We combine theoretical, computational and experimental methods to gain deeper insight into the non-linear behavior of materials and structures.

**Non-linear response of materials**

- **Dielectrics**
- **Bio-hybrid materials**
- **Fibronectin nanotextiles**

Henann, Chester, Bertoldi, JMPS 2013

Shim, Grosberg, Nawroth, Parker, Bertoldi, J Biomechanics, 2012

Deravi, Su, Paten, Ruberti, Bertoldi, Parker. NanoLetters, 2012

**Non-linear response of structures**

Deformation

Deformation
Adaptive Materials

Periodic structures with deliberately designed patterns

Dramatic geometric rearrangements induced by instabilities

Shim et al., PNAS, 2012

..... can we exploit deformation and instabilities to tune and control the propagation of elastic waves?
Large deformation & Instabilities

……traditionally we want to avoid them

……but they can be exploited to
•control adhesion
•facilitate flexible electronics
•fabricate micro-fluidic structures
•control surface wettability

Chan et al., Adv. Mat, 2008

Rogers et al., Science, 2010

Chung et al., Soft Matter, 2007
Highly non-linear response

- Initial linear elastic behavior with a sudden departure from linearity to a plateau stress
- Completely homogeneous pattern transformation corresponds to the plateau region
- The transformed pattern is accentuated with continuing deformation
- The critical triggering stress level scales consistently with ligament buckling

Mullin et al., PRL, 2007
Bertoldi et al., JMPS, 2008
Analysis of instabilities: Global and Local modes

• Macroscopic (global) instabilities with wavelengths much larger than the characteristic size of the microstructure. They can be computed from the loss of strong ellipticity of the corresponding homogenized properties (Geymonat et al., 1993).

• Microscopic (local) instabilities with finite wavelengths. They alter the periodicity of the solid, but are investigated on the primitive cell through a Bloch wave analysis. Instability → wave of vanishing frequency

Bloch wave analysis provides:
• point along the loading path where instability occurs
• the new periodicity of the structure ($p_1$, $p_2$).

We are interested in triggering microscopic (local) instabilities
Arrangement of Holes


Two buckled structures have a chiral pattern!
Pore shape

Holes with four-fold symmetry. Fourier series expansion to describe their contour as

\[ x_1(\theta) = r(\theta)\cos(\theta) \]
\[ y_1(\theta) = r(\theta)\sin(\theta) \]
\[ r(\theta) = r_0\left[1 + c_1 \cos(4\theta) + c_2 \cos(8\theta)\right], \]

Here **macroscopic** instabilities are critical.

Here **microscopic** instabilities are critical.

Overvelde JTB, Shan S., Bertoldi K, Advanced Materials, 2012
Overvelde JTB, Bertoldi K, JMPS, 2014
Loading direction

Simulations Experiments

A

B

C

\( \varepsilon_{\text{Area}} = -0.24 \)

\( 5\text{mm} \)
Applications:

Can we exploit the pattern transformation induced by buckling to design a new class of adaptive materials and devices?

- materials with unusual properties
- color displays
- **phononic switches/ tunable phononic crystals**
- formation of complex pattern
Manipulating elastic waves

**Phononic Crystals**

**Locally Resonant Metamaterials**

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**Question:** Can we exploit deformation and instabilities to manipulate the propagation of elastic waves?
We exploit large deformation and buckling in phononic crystals to tune the band gaps of the structures.

**Simulation steps** (Finite Element Method):
- Buckling analysis
- Postbuckling analysis (finite deformations, periodic boundary conditions)
- Wave propagations (small amplitude waves. Perturbation step)
Tunable Phononic Band-gaps

Effect of pattern transformation on phononic band gaps.
Tunable Phononic Band-gaps

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Effect of pattern transformation on phononic band gaps.
Tunable Phononic Band-gaps

Evolution of the phononic band gaps as a function of the applied strain.

*Initial linear response:* Band gaps affected marginally by deformation, evolving in an affine and monotonic way

*Pattern transformation:* The in-plane modes undergo a transformation and 2 new band gaps are opened

Geometric non-linearities play a major role
Phononic crystals are characterized by a low frequency directional behavior that can be exploited to steer or redirect waves in specific directions.
Simulations (Finite Element Method):

- Buckling analysis
- Postbuckling analysis (finite deformations, periodic boundary conditions)
- Wave propagations (small amplitude waves. Perturbation step)

\[ \varepsilon_{xx} = -0.24 \]
\[ \varepsilon_{yy} = -0.14 \]
\[ \varepsilon_{xx} = \varepsilon_{yy} = -0.14 \]
\[ \varepsilon_{yy} = -0.24 \]
Experiments

- Computer
- Amplifier
- Accelerometer
- Shaker
- Data Acquisition Module
- Sample
Experiments
Experiments

Instabilities and large deformation can be effectively used to manipulate the propagation of elastic waves in periodic structures.
Tunable band-gaps in locally resonant metamaterials

Core (metal)  Coating (rubber)

Matrix (epoxy)

(Liu et al., Science, 2000)

Our design: structural coating

Structural coating:
- Use elastic and highly deformable beams
- Use buckling to significantly alter the beam stiffness and – in turns – the frequency of the bandgap
Our design

Undeformed

Structural coating: array of beams

Deformed

20 mm

A₀

A

Core

Matrix
Buckling and large deformation significantly alter the stiffness of the beams.
Dynamic response: undeformed configuration

FEM: Undeformed configuration

Flat band $\rightarrow$ Local mode
Experimental results: Undeformed configuration

Undeformed configuration

![Graph showing experimental results with frequency (Hz) on the y-axis and reduced wave vector, k, on the x-axis. The graph includes multiple curves and a shaded area representing 50 mm.]
Dynamic response: deformed configuration

Effect of deformation:

Because of the softening of the two vertical beams induced by buckling, the bangap frequency decreases.

Because of the increase in tangential stiffness induced by buckling, the rotational mode rises.

The bandgap is completely suppressed → phononic switch.
Experimental results
Experimental results

No strain

4% strain

4% strain

Transmission (dB)

Frequency (Hz)
Experimental results
Experimental results

No strain

10% strain

Frequency (Hz)

Transmission (dB)
Experimental results

No strain
2% strain
4% strain
6% strain

50 mm

50 mm

Frequency (Hz)

Transmission (dB)

Bandgap Frequency (Hz)

Experiment
FEM Simulation

Strain (%)

0 5 10
Structured Spherical Shells

2D Continuum Structure

3D Continuum Structure

Hoberman Twist-O

Continuum Version?
Continuum structure

……from “rigid” Hoberman Twist-O

- Opportunities for reversible encapsulation
- Large number of hinges required
- Challenging fabrication at the micro and nano scale

……to Buckliball: a continuum shell that uses buckling to expand/contract

- No hinges
- Actuation mechanism works over a wide range of scales

Design parameters:

<table>
<thead>
<tr>
<th>Arrangement of holes</th>
<th>Volume of Void</th>
<th>Shell Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volume of Sphere</td>
<td>Sphere Inner Radius</td>
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</table>

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## Arrangement of holes


**Folded sphere:**
1) spherical shape
2) all holes with the same shape
3) equally distributed holes
4) closed holes

**Polyhedra**
1) convex polyhedra
2) vertex-transitive
3) regular face
4) quadrilateral vertex figure

<table>
<thead>
<tr>
<th>N</th>
<th>6</th>
<th>12</th>
<th>24</th>
<th>30</th>
<th>60</th>
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<td><img src="image7" alt="Image" /></td>
<td><img src="image8" alt="Image" /></td>
<td><img src="image9" alt="Image" /></td>
<td><img src="image10" alt="Image" /></td>
</tr>
</tbody>
</table>
24 holes: Shell thickness and Void volume fraction

Finite Element Simulations: Buckling analysis

Out-of-sphere Buckling

On-sphere Buckling
Experiment: Buckliball

\[ R_{in} = 22.5 \, mm \]

\[ t_{shell} = 5.0 \, mm \]

\[ \bar{h}_{membrane} = 0.4 \pm 0.1 \, mm \]

Porosity = 60%
**Goal:** design a new class of 3-D materials whose architecture can be dramatically changed in response to an external stimulus

**Idea:** We use elastomeric Buckliballs as building blocks for 3-D reconfigurable structures.
Building blocks: Buckliballs
Packing: cubic lattice systems (sc, bcc, fcc)

→ 6 bucklicrystals
Experiments

Building block (6H)

\[
\tau = \frac{t}{R_i} = \frac{7.1 \text{ mm}}{9.9 \text{ mm}} = 0.71
\]

\[
\psi = 0.73
\]

Bucklicrystal

91 building blocks

Cube with edge length = 141.3 mm
Can we exploit the large deformation induced by buckling to tune the propagation of waves both in the matrix and in air?
Harnessing fluid-structure interactions

Casadei F., Bertoldi K.  Journal of Applied Physics, 2014
Airfoil Resonating Unit

The resonating unit is modeled as a rigid airfoil with two degrees of freedom

\[ m_a \ddot{h} + m_a b x_\theta \dot{\theta}(t) + k_h h = \ell(t), \]

\[ m_a b x_\theta \ddot{h}(t) + I_\alpha \dot{\theta}(t) + k_\theta \theta(t) = m(t) + b \left( \frac{1}{2} + a_f \right) \ell(t), \]

A finite-state induced flow theory is used to model the unsteady aerodynamic loads on the airfoil (Peters et al. 1995)

\[ \ell(t) = \pi \rho_\infty b^2 \left( \dot{h}(t) + V_\infty \dot{\theta}(t) - ba\ddot{\theta}(t) \right) + 2\pi \rho_\infty sV_\infty b \left[ \dot{h}(t) + V_\infty \theta(t) + b \left( \frac{1}{2} - a \right) \dot{\theta}(t) - \frac{1}{2} b^T \lambda(t) \right], \]

\[ m(t) = -\pi \rho_\infty s b^3 \left[ \frac{1}{2} \ddot{h}(t) + V_\infty \dot{\theta}(t) + b \left( \frac{1}{8} - \frac{a}{2} \right) \ddot{\theta}(t) \right], \]

with \[ A \dot{\lambda}(t) + \frac{V_\infty}{b} \lambda = c \left[ \dot{h}(t) + V_\infty \dot{\theta}(t) + b \left( \frac{1}{2} - a \right) \dot{\theta}(t) \right]. \]
Aeroelastic response of the Flaps
Dispersion Relations of the Beam

A transfer matrix approach is used to compute the dispersion relations of a beam with periodic airfoil-type resonators.

\[
\left[ K_a + i\omega C_a - \omega^2 M_a \right] z(\omega) = f(\omega)
\]

\[
\begin{pmatrix}
z_r \\
f_r
\end{pmatrix} = T(\omega, V_\infty) \begin{pmatrix}
z_l \\
f_l
\end{pmatrix} = e^{i\mu(\omega)} \begin{pmatrix}
z_l \\
f_l
\end{pmatrix}
\]

Complex propagation constant
Numerical Results

$V_\infty = 10 \text{ m/s}$

$V_\infty = 12 \text{ m/s}$

$V_\infty = 14 \text{ m/s}$

$V_\infty = 2 \text{ m/s}$

$V_\infty = 4 \text{ m/s}$

$V_\infty = 0 \text{ m/s}$

Heave mode

$L_b = 50 \text{ mm}$

$s = 45 \text{ mm}$

Pitch mode

Heave mode

$L_b = 95 \text{ mm}$

$s = 45 \text{ mm}$

Pitch mode
Experimental Setup

Aluminum beam (L=360mm, w=25.4mm, t=1.27mm)

- Measure the transmission coefficient of a beam with six airfoil-type resonators
- Repeat measurements at different wind speeds
Experimental Results

\[ V_\infty = 14 \text{ m/s} \]

FEM

\[ V_\infty = 0 \text{ m/s} \]

Experiments

\[ L_t = 50 \text{ mm} \]

\[ L_t = 95 \text{ mm} \]
Conclusions

Non linear response of structures: Exciting playground due to the interplay of geometry and large deformation – buckling

Large deformation / instabilities and other stimuli (such as flow speed) can be exploited to design material with novel and tunable properties
Thanks

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