0.4	0.88	0.8	0.5
Other parameters	$c=0.2, r = 0.05, \kappa(x) = x^2/2$			$c=0.13, r = 0.03, \kappa(x) = x^2/2$		
Innovation type	Increm.	Increm.	Rad.	Increm.	Increm.	Rad.
Existence high eq.	Yes	No	-	Yes	No	-
Uniqueness low eq.	Yes	Yes	Yes	Yes	Yes	Yes
$p^s$	0.4667	0.4667	0.4667	1.3379	1.3379	1.3379
$p^m$	0.6	0.6	0.6	1.5532	1.5532	1.5532
$p_{low}^a$	0.5015	0.4894	0.4897	1.4119	1.3896	1.3883
$p_{high}^a$	0.6641	-	-	1.5961	-	-
$pace^{na}$	0.2391	0.5973	1.4803	0.1481	0.2430	0.6022
$pace_{low}^a$	0.1836	0.5824	1.4739	0.1123	0.2276	0.5957
$pace_{high}^a$	0.0613	-	-	0.0403	-	-

Table 1: R&D Deterrence Effect: Numerical Examples

Note: An innovation is incremental (Increm.) or radical (Rad.) when  $\pi^m > \pi^l$  and  $\pi^m \leq \pi^l$ , respectively. Existence high eq. indicates whether a high deterrence equilibrium exists. Uniqueness low eq. indicates whether the condition for low deterrence equilibrium uniqueness in [Proposition 2](#) is satisfied.  $p^s, pace^{na}$  are the equilibrium outcomes under no price announcements, where  $pace$  is defined as  $2x$ .  $p_{low}^a, pace_{low}^a$  and  $p_{high}^a, pace_{high}^a$  are the equilibrium outcomes with price announcements in a low and high deterrence equilibrium, respectively.

To illustrate our results and the existence of multiple equilibria with different levels of R&D deterrence, we present a series of examples in [Table 1](#). In Panels A and B of [Table 1](#), we present examples with linear and logit demand functions, respectively. In both panels we assume that the R&D cost function is  $\kappa(x) = x^2/2$ , and we keep the demand function, the marginal cost ( $c$ ), and the discount rate ( $r$ ) fixed throughout the examples in each panel. The magnitude of the innovation ( $\beta$ ) is the only parameter that varies across examples. Each panel has the same taxonomy. Columns I and II present examples with incremental innovations ( $\pi^m > \pi^l$ ). From [Lemma 5](#) we know that a high deterrence equilibrium may exist whenever the innovation is incremental, however, we find that a high deterrence equilibrium only exists in Column I of each panel (i.e., a high deterrence equilibrium is not guaranteed to exist when the innovation is incremental). Column III presents an example with a radical innovation ( $\pi^m \leq \pi^l$ ), where we know from [Lemma 5](#) that only a low deterrence equilibrium may exist. In all of the examples we have that  $\Psi(p) < 0$  for all  $p \in (p^s, p^m)$  (see [Proposition 2](#)), implying a unique low deterrence equilibrium.

### 4.3 Consumer Welfare

Propositions 2 and 3 imply that consumers face both higher prices in the pre-innovation period and a lower pace of innovation when firms make price announcements (i.e.,  $p_{low}^a, p_{high}^a > p^s$  and  $x_{low}^a, x_{high}^a < x^{na}$ ). A lower pace of innovation implies that the innovation—and its negative impact on prices,  $p^l < p^f < p^s < p^a$  (see Lemma 1)—will on average reach the market later in time. These observations combined imply that consumers face weakly greater prices throughout the industry lifetime, and thus earn less consumer surplus when firms make price announcements.

To see this more formally, define  $cs(\mathbf{p})$  to be the consumer surplus flow at prices  $\mathbf{p}$ . We assume that  $cs(\mathbf{p})$  is strictly decreasing in each dimension of  $\mathbf{p}$ , capturing that higher prices lead to a lower consumer surplus flow. Define the expected discounted consumer surplus in the market as a function of pre-innovation prices  $\mathbf{p}$  and innovation pace  $\lambda$  by

$$CS(\mathbf{p}, \lambda) = \frac{1}{r} \left( \frac{rcs(\mathbf{p}) + \lambda cs(p^l, p^f)}{r + \lambda} \right), \quad (9)$$

where  $cs(\mathbf{p})$  is the consumer surplus flow at the vector of pre-innovation prices  $\mathbf{p}$ ,  $cs(p^l, p^f)$  is the consumer surplus flow after the innovation reaches the market, and  $\lambda = 2x$  is the pace of innovation in the pre-innovation period (see Appendix D for details).

It is not hard to verify that  $CS(\mathbf{p}, \lambda)$  is decreasing in the pre-innovation prices  $\mathbf{p}$ . Also, observe that  $CS(\mathbf{p}, \lambda)$  equals the value of a perpetuity that pays consumers a convex combination of  $cs(\mathbf{p})$  and  $cs(p^l, p^f)$ , where the weight on  $cs(p^l, p^f)$  increases as a function of the pace of innovation  $\lambda$ . Therefore, for any vector  $\mathbf{p}$  such that  $cs(\mathbf{p}) < cs(p^l, p^f)$ ,  $CS(\mathbf{p}, \lambda)$  is increasing in  $\lambda$ . These two properties imply that in any equilibrium with price announcements we have that  $CS(\mathbf{p}^s, \lambda^{na}) > CS(\mathbf{p}^a, \lambda^a)$ .

**Proposition 4** (R&D Deterrence Harms Consumers). *Consumer welfare with price announcements is less than consumer welfare without price announcements.*

### 4.4 Innovation Uncertainty and Commitment

Innovation uncertainty plays an important role in understanding why firms do not face a commitment problem in announcing R&D-detering prices. Since firms do not know when the innovation will arrive and the stochastic process is memo-



ryless, there is no terminal period in the pre-innovation phase of the game that would trigger deviations and make the pricing game unravel to the static price equilibrium in every period. Unless some firm successfully innovates, the firms will continue earning the default profit flows that correspond to their pre-innovation price choices, which is why the pre-innovation prices impact the incremental value of the innovation (and thus pricing incentives).

Compare this with the case where the arrival time of the innovation is certain. With arrival time certainty, the post-innovation values are the only relevant value functions for firms when choosing their R&D investments, as firms exit the pre-innovation phase of the game with certainty. The incremental value of the innovation is then the benefit of inventing the innovation minus the value of not inventing the product (as opposed to the value of inventing the innovation minus the value of remaining in the race). Since the pre-innovation prices have no impact on the values of inventing the product or not, pre innovation prices play no role in the incremental value of the innovation, and equilibrium prices with innovation certainty therefore simply unravel to the equilibrium prices of the static price game.

## 5 Properties of the R&D Deterrence Effect

In this section, we study two aspects of the R&D deterrence effect. First, how the R&D deterrence effect varies with the degree of product differentiation. Second, whether the higher prices caused by the R&D deterrence effect imply that consumers would be better off if firms did not engage in R&D (i.e., consumer surplus dissipation). We henceforth focus on the low deterrence equilibrium and drop the *low* subindex that corresponds to low deterrence equilibrium objects.

To answer these questions we parameterize the demand system. Consumers derive utility from the products sold by each firm as well as from a numeraire good. Consumers maximize  $U(q_1, q_2) + m$  subject to  $I = m + p_1q_1 + p_2q_2$  where  $q_i$  and  $p_i$  are the quantity and price of good  $i$ ,  $m$  is the numeraire good (with a price normalized to one), and  $U(q_1, q_2) = q_1 + q_2 - (q_1^2 + \sigma q_1q_2 + q_2^2)/2$ .<sup>13</sup> The demand for good  $i$  is therefore given by

$$q_i = \frac{1}{1 + \sigma} - \frac{1}{1 - \sigma^2}p_i + \frac{\sigma}{1 - \sigma^2}p_j, \quad (10)$$

---

<sup>13</sup> $U(q_1, q_2)$  is quadratic and strictly concave.

where we restrict  $\sigma$  to be in the unit interval to capture the case of substitute goods.

As discussed in [Singh and Vives \(1984\)](#), this demand system can capture various degrees of product differentiation. The two extreme cases of this system are the case of independent goods when  $\sigma = 0$  (full differentiation) and the case of homogeneous goods when  $\sigma \rightarrow 1$  (no differentiation). Henceforth, we use  $\sigma$  as our measure of product differentiation.

Lastly, we use the indirect utility function to measure how price changes impact welfare. Because income does not affect the demand for goods 1 and 2, the compensating variation is equivalent to changes in consumer surplus.

## 5.1 Product Substitution

In this subsection we analyze how the degree of product differentiation impacts the magnitude of the R&D deterrence effect ( $p^a - p^s$ ). Using equations (7) and (10), we can write the equilibrium markup as

$$p^a - c = \frac{(1 - \sigma^2)q(\mathbf{p}^a)}{1 - \sigma R(\mathbf{p}^a)}. \quad (11)$$

Similarly, we can write the equilibrium markup under no price announcements as  $p^s - c = (1 - \sigma^2)q(\mathbf{p}^s)$ .

Equation (11) captures how the degree of product differentiation affects the R&D deterrence effect. Under full product differentiation, represented by  $\sigma = 0$ , firms do not have incentives to deter their rivals' R&D investments, as rival prices do not impact their profits. Similarly, as  $\sigma$  approaches one, the degree of product differentiation vanishes and the goods become homogeneous. In this case, the incentives to undercut the rival in the product market outweigh the R&D deterrence effect and the price markup goes to zero. In both of these extreme cases, equation (11) equals the equilibrium markup under no price announcements, implying that the R&D deterrence effect vanishes. Lastly, equation (11) captures the upward pricing pressure caused by the R&D deterrence effect that exists for intermediate values of product differentiation  $\sigma \in (0, 1)$ . Since  $R(\mathbf{p}) < 1$  holds in any low deterrence equilibrium, the denominator in (11) is strictly less than one, implying that  $p^a > p^s$ .

These observations combined lead us to conclude that the R&D deterrence effect is non-monotonic in the degree of product differentiation. The following

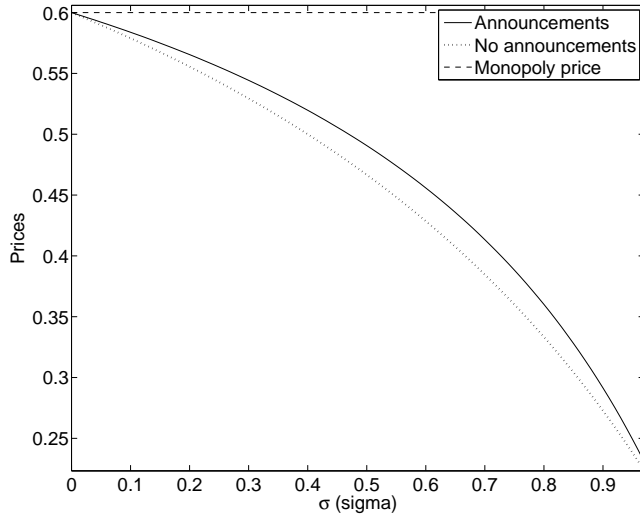


Figure 2: R&D Deterrence Effect and Product Differentiation

Note: The figure shows the equilibrium prices with price announcements ( $p^a$ ), without price announcements ( $p^s$ ), and the multiproduct monopoly price for the following parameters:  $\kappa(x) = x^2/2$ ,  $r = 0.02$ ,  $\beta = 0.65$ , and  $c = 0.2$ ,  $\sigma \in [0, 0.97]$ . The sufficiency condition for equilibrium uniqueness,  $\Psi(p) < 0$  for  $p \in [p^s, p^m]$ , is satisfied for every value of sigma.

proposition summarizes these results, and Figure 2 illustrates them with a numerical example.

**Proposition 5** (The Magnitude of the R&D Deterrence Effect). *The R&D deterrence effect is non-monotonic in the degree of product differentiation ( $\sigma$ ). The effect is positive for  $\sigma \in (0, 1)$ , and it vanishes both when products are independent ( $\sigma = 0$ ) and when products are homogeneous ( $\sigma \rightarrow 1$ ).*

## 5.2 Consumer Benefits of Innovation

From Proposition 2 and Proposition 3 we know that the prices faced by consumers in the period before a firm successfully innovates ( $p^a$ ) are higher than the equilibrium prices when firms maximize static product market profits ( $p^s$ ). We can also establish that consumers directly benefit when the innovation reaches the market, as prices fall:  $p^l < p^f < p^s < p^a$  (see Lemma 1). Motivated by these opposing welfare effects, we ask whether the R&D deterrence effect ( $p^a > p^s$ ) outweighs the positive impact of innovation on consumer welfare ( $p^l, p^f < p^s$ ). That is, whether consumers would be better off if the firms did not perform R&D and the innovation never reached the market.

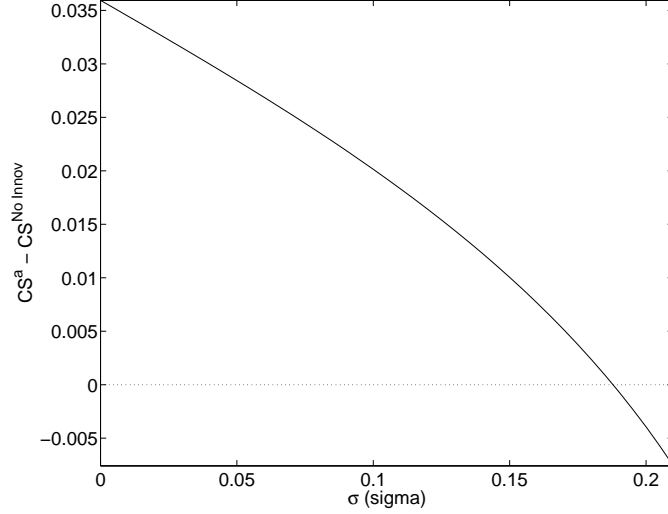


Figure 3: R&D Deterrence Effect and Consumer Benefits of Innovation

Note: The figure shows  $\Delta CS = CS^a - CS^{\text{No Innov}}$  for the following parameters:  $\kappa(x) = x^2/2$ ,  $r = 0.04$ ,  $\beta = 0.9$ , and  $c = 0.08$ ,  $\sigma \in [0, 0.24]$ . The sufficiency condition for equilibrium uniqueness,  $\Psi(p) < 0$  for  $p \in [p^s, p^m]$ , is satisfied for every value of sigma.

To answer this question, we compare the expected discounted consumer surplus under price announcements  $CS(\mathbf{p}^a, \lambda^a)$  (see equation (9)) to the consumer surplus that exists with no R&D competition whatsoever; i.e.,  $CS^{\text{No Innov}} = cs(\mathbf{p}^s)/r$ . The difference between  $CS(\mathbf{p}^a, \lambda^a)$  and  $CS^{\text{No Innov}}$  can be written as

$$\Delta CS = \frac{1}{r(r + \lambda)} \left( \underbrace{r(cs(\mathbf{p}^a) - cs(\mathbf{p}^s))}_{<0} + \lambda \underbrace{(cs(p^l, p^f) - cs(\mathbf{p}^s))}_{>0} \right).$$

The first term in the parentheses is negative because of the R&D deterrence effect ( $p^a > p^{na}$ ), and captures the loss in welfare due to higher prices during the period when the firms are still performing R&D. The second term in the parentheses is positive because consumers face lower prices once the innovation reaches the market ( $p^l, p^f < p^a$ ). Depending on the relative magnitude of each of these terms, consumers may be worse off when firms perform R&D.

Figure 3 shows that depending on the parameters of the model, consumers may benefit or lose with R&D competition. For some parameters, the R&D deterrence effect fully dissipates the utility gains of innovation (e.g.,  $\sigma = 0.2$ ). While for other parameters, consumers are still better off with innovation competition despite

the R&D deterrence effect (e.g.,  $\sigma = 0.1$ ). Regardless, we conclude that the R&D deterrence effect is of first order in understanding the consumer benefits of innovation.

**Result 1.** *The R&D deterrence effect of price may dissipate the consumer benefits of innovation.*

## 6 Extensions

In this section, we extend the model to show that the R&D deterrence effect exists more generally. We first consider the case when there are  $n$  firms in the industry (as opposed to a duopoly). We then analyze the case of asymmetric firms competing to develop the innovation. Lastly, we discuss the case of sequential innovations.

### 6.1 $n$ Symmetric Competitors

Consider a industry served by  $n$  symmetric firms. As before, let  $\pi^l$  and  $\pi^f$  represent the profit flow obtained by the innovating firm and the non-successful followers after the innovation occurred. Likewise, let  $L = \pi^l/r$  and  $F = \pi^f/r$  represent the present value of being the technology leader and follower after the innovation arrives.

$V_i(\mathbf{p})$  represents the value of firm  $i$  at time  $t$  before any firm has successfully innovated as a function of the vector of Markov strategies  $\mathbf{p}$ ,

$$V_i(\mathbf{p}) = \max_{x_i} \frac{\pi_i(\mathbf{p}) + x_i L + \sum_{j \neq i} x_j F - \kappa(x_i)}{r + \sum_k x_k},$$

where  $V_i(\mathbf{p})$  now captures that an invention by any of the  $n - 1$  rivals makes firm  $i$  become a follower. Firm  $i$ 's first order condition with respect to the R&D investment is given by equation (4).

Firm  $i$  chooses its price announcements by maximizing  $V_i(\mathbf{p})$  given beliefs about its rival's strategies. The first order condition with respect to price is equivalent to

$$\frac{dV_i}{dp_i} = 0 \Leftrightarrow \underbrace{\frac{d\pi_i(\mathbf{p})}{dp_i} - \sum_{j \neq i} \frac{dx_j}{dp_i} (V_i(\mathbf{p}) - F)}_{\text{R\&D Deterrence Effect}} = \frac{d\pi_i(\mathbf{p})}{dp_i} + \underbrace{\sum_{j \neq i} R(\mathbf{p}) \frac{d\pi_j}{dp_i}}_{> 0} = 0, \quad (12)$$

where

$$R(\mathbf{p}) = \frac{f'(L - V(\mathbf{p}))(V(\mathbf{p}) - F)}{r + nf(L - V(\mathbf{p})) - (n - 2)f'(L - V(\mathbf{p}))(V(\mathbf{p}) - F)} > 0.$$

As before, firm  $i$  takes into consideration how its price will impact its product market profit as well as the R&D investment of each of its rivals. It is possible to show that  $R(\mathbf{p}) \in (0, 1)$  in any low deterrence equilibrium. Thus, the existence of a low deterrence equilibrium with prices satisfying  $p_{low}^a \in (p^s, p^m)$  follows from analogous arguments to those presented in [Proposition 2](#).

## 6.2 Asymmetric Firms

Consider an asymmetric duopoly where firm 1 is investing in R&D to increase its cost advantage (i.e., lower its marginal cost from  $\beta c$  to  $\beta^2 c$ ), and firm 2 is investing to match firm 1's marginal cost (i.e., lower its marginal cost from  $c$  to  $\beta c$ ). Once one of the two firms succeeds, the industry reaches maturity and the firms no longer invest in R&D. The firms' post-innovation values differ depending on which firm successfully innovates. If firm  $i$  innovates, the values are given by  $L_i$  and  $F_i$ , with  $L_1 > L_2 = F_2 > F_1$ .<sup>14</sup>

Let  $V_i(\mathbf{p})$  represent the value of firm  $i$  at time  $t$  before any firm has successfully innovated as a function of the firms' Markov strategies,

$$V_i(\mathbf{p}) = \max_{x_i} \frac{\pi_i(\mathbf{p}) + x_i L_i + x_j F_j - \kappa(x_i)}{r + x_i + x_j},$$

where  $\pi_1(\mathbf{p}) = (p_1 - \beta c)q_1(\mathbf{p})$  and  $\pi_2(\mathbf{p}) = (p_2 - c)q_2(\mathbf{p})$ . Firm  $i$ 's first order condition with respect to the R&D investment is given by  $\kappa'(x_i) = L_i - V_i(\mathbf{p})$ .

The price announcements are chosen by maximizing  $V_i(\mathbf{p})$  given beliefs about  $p_j$ , and must satisfy

$$\frac{dV_i}{dp_i} = 0 \Leftrightarrow \frac{d\pi_i(\mathbf{p})}{dp_i} - \underbrace{\frac{dx_j}{dp_i}(V_i(\mathbf{p}) - F_i)}_{\text{R\&D Deterrence Effect}} = \frac{d\pi_i(\mathbf{p})}{dp_i} + \underbrace{R_i(\mathbf{p}) \frac{dp_j}{dp_i}}_{> 0} = 0, \quad (13)$$

with

$$R_i(\mathbf{p}) = \frac{f'(L - V_j(\mathbf{p}))(V_i(\mathbf{p}) - F)}{r + f(L - V_i(\mathbf{p})) + f(L - V_j(\mathbf{p}))} > 0.$$

<sup>14</sup>These inequalities follow from arguments analogous to those in [Lemma 1](#).

Arguments analogous to those in [Proposition 2](#) establish existence of a low deterrence equilibrium with the properties discussed in [Section 4](#).

### 6.3 Sequential Innovations

We next use the analysis above to argue that the existence of the R&D deterrence effect extends to scenarios where firms compete to develop a sequence of innovations. Consider the case where two symmetric firms compete to develop a sequence of two innovations.<sup>15</sup> Each innovation reduces a firm's marginal cost by a factor of  $\beta$ . Once the two innovations have been invented, the industry reaches maturity, and the firms no longer invest in R&D.

The analysis of the game after the first innovation has been invented is equivalent to the analysis in [Section 6.2](#), where we studied the asymmetric case where one of the firms has a cost advantage of magnitude  $\beta$ . The analysis of the game before the first innovation has been invented is equivalent to the symmetric analysis in [Section 2](#) when replacing  $L$  and  $F$  for  $V_1$  and  $V_2$ , respectively, and where  $V_1$  and  $V_2$  are the pre-innovation equilibrium values in [Section 6.2](#). Because the R&D deterrence effect exists in both of these versions of the model, it follows that the R&D deterrence effect and its properties extend to the case of a sequence of innovations.

## 7 Concluding Remarks

We show that price announcements can be used to manipulate rivals' incentives to innovate by inducing complacency. In equilibrium, firms set prices that are higher than the prices when price announcements cannot be used strategically, and these higher prices lead to lower innovation rates. Firms sacrifice short-run profits to deter their rivals' R&D investments, but earn greater discounted profits in the equilibrium with price announcements. These results provide an explanation for the use of price announcements in media events by tech firms and suggest that the measurement of the welfare benefits of innovation will be biased unless the strategic price effects associated to R&D deterrence are considered.

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<sup>15</sup>The argument provided here extends to the case of  $k$  innovations. We consider the case of two innovations for ease of exposition.

## References

- ARROW, K. (1962). Economic welfare and the allocation of resources for invention. In *The Rate and Direction of Inventive Activity: Economic and Social Factors*, NBER Chapters, National Bureau of Economic Research, Inc, pp. 609–626.
- BESANKO, D., DORASZELSKI, U. and KRYUKOV, Y. (2014). The economics of predation: What drives pricing when there is learning-by-doing? *The American Economic Review*, **104** (3), 868–897.
- CHEN, Z. and ROSS, T. W. (2000). Strategic alliances, shared facilities, and entry deterrence. *The RAND Journal of Economics*, pp. 326–344.
- CHRISTENSEN, C. (2013). *The innovator’s dilemma: when new technologies cause great firms to fail*. Harvard Business Review Press.
- DIXIT, A. (1980). The role of investment in entry-deterrence. *The Economic Journal*, **90** (357), 95–106.
- ELLISON, G. and ELLISON, S. F. (2011). Strategic entry deterrence and the behavior of pharmaceutical incumbents prior to patent expiration. *American Economic Journal: Microeconomics*, **3** (1), 1–36.
- GALLINI, N. T. (1984). Deterrence by market sharing: A strategic incentive for licensing. *The American Economic Review*, **74** (5), 931–941.
- GILBERT, R. J. and NEWBERY, D. M. (1982). Preemptive patenting and the persistence of monopoly. *The American Economic Review*, **72** (3), 514–526.
- GOOLSBEE, A. and SYVERSON, C. (2008). How do incumbents respond to the threat of entry? evidence from the major airlines. *The Quarterly Journal of Economics*, **123** (4), 1611–1633.
- HARRINGTON, J. E. (1986). Limit pricing when the potential entrant is uncertain of its cost function. *Econometrica*, **54** (2), 429–437.
- IGAMI, M. (2017). Estimating the innovators dilemma: Structural analysis of creative destruction in the hard disk drive industry, 1981–1998. *Journal of Political Economy*, **125** (3), 798–847.



- KREPS, D. M. and WILSON, R. (1982). Reputation and imperfect information. *Journal of Economic Theory*, **27** (2), 253–279.
- LEE, T. and WILDE, L. L. (1980). Market structure and innovation: A reformulation. *The Quarterly Journal of Economics*, **94** (2), pp. 429–436.
- LOURY, G. C. (1979). Market structure and innovation. *The Quarterly Journal of Economics*, **93** (3), pp. 395–410.
- MARSHALL, G. and PARRA, A. (2018). Innovation and competition: The role of the product market. *Working Paper, University of Illinois at Urbana-Champaign*.
- MILGROM, P. and ROBERTS, J. (1982). Predation, reputation, and entry deterrence. *Journal of Economic Theory*, **27** (2), 280–312.
- REINGANUM, J. F. (1982). A dynamic game of R and D: Patent protection and competitive behavior. *Econometrica*, **50** (3), pp. 671–688.
- SINGH, N. and VIVES, X. (1984). Price and quantity competition in a differentiated duopoly. *The RAND Journal of Economics*, **15** (4), 546–554.
- SPENCE, A. M. (1977). Entry, capacity, investment and oligopolistic pricing. *The Bell Journal of Economics*, **8** (2), 534–544.

# Appendix

## A. Omitted Proofs

**Proof of Lemma 1.** Using the envelope theorem and the demand regularity conditions (A), (B), and (D) (see section C of the Appendix) we find:

$$\frac{dp_i}{dc_i} = -\frac{-\frac{dq_i}{dp_i}}{2\frac{dq_i}{dp_i} + \frac{d^2q_i}{dp_i^2}(p_i - c)} \in (0, 1), \quad \frac{dp_j}{dp_i} = -\frac{\frac{dq_j}{dp_i} + \frac{d^2q_j}{dp_i dp_j}(p_j - c)}{2\frac{dq_j}{dp_j} + \frac{d^2q_j}{dp_j^2}(p_j - c)} \in (0, 1)$$

Since  $\frac{dp_j}{dc_i} = \frac{dp_j}{dp_i} \frac{dp_i}{dc_i} < \frac{dp_i}{dc_i}$ , prices are lower when the cost-saving innovation arrives. In particular,  $p^l < p^f < p^s$ . Similarly for the profits we find:

$$\frac{d\pi_j}{dc_i} = \frac{dq_j}{dp_i} \frac{dp_i}{dc_i} (p_j - c) > 0, \quad \frac{d\pi_i}{dc_i} = -q_i + \frac{dq_i}{dp_j} \frac{dp_j}{dp_i} \frac{dp_i}{dc_i} (p_i - c)$$

Given the results above, it is easy to see that  $d\pi_j/dc_i$  is positive. For  $d\pi_i/dc_i$ , observe that at prices satisfying the first order condition can be rewritten as:

$$\frac{d\pi_i}{dc_i} = \left( \frac{dq_i}{dp_i} + \frac{dq_i}{dp_j} \frac{dp_j}{dp_i} \frac{dp_i}{dc_i} \right) (p_i - c) < \left( \frac{dq_i}{dp_i} + \frac{dq_i}{dp_j} \right) (p_i - c) < 0,$$

where we use  $dp_j/dp_i$  and  $dp_i/dc_i \in (0, 1)$  for the first inequality, and demand regularity condition (C) for the second. We conclude that  $\pi^f < \pi^s < \pi^l$ . ■

**Proof of Lemma 2.** (i) and (ii): Observe that  $\phi(F, \pi) = \pi - \pi^f + f(L - F)(L - F) - \kappa(f(L - F))$ . Due to the convexity of  $\kappa$  and  $\kappa(0) = 0$ , we have that  $x\kappa'(x) > \kappa(x)$  for all  $x > 0$ . Equation (4) implies  $\kappa'(f(L - F)) = (L - F)$ . Replacing back, we obtain  $\phi(F, \pi) = \pi - \pi^f + x\kappa'(x) - \kappa(x)$ . Thus,  $\pi > \pi^f$  is sufficient for  $\phi(F, \pi) > 0$ . Similarly,  $\phi(L, \pi) = \pi - \pi^l$ . Thus  $\phi(L, \pi) < 0$  whenever  $\pi < \pi^l$  and  $\phi(L, \pi) = 0$  when  $\pi = \pi^l$ .

(iii) Differentiating  $\phi$  with respect to  $V$  we obtain:  $\phi'(V, \pi) = -r - 2f(L - V) + f'(L - V)(V - F)$ ; and  $\phi''(V, \pi) = 3f'(L - V) - f''(L - V)(V - F)$ , which is positive for  $V \in (F, L)$  as  $f'(x) > 0$  and  $f''(x) \leq 0$  (see Lemma 6). Then,  $\phi'(F, \pi) = -r - 2f(L - F) < 0$ . On the other hand,  $\lim_{V \rightarrow L} \phi'(V, \pi) = -r + (L - F) \lim_{V \rightarrow L} f'(L - V)$ . Here we have two cases (see Lemma 6), if  $\gamma > 2$  then  $\lim_{V \rightarrow L} f'(L - V) = \infty$  and  $\phi'(L, \pi) > 0$ . If  $\gamma = 2$ , then  $\phi'(L, \pi) = -r + (L - F)$  and  $\phi'(L, \pi) > 0$  whenever  $L - F > r$ .<sup>16</sup>

(iv) Observe that  $\phi'(V, \pi)$  is independent of  $\pi$ . Also, since  $\phi'(F, \pi) < 0$  and  $\phi'(L, \pi) > 0$ , the intermediate value theorem implies that there exists a  $\underline{V} \in (F, L)$  such that  $\phi'(\underline{V}, \pi) = 0$ . Finally, because  $\phi'' > 0$ ,  $\underline{V}$  is unique and corresponds to a minimum.<sup>17</sup> ■

**Proof of Lemma 3.** Since  $\phi(\underline{V}, \bar{\pi}) = 0$  and  $\phi''(V, \pi) > 0$ , there is a unique solution when  $\pi = \bar{\pi}$ . Because  $\phi(\underline{V}, \pi) < 0$  for  $\pi < \bar{\pi}$ , and  $\phi(F, \pi) > 0$  for  $\pi \geq \pi^f$ , the

<sup>16</sup>This implies that  $\phi'(L, \pi) < 0$  (and consequently  $\phi'(V, \pi) < 0$  for all  $V \in [F, L]$ ) when  $L - F < r$ . Because our results still apply in this scenario, and since in all relevant economic applications we normally have  $L - F > r$ , we chose not to overwhelm the reader with technical details in the main text and present the arguments for this case in footnotes 17 and 18.

<sup>17</sup>In the  $L - F < r$  and  $\gamma = 2$  scenario,  $\underline{V} = L$  and  $\bar{\pi} = \pi^l$ .

intermediate value theorem implies that there exists a solution to the left of  $\underline{V}$  when  $\pi \in [\pi^f, \bar{\pi}]$ . Because  $\phi$  is monotone in  $V \in [F, \underline{V}]$ , the solution to the left is unique. Similarly, because of the monotonicity of  $\phi$  to the right of  $\underline{V}$ , there is no solution larger than  $\underline{V}$  when  $\phi(L, \pi) < 0$  (i.e.,  $\pi < \pi^l$ ) and a unique solution when  $\phi(L, \pi) \geq 0$  (i.e.,  $\pi \geq \pi^l$ ).<sup>18</sup> ■

**Proof of Lemma 4.** From the proof of Lemma 3 we know that for any  $\pi < \bar{\pi}$ ,  $V_{low} < \underline{V} < V_{high}$ , which proves the first claim. For the second claim simply observe that  $\phi'(V, \pi) < 0$  (or  $\phi'(V, \pi) = 0$  or  $\phi'(V, \pi) > 0$ ) is equivalent to  $R(\mathbf{p}) < 1$  (or  $R(\mathbf{p}) = 1$  or  $R(\mathbf{p}) > 1$ , respectively) after manipulating the inequality and using the definition of  $R(\mathbf{p})$ . Since  $R(\mathbf{p}) > 0$ ,  $\phi'(\underline{V}, \pi) = 0$  and  $\phi''(V, \pi) > 0$ , it follows that  $R(\mathbf{p}) \in (0, 1)$  at  $V_{low}$  and  $R(\mathbf{p}) > 1$  at  $V_{high}$ . For the last statement, at any solution  $V^a \in \{V_{low}, V_{high}\}$  of  $\phi(V, \pi)$ , observe that  $dV^a/d\pi = -\phi'(V^a, \pi)^{-1}$ . The result follows. ■

**Proof of Lemma 5.** See argument in the text. ■

**Proof of Proposition 1.** The first order condition for price in (2) and the demand regularity conditions imply that  $\mathbf{p}^s = (p^s, p^s)$  is the unique equilibrium price vector and  $\pi^s = \pi_i(\mathbf{p}^s)$  is the equilibrium profit earned by the firms. Using the condition (2) for R&D investments, we can construct  $\phi(V, \pi^s)$  which, by Lemma 3 single crosses zero at a value  $V^{na} \in (F, L)$  since  $\pi^s \in (\pi^f, \pi^l)$  (see Lemma 1). Lastly, strict convexity of  $\kappa(x)$  guarantees that  $x^{na}$  is the unique solution to equation (2) given  $V^{na}$  and  $L$ . ■

**Proof of Proposition 2.** The existence of the function  $V_{low}(\mathbf{p})$  follows from Lemmas 2 and 3. The existence of  $p_{low}^a \in (p^s, p^m)$  follows from Lemma 4 and the application of the intermediate value theorem, as discussed in the text. For completeness and to better show the differences between a *high* and *low* deterrence equilibrium, we present a complete derivation of the first order conditions. Then, we prove that  $p_{low}^a$  is indeed an equilibrium by showing that the second order condition holds at all solutions of the first order condition (hence, the equilibrium is unique). Then, we show that  $\pi(\mathbf{p}_{low}^a) > \pi^s$  which, by Lemma 4, immediately implies  $V_{low}^a > V^{na}$  and  $x_{low}^a < x^{na}$ .

*First order condition:* Using implicit differentiation, the derivative of  $V_i$  with respect to  $p_i$  is

$$\frac{dV_i}{dp_i} = \frac{\frac{d\pi_i}{dp_i} + f'(L - V_j) \frac{dV_j}{dp_i} (V_i - F)}{r + f(L - V_j) + f(L - V_i)}, \quad \text{where} \quad \frac{dV_j}{dp_i} = \frac{\frac{d\pi_j}{dp_i} + f'(L - V_i) \frac{dV_i}{dp_i} (V_j - F)}{r + f(L - V_j) + f(L - V_i)}.$$

Observe that at the optimum  $dV_i/dp_i = 0$ . Using this condition in  $dV_j/dp_i$  and replacing back into  $dV_i/dp_i$  delivers equation (7). More generally (i.e., not necessarily at the optimum), we obtain

$$\frac{dV_i}{dp_i} = \frac{1}{K} \frac{\frac{d\pi_i}{dp_i} + R_i \frac{d\pi_j}{dp_i}}{(1 - R_i R_j)} \quad (14)$$

where  $K = r + f(L - V_i) + f(L - V_j) > 0$  and  $R_i = f'(L - V_j)(V_i - F)/K > 0$ . The expression above captures how in a *high* deterrence equilibrium the derivative of  $V_i$  with

<sup>18</sup> In the  $L - F < r$  and  $\gamma = 2$  scenario,  $\phi(V, \pi)$  is strictly decreasing for all relevant  $V$ . Since  $\phi(L, \pi^l) = 0$ , this means that  $\phi(V, \pi)$  single-crosses zero to the left of  $\underline{V} = L$  whenever  $\pi < \pi^l$ , and it never crosses zero when  $\pi > \pi^l$ . That is, only low deterrence equilibria exist.

respect to  $p_i$  has the opposite sign of the same derivative in a *low* deterrence equilibrium, since  $R_i > 1$  for  $i \in \{1, 2\}$  in any high deterrence equilibrium.

*Second order condition:* Differentiating (14) with respect to  $p_i$  and using that in equilibrium  $-d\pi_i/dp_i = R_i d\pi_j/dp_i$ , we obtain:

$$\frac{d^2 V_i}{dp_i^2} = \frac{\frac{d^2 \pi_i}{dp_i^2} + R_i \frac{d^2 \pi_j}{dp_i^2} - \frac{1}{K^2} \left( \frac{f'(L-V_j)^2 - f''(L-V_j)K}{f'(L-V_j)} \right) \frac{d\pi_i}{dp_i} \frac{d\pi_j}{dp_i}}{K(1 - R_i R_j)}. \quad (15)$$

The denominator is positive in a low deterrence equilibrium whereas it is negative in a high deterrence equilibrium (Lemma 4). We show that  $\Psi(p) < 0$  for every  $p \in (p^s, p^m)$  implies that at every price where (7) is satisfied, an upper-bound of (15) is negative.

Because the denominator is positive, we ignore it to determine the sign of  $d^2 V_i/dp_i^2$ . We know that  $d^2 \pi_i/dp_i^2 < 0$  because of demand regularity condition (A), which guarantees uniqueness of the static oligopoly game. To bound the second term from above, we use  $\max\{0, d^2 \pi_j/dp_i^2\}$  (if  $d^2 \pi_j/dp_i^2 > 0$  take  $R_i = 1$ , if  $d^2 \pi_j/dp_i^2 < 0$  take  $R_i = 0$ ). Before bounding the third (and last) term, we show that it is positive (i.e., the negative 1 multiplies a negative term).  $d\pi_i/dp_i < 0$  because every price that satisfies (7) is greater than  $p^s$ , and we have that  $d\pi_i(\mathbf{p}^s)/dp_i = 0$  and  $d^2 \pi_i/dp_i^2 < 0$ .  $d\pi_j/dp_i > 0$  because  $dq_j/dp_i > 0$ . Finally the term in parenthesis is positive by Lemma 6. We bound the term in parenthesis using the parametric specification of the cost function,  $\kappa(x) = x^\gamma/\gamma$ , which implies  $f(z) = z^{1/(\gamma-1)}$ , and

$$\frac{1}{K^2} \left( \frac{f'(L-V)^2 - f''(L-V)K}{f'(L-V)} \right) = \frac{(\gamma-1)^{-1}}{(r+2f(L-V))^2} \left( \frac{1}{(L-V)^{\frac{\gamma-2}{\gamma-1}}} + \frac{\gamma-2}{L-V} \right).$$

The expression above is increasing in  $V$ . Because every low deterrence equilibrium satisfies  $V_{low} \leq \underline{V}$ , we use  $\underline{V}$  to bound the expression above. Thus,  $d^2 V_i/dp_i^2 < \Psi(p_{low}^a)/(K(1 - R_i R_j)) < 0$ , where the second inequality follows from the assumption that  $\Psi(p) < 0$  for all  $p \in (p^s, p^m)$ .  $d^2 V_i/dp_i^2 < 0$  implies that  $p_{low}^a$  is a local maximum, and because this is true for every  $p$  satisfying (7), there is no local minimum satisfying the first order condition. Hence, the equilibrium is unique.

*Proof of  $\pi(\mathbf{p}_{low}^a) > \pi^s$ :* The solution to the problem of a firm that controls the price of all products in the case of symmetric demand functions (i.e.,  $\max_p \pi(p, p)$ ) is given by the multiproduct monopoly price,  $p^m$ . This implies that  $\pi(p, p)$  is increasing in  $p$  until the price reaches the monopoly price,  $p^m$ . Because  $p^s < p_{low}^a < p^m$ , the result follows. ■

**Proof of Proposition 3.** High deterrence equilibria are shown in Table 1. For  $p_{high}^a > p^m$  (and therefore  $p_{high}^a > p_{low}^a$ ) see the discussion in the text. By Lemma 1, we know  $V_{high}^a$  (if it exists) is larger than  $V_{low}^a$ . This, in conjunction with equation (4), implies  $x_{high}^a < x_{low}^a$ . Finally, to check that a price satisfying equation (7) is indeed a high deterrence equilibrium, we need to check the second order condition (15) (we do so numerically in the examples in the table). Since  $R_i > 1$ , the denominator is negative, thus the numerator has to be positive. Examples suggest show that it is possible to simultaneously satisfy  $\Psi(p) < 0$  for  $p \in (p^s, p^m)$  and have the numerator of (15) be positive. ■

**Proof of Proposition 4.** See argument in the text. ■

**Proof of Proposition 5.** See argument following equation (11) in the main text. ■

## B. Auxiliary Results

**Lemma 6.** *The function  $f(z)$  implicitly defined by  $\kappa'(f(z)) = z$  satisfies  $f(0) = 0$  and is increasing ( $f'(z) > 0$  for all  $z > 0$ ). When  $\gamma > 2$ ,  $\lim_{z \rightarrow 0} f'(z) = \infty$ . When  $\gamma = 2$ ,  $f(z) = 1$  for all  $z \geq 0$ . When  $\kappa'''(x) > 0$ ,  $f(z)$  is concave  $f''(z) < 0$  for all  $z > 0$ , and when  $\kappa'''(x) = 0$ ,  $f''(z) = 0$  for all  $z > 0$ .*

**Proof.** The first statement follows from  $\kappa'(0) = 0$ . The second follows from the derivative of  $f(z)$  being equal to  $f'(z) = 1/\kappa''(f(z))$  and the fact that  $\kappa(x)$  is strictly convex (i.e.,  $\kappa''(x) > 0$ ). The limiting result follows from  $\kappa''(0) = 0$  when  $\gamma > 2$  and  $\kappa''(0)$  is a positive constant when  $\gamma = 2$ . Similarly,  $f''(z) = -\kappa'''(f(z))/(\kappa''(f(z)))^3$  and the results follows from the value of  $\kappa'''(x)$ . ■

## C. Demand Regularity Conditions

We assume that the following conditions hold for every symmetric price vector such that the first order condition  $q_i + dq_i/dp_i (p_i - c) = 0$  is satisfied.

$$-\frac{dq_i}{dp_i} > \frac{d^2q_i}{dp_i^2}(p_i - c), \quad (\text{A}) \quad \frac{dq_i}{dp_j} > -\frac{d^2q_i}{dp_j dp_i}(p_i - c), \quad (\text{B}) \quad -\frac{dq_i}{dp_i} > \frac{dq_i}{dp_j}, \quad (\text{C})$$

$$-\frac{dq_i}{dp_i} > \frac{dq_i}{dp_j} + \left( \frac{d^2q_i}{dp_i^2} + \frac{d^2q_i}{dp_j dp_i} \right) (p_i - c). \quad (\text{D})$$

(C) requires that own price effects are larger than those of the opponent. (A) guarantees both the second order condition for the oligopolist firm problem, and that  $dp_i/dc_i < 1$ . Finally, (B) and (D) are required for  $j$  to not over react to an  $i$ 's price change; i.e.,  $dp_j/dp_i \in (0, 1)$ .

To determine the conditions for when the multiproduct monopolist's problem has a unique solution, define the function:  $F(p_1, p_2) \equiv \pi_1(p_1, p_2) + \pi_2(p_1, p_2)$ . Let  $F_i$  represent the derivative of  $F$  with respect to  $p_i$ . Uniqueness is guaranteed if  $F_{i,i} < 0$  and  $F_{1,1}F_{2,2} - (F_{1,2})^2 > 0$  at any price vector such that  $F_i = 0$ .

## D. Discounted Expected Consumer Surplus

The discounted expected consumer surplus at time  $t$ , before the innovation has arrived, and given a vector of price announcements,  $\mathbf{p}$ , is given by

$$CS(\mathbf{p}) = \int_t^\infty e^{-(r+\lambda)(s-t)} \left( cs(\mathbf{p}) + \lambda cs(p^l, p^f)/r \right) ds = \frac{rcs(\mathbf{p}) + \lambda cs(p^l, p^f)}{r(r+\lambda)},$$

where  $cs(\mathbf{p})$  is the consumer surplus flow at prices  $\mathbf{p}$ ,  $cs(p^l, p^f)/r$  is the discounted consumer surplus after the innovation arrives,  $\lambda$  is the pace of innovation, and  $r$  is the discount rate. The interpretation of  $CS(\mathbf{p})$  is similar to that of equation (1).