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# RAMSEY OPTIMAL TWO PART TARIFFS: THE CASE OF MANY HETEROGENEOUS GROUPS*

by

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## I. INTRODUCTION

This paper examines efficient pricing for a public enterprise that is subject to a minimum profit constraint and that is able to set different two part tariffs for different, internally heterogeneous consumer groups. Ng and Weisser [1974] (subsequently referred to as NW) and Schmalensee [1981], among others, consider efficient two part tariffs for the single group case. Public enterprises are, however, usually able to segment consumers into different groups and charge different fee schedules to different groups. For example, segmentation according to location is a common practise for telephone companies and power authorities.

Many aspects of efficient pricing carry over directly to the multi-group case, but interesting new issues associated with cross subsidization arise. In addition, the new dimension concerning how the price and license fee (or access) parts of the two part tariff vary over consuming groups is of interest. Since the actual efficiency gains of moving from (for example) simple average cost pricing to two-part tariffs are generally small (as a manifestation of the "iron law of deadweight loss"), while implicit transfers involved in price discrimination between groups may be large, it seems worthwhile to explicitly consider multi-group pricing schedules and the extent of cross subsidization.

The efficient price structure will be subsidy free for certain classes of cost function including the simple case of a fixed cost plus constant marginal cost. In other cases efficiency involves cross subsidization, particularly if marginal cost is decreasing at the solution. Three expressions are derived relating prices and license fees to various elasticities. One expression shows that percentage markups conform to a two stage rule, the first stage of which is the standard Ramsey rule. Rules are also derived for the case in which discrimination is possible only over the license fee.

In addition to analyzing these cross subsidy and discrimination[1] questions, we generalize the main result of NW concerning the sign of the excess price (price minus marginal cost) to the many group case. The generalization is not immediate because a proposition concerning the extent to which cross subsidy can occur is central to the generalized result. An efficient price below marginal cost, while ruled out by the strong monotonicity assumption used in much of the literature that demand curves of different consumers do not cross, is an important possibility with the weaker monotonicity assumption used by NW and with the formulation used here.

This paper is related to a very large literature on public pricing. The two themes we draw from are Ramsey pricing, which grew out of Ramsey's [1927] tax problem and which includes Boiteux [1956, 1971], Dreze [1964], Baumol and Bradford [1970], and Hartwick [1978], and the two part tariff and non-uniform pricing literature. Aside from NW and Schmalensee, this literature includes Feldstein [1972], Littlechild [1975], and Leland and Meyer [1976], Faulhaber and Panzar [1977], Spence [1977, 1980], Roberts [1979], and Mirman and Sibley [1980], among others.

Section II sets out the model and its basic properties, Section III concerns cross subsidy, Section IV contains results on the signs of excess prices and license fees, and reports some expressions relating prices and license fees to various elasticities, including the special case of a discriminating flat rate. Section V contains some concluding remarks.

## II. THE MODEL

There are $n$ groups of consumers, indexed by letter $i$. Within each group consumers vary only according to a single characteristic $h_i$. Consumers in group $i$ have (identical) utility functions of the form

$$(1) \qquad U^i = u_i(x;h_i) + v$$

where $x$ is consumption of the publicly produced good and $v$ is income spent on other goods.[2] The value of $x$ chosen by a consumer in group $i$ depends on that consumer's value of $h_i$. The assumption that each group is heterogeneous is important. With homogeneous groups the two part tariff achieves the first best result with all prices equal to marginal cost and with license fees to cover any deficit.

The two part tariff facing group $i$ is denoted $(P^i, L^i)$ where $P^i$ is the per unit price and $L^i$ is the license fee which must be paid before a consumer is allowed to purchase the publicly produced good. For $L^i = 0$, maximization of (1) subject to the consumer's budget constraint yields an ordinary demand function $x_i = f^i(P^i, h_i)$, which is assumed differentiable in $h_i$ and $P^i$ with a strictly negative slope $f^i_p$. (At $P^i = 0$, $f^i_p$ is the right hand derivative). Define

(2) $\quad s^i(P^i, L^i, h_i) \equiv \int_{P^i}^{\infty} f^i(p, h_i)dp - L^i$

With utility function (1) there are no income effects on $x_i$ so the license fee affects only the decision of whether or not to consume. A consumer chooses to pay the license fee and consume if (and only if) $s^i \geqslant 0$ so demand is $f^i$ if $s^i \geqslant 0$ and $0$ if $s^i < 0$.

Marginal consumers are those for whom $s^i = 0$. We make the monotonicity assumption that $s^i$ is strictly increasing in $h_i$ provided $f^i > 0$. The attribute $h_i$ is assumed to have continuous and strictly positive density $m(h_i)$ defined for $h_i \epsilon [0,1]$. If a marginal consumer exists and if $L^i > 0$, then $x_i > 0$ and from monotonicity there exists a unique $h_i^*(P^i, L^i) \epsilon [0,1]$ which satisfies $s^i(P^i, L^i, h_i) = 0$. The associated demand of a marginal consumer is $x_i^* = f^i(P^i, h_i^*)$, which is strictly positive, and unique given $(p^i, L^i)$.[3] Although this monotonocity assumption is restrictive, it does allow demand curves to cross.[4]

There are values of $P^i$ and $L^i$ for which $s^i$ is positive even for $h^i = 0$, in which case all consumers in group $i$ would be inframarginal. We refer to consumers who pay the license fee as "members". If there are marginal consumers, the number of members, $M$, in group $i$ is

(3) $\quad M^i(P^i, L^i) = \int_{h_i^*(P^i,L^i)}^{1} m_i(h)dh$

$M^i$ may be discontinuous at $P^i = 0$ or $L^i = 0$ since a negative $P^i$ or $L^i$ will induce full membership, but the derivative is taken to be the (well-defined) right hand derivative. Using subscripts to denote derivatives, it follows that if there are marginal consumers, $M_P^i = -m_i(h_i)\partial h_i^* / \partial P^i < 0$ and similarly $M_L^i < 0$. If all individuals in group $i$ are inframarginal then $M^i$ is equal to the integral over $[0,1]$ and $M_P^i = M_L^i = 0$.

Total demand forthcoming from group $i$ is $X^i(P^i, L^i)$, which is the integral of demand over the inframarginal members given $(P^i, L^i)$. The derivative with respect to price can be decomposed as follows.

(4) $\quad X_P^i = D_P^i + x_i^* M_P^i$

where $D_P^i$ is the partial derivative of $X^i$ holding membership constant.

The structure described so far is similar to that in several papers dealing with the single group case. For some purposes, generalization to many groups merely requires the use of superscripts to denote different groups. Accordingly, several very useful single group properties carry over directly. The following result is available in Schmalensee [1981] (for the single group case).

*A. Lemma 1*

(5) $\quad M_P^i = x_i^* M_L^i = X_L^i$

The second equality in (5) is intuitively clear: the fall in demand associated with a license fee increase is just the effect on membership times the amount consumed by a marginal consumer who is leaving. The first equality is less obvious but reflects the relationship between license fee and price changes. In particular, a price increase of $1/x^*$ reduces the surplus of the marginal consumer by the same amount as an increase of $1$ in $L$.

Total surplus for group $i$ is

$$S^i(P^i, L^i) = \int_{h_i^*}^{1} s^i(P^i, L^i, h_i) m_i(h) dh \qquad (6)$$

if there are marginal consumers, and is equal to the integral over $[0,1]$ if all consumers in group $i$ are inframarginal. Since $x$ is chosen by maximizing consumers, differentiation of (6) produces (by Roy's identity) the results

$$S^i_P = -X^i; \; S^i_L = -M^i \qquad (7)$$

The second expression in (7) can be understood from the observation that marginal consumers have no surplus net of the license fee so the only loss in surplus as $L$ increases infinitesimally is the extra license fee paid by continuing members.

Using $\pi$ to denote profit and using the obvious vector notation

$$\pi(P, L) = P \cdot X + L \cdot M - C(Y(P, L))$$

where $Y = \Sigma_{i=1}^n X^i$ and $C(Y)$ is the cost function. ($C'$ is marginal cost). Efficient pricing rules are found by maximizing the sum of producer and consumer surplus subject to the constraint that profits be nonnegative. The Lagrangian function, $H$, is

$$H(P, L, \lambda) = S(P, L) + (1 + \lambda)\pi(P, L)$$

where $S = \Sigma_{i=1}^n S^i$.

Assuming that a maximum exists, the solution can be characterized by setting the partial derivatives of $H$ equal to zero if it is differentiable there. However, as mentioned, $M^i$ will normally be discontinuous at $L^i = 0$ and $P^i = 0$. Similarly $X^i$ will be discontinuous at $P^i = 0$. Consequently, before writing down the first order conditions we note the following lemma (which also follows directly from Schmalensee [1981] or Leland and Meyer [1976]).

*B. Lemma 2*

At the efficient solution $P^i \geqslant 0$. Also, if $\lambda > 0$, then $L^i > 0$.

Efficient solutions are characterized by the following first order conditions for $\lambda \geqslant 0$, $P^i \geqslant 0$, and $L^i \geqslant 0$.

$$H^i_P = [(P^i - C')X^i_P + L^i M^i_P](1 + \lambda) + \lambda X^i = 0 \text{ for } P^i > 0 \; (\leqslant 0 \text{ if } P^i = 0) \qquad (8)$$

(9) $H^i_L = [(P^i - C')X^i_L + L^i M^i_L](1 + \lambda) + \lambda M^i = 0$

(10) $H^i_\lambda = \pi(P, L) = 0$ if $\lambda > 0$ ($\geqslant 0$ if $\lambda = 0$)

Expression (9) holds even if $L^i = 0$ (which may occur if $\lambda = 0$). At $L^i = 0$, the consumption of the marginal consumer, $x^*_i$, equals zero so that from (5), $X^i_L = 0$, which implies that the L.H.S. of (9) is zero.

## III. CROSS SUBSIDY

One important feature of public pricing theory is that the distributional effects of efficiency pricing may be large compared to the efficiency gain.[5] Consequently, the issue of cross subsidization is of considerable relevance: does one group of consumers subsidize another. We define a pricing structure as subsidy free if each group pays no less than its incremental cost and no more than its stand-alone cost. The incremental cost for group $i$ is the extra cost of its consumption given the consumption of other groups and the stand-alone cost is the cost of its consumption if other groups consume nothing. (Faulhaber [1975] suggests, in the multi-product case, a stronger definition requiring that all coalitions of groups pay an amount between incremental and stand-alone costs).

An extreme case occurs if all consumers in one particular group are inframarginal at the solution.

### *A. Proposition 1*

i) If, at the optimum, $M^i_L = 0$ for some group $i$ (with $M^i > 0$) then $\lambda = 0$ and $P^i = C'$.
ii) If $\lambda = 0$ and $M^j_L < 0$ for some $j$ then $L^j = 0$ and $P^j = C'$.
Proof: i) If $M^i_L = 0$ (with $M^i > 0$), all individuals in group $i$ are inframarginal and $X^i_L = M^i_L = 0$. From first order condition (9), $H_L = \lambda M^i = 0$, which implies $\lambda = 0$. Also $X^i_P = D^i_P < 0$ and $M^i_P = 0$, so (8) implies $P^i \geqslant C'$. Since $P^i > 0$ implies $H^i_P = 0$, we have $P^i = C'$.
(ii) Let $\lambda = 0$ and $M^j_L < 0$. $L^j > 0$ leads to a contradiction as follows. Using (5) and $\lambda = 0$, (9) becomes $H^j_L = k^j M^j_L = 0$ where $k^j = (P^j - C')x_{j^*} + L^j$. Since $M^j_L < 0$, $k^j = 0$ and $P^j - C' < 0$. Using (4) and $\lambda = 0$, (8) becomes $H^j_P = (P^j - C')D^j_P + k^j M^j_P \leqslant 0$. Since $k^j = 0$ and $P^j - C' < 0$, noting that $D^j_P < 0$ then yields a contradiction. $L^j$ must be zero. ($L^j < 0$ is ruled out since $L^j < 0$ implies $M^j_L = 0$). Given $L^j = 0$, (8) can be written $H^j_P = (P^j - C')(D^j_P + x_{j^*}M^j_P) \leqslant 0$, which implies $P^j \geqslant C'$. Since $P^j > 0$ implies $H^j_P = 0$, we have $P^j = C'$.

Taken together the two parts of Proposition 1 imply that if $M^i_L = 0$ for some $i$, then $P^j = C'$ for groups $j$ and if, in addition, $M^j_L < 0$ for some $j$, the members of group $j$ would pay no license fee. In essence, if $M^i_L = 0$, a license fee on group $i$ is (locally) a nondistorting lump sum transfer. The profit constraint is no longer binding and all distortions, such as $P^j \neq C'$ or $L^j > 0$, should be eliminated.

Groups $j$ with $P^j = C'$ and $L^j = 0$ pay $C'(Y^*)X^j$, where $Y^*$ is total consumption. If $C'$ is decreasing over the interval $(Y^* - X^i, Y^*)$, such groups are subsidized in that they pay less than their incremental costs. The cost functions can be written $C(Y^*) = F + \int_0^{Y^*} C'(Y)dY$ where $F$ is fixed or overhead cost and $\int_0^{Y^*} C'(Y)dY$ is variable cost. The payment by group $i$, denoted $R^i$, is $C(Y^*) - \Sigma_{j\neq i}R^j$, which, if group $i$ is the only group with $M_L^i = 0$, is $C(Y^*) - C'(Y^*)(Y^* - X^i)$. The stand-alone cost, denoted $C^{is}$, is $F + \int_0^{X^i} C'(Y)dY$. Therefore, $R^i - C^{is} = \int_{X^i}^{Y^*} C'(Y)dY - C'(Y)(Y^* - X^i)$, which is positive if average variable cost over the interval $[X^i, Y^*]$ exceeds $C'(Y^*)$. Under this condition group $i$ pays more than its stand-alone cost. Thus cross subsidization may easily arise in this extreme case with $M_L^i = 0$. However, if $M_L^i < 0$ for every $i$, ensuring that the profit constraint is binding, each group does at least contribute above marginal cost, as stated in the following proposition.

### *B. Proposition 2*

Let $K^i = (P^i - C')X^i + L^iM^i$, the contribution above marginal cost by group $i$. If $\lambda > 0$, then $K^i > 0$.

*Proof:* The first step is to show that if $\lambda > 0$ then $k^i \equiv (P^i - C')x_i^* + L^i > 0$. Using (5) and (9), $H_L^i \equiv k^i M_L^i (1 + \lambda) + \lambda M^i = 0$. Since $M_L^i = 0$ implies $\lambda = 0$ by Proposition 1, then $M_L^i < 0$ and $k^i$ must be positive.

Now $K^i \leqslant 0$ leads to a contradiction as follows. With $\lambda > 0$ and $L^i > 0$ from Lemma 2, $K^i \leqslant 0$ implies $P^i - C' < 0$. Also $K^i = M^i((P^i - C')\bar{X}^i + L^i) \leqslant 0$ (where $\bar{X}^i = X^i/M$) implies $(P^i - C')\bar{X}^i + L^i \leqslant 0$. With $P^i - C' < 0$ and $k^i > 0$ this requires $\bar{X}^i > x_i^*$. Using (4), (8) becomes $H_P^i = ((P^i - C')D_P^i + k^iM_P^i)(1 + \lambda) + \lambda X^i$. Taking (8) and (9) and eliminating $\lambda$, $(P^i - C')D_P^i + k^iM_P^i = \bar{X}^i k^i M_L^i$ if $P^i > 0$ ($\leqslant$ if $P^i = 0$). Since $\lambda > 0$, $M_L^i < 0$ from Proposition 1. Dividing through by $M_L^i$ and using Lemma 1 we obtain $(P^i - C')D_P^i/M_L^i + x_i^* k^i = \bar{X}^i k^i$ if $P^i > 0$ ($\geqslant$ if $P^i = 0$). Since $(P^i - C') < 0$, this requires $\bar{X}^i < x_i^*$ which is a contradiction. Therefore $K^i > 0$ which completes the proof.

Proposition 2 limits the extent of cross subsidization and rules it out entirely for certain cost structures. In particular if $C'' \geqslant 0$ for all $Y$ (but overhead costs are sufficiently high that $\lambda > 0$), then the pricing structure is subsidy free. The incremental cost for group $i$ is $\int_{Y^*-X^i}^{Y^*} C'(Y)dY$. Provided $C'' > 0$, this incremental cost must be less than or equal to $X^iC'(Y^*)$ which, since $K^i > 0$, is in turn strictly less than the payment $R^i$ by group $i$. (Note that $K^i = R^i - C'(Y^*)X^i$). Thus all groups pay above incremental cost.

As for stand-alone cost, we have $R^i - C^{is} \neq C(Y^*) - \Sigma_{j\neq i}R^j - (F + \int_0^{X^i} C'(Y)dy)$. Since by Proposition 2, $K^j > 0$ for every $j$, $\Sigma_{j\neq i}R^j > C'(Y^*)(Y^* - X^i)$ so that $R^i - C^{is} <$

$\int_{X^i}^{Y^*} C'(Y)dY - C'(Y^*)(Y^* - X^i)$, which is negative if $C'' \geqslant 0$. Thus each group $i$ pays less than stand-alone costs. Having $C'' \geqslant 0$ is a strong sufficient condition for subsidy free pricing which, incidentally, also implies that the adaptation to this case of Faulhaber's [1975] strong definition of subsidy free pricing is satisfied. Weaker sufficient conditions are that $C'' \geqslant 0$ over the interval $[Y^* - X^i, Y^*]$ and that average variable cost over $[X^i, Y^*]$ be not greater than $C'(Y^*)$.

If however, $C'' < 0$ over a significant range, cross subsidization may easily occur. Thus economies of scale induced by decreasing marginal cost can cause cross subsidy problems whereas economics of scale arising from large overhead costs will not. Declining marginal cost generally leads to cross-subsidy and consequent lack of sustainability. (See also Faulhaber and Levinson [1981] for further interesting analysis of subsidy free pricing).

## IV. EFFICIENT PRICES AND LICENSE FEES

First we extend to the many group case the result of NW that efficient prices will usually exceed marginal cost, but that prices below marginal cost are possible. Proof of the theorem requires the following lemma.

### *A. Lemma 3*

If $\lambda > 0$ and $P^i, L^i > 0$, then

$$(11) \qquad (P^i - C')X^i/K^i = \frac{\epsilon_P^i(\bar{X}_i - x_i^*)\bar{X}_i/x_i^*}{\mu_P^i\bar{X}^i + \epsilon(\bar{X}^i + x_i^*)^2/x_i^*}$$

where $\epsilon_P^i = -M_P^iP^i/M^i$ and $\mu_P^i = -D_P^i/X^i$.

*Proof:* The derivation of this expression from first order conditions (8) and (9) is largely mechanical and, despite our different formulation of the problem, is similar to the derivation of expression (13) in NW.

Proposition 3, which follows, corresponds to Theorem 1 of NW. The main new element is that, while $K > 0$ follows immediately from the budget constraint in the single group case, here we have to use Proposition 2 to ensure $K^i > 0$.

### *B. Proposition 3*

If $\lambda > 0$

i) $P^i = C'$ if $\bar{X}^i = x_i^*$

ii) $P^i > C'$ if $\bar{X}^i > x_i^*$

iii) $P^i < C'$ if $\bar{X}^i < x_i^*$

Proof: From Proposition 2, $\lambda > 0$ implies $K^i > 0$. Also $\mu_P^i \equiv -D_P^i P^i / X^i > 0$. In addition $\lambda > 0$ implies $\epsilon_P^i \equiv -M_P^i P^i / M^i > 0$ since, if $\epsilon_P^i = 0$ then $M_P^i = 0$ (for $M^i > 0$) and $M_L^i = 0$, which would imply $\lambda = 0$, a contradiction.
(i) With $\lambda > 0$, $K^i > 0$, $\epsilon_P^i > 0$, and $\mu_P^i > 0$, then expression (11) implies that $P^i = C'$ if $\bar{X}^i = x_i^*$.
(ii) If $\bar{X}^i > x_i^*$, expression (11) implies $P^i > C'$.
(iii) If $\bar{X}^i < x_i^*$ and $P^i > 0$, then (11) implies $P^i < C'$. If $P^i = 0$ then $P^i$ is obviously less than $C'$.

Proposition 3 is expressed rather differently from Theorem 1 of NW. We use $\lambda > 0$ as an underlying condition. If there is only one group this is equivalent to the assumption of NW that $\epsilon_P \equiv -M_P P/M > 0$. However, with more than one group, $\epsilon_P^i > 0$ does not imply $\lambda > 0$, and $\epsilon_P^i > 0$ together with $\bar{X}^i > x_i^*$ would not ensure that $P^i > 0$ since $\lambda$ and $K^i$ might be zero so that Lemma 3 would not apply.

Perhaps the most well-known result of public enterprise pricing is that efficient markups are inversely related to demand elasticities. With two part tariffs, markup (or excess price) equations are complicated by interaction effects between membership and demand. The following three expressions relate the efficient two part tariffs to demand elasticities. Let $e_i = P^i - C'$, $k^i = e_i x_i^* + L^i$, $\alpha_i = P^i X^i / L^i M^i$, and $\eta_P^i = -X_P^i P^i / X^i$.

*C. Proposition 4*

Given $\lambda > 0$, $P^i, P^j > 0$ and $L^i, L^j > 0$

$$e_i/L^i = \frac{(\bar{X}^i - x_i^*)/x_i^*}{(\mu_P^i/\epsilon_P^i)\bar{X}^i - (\bar{X}^i - x_i^*)} \tag{12}$$

$$\frac{e_i/P^i}{e_j/P^j} = \eta_P^j/\eta_P^i + \frac{\alpha_i\epsilon_P^j - \alpha_j\epsilon_P^i}{\alpha_i\alpha_j\eta_P^i e_j/P^j} \tag{13}$$

$$\frac{k^i/P^i x_i^*}{k^j/P^j x_j^*} = \epsilon_P^j/\epsilon_P^i \tag{14}$$

*Proof:* With $\lambda > 0$, $P^i > 0$ and $L^i > 0$, eliminating $\lambda$ from first order conditions (8) and (9) implies $e_i X_P^i + L^i M_P^i = \bar{X}\,(e_i X_L^i + L^i M_L^i)$. Rearrangement yields $e_i/L^i = (\bar{X}^i M_L^i - M_P^i)/(X_P^i - \bar{X}^i X_L^i)$. Using (4) and (5), $X_P^i - \bar{X}^i X_L^i = D_P^i + x_i^* M_P^i - \bar{X}^i x_i^* M_L^i = D_P^i - x_i^*\,(\bar{X}^i M_L^i - M_P^i)$. From Lemma 1, $\bar{X}^i M_L^i - M_P^i = (\bar{X}^i - x_i^*)\,M_P^i/x_i^*$. Therefore, $e_i/L^i = [(\bar{X}^i - x_i^*)\,M_P^i/x_i^*]/[D_P^i - (\bar{X}^i - x_i^*)\,M_P^i]$. Rewriting in terms of elasticities and rearranging yields (12).

With $\lambda > 0$ and $P^i > 0$, eliminating $\lambda$ from (8) yields $[e_i X_P^i + L^i M_P^i]/X^i = [e_j X_P^j + L^j M_P^j]/X^j$. In terms of elasticities, this is $(e_i\eta_P^i/P^i) + \epsilon_P^i/\alpha^i = (e_j\eta_P^j/P^j) + \epsilon_P^j/\alpha^j$ which becomes (13) after some rearrangement.

Using (5), $H^i_L = k^i M^i_L(1+\lambda) + \lambda M^i$. With $\lambda > 0$ and $L^i > 0$, eliminating $\lambda$ from (9) yields $k^i M^i_L/M^i = k^j M^j_L/M^j$. Substituting $M^i_P = x^*_i M^i_L$ (Lemma 1) and $\epsilon^i_P$, we obtain $k^i \epsilon^i_P / x^*_i P^i = k^j \epsilon^j_P / x^*_j P^j$, which rearranged yields (14).

Proposition 4 preserves the basic Ramsey insight, but in weakened form. Expression (12) shows that, for equal values of $x^*_i$ and $\bar{X}^i$, low values of $\mu^i_P/\epsilon^i_P$ are associated with high absolute values of $e_i/L^i$: if demand is inelastic but membership elastic, it is efficient to use price relatively more as a revenue raising tool. A small percentage reduction in the license fee brings in a relatively large percentage of new consumers who pay the license fee and also purchase the product.

Expression (13) shows that the ratio of percentage markups for any two groups can be regarded as a two stage Ramsey rule. In the case of simple price discrimination (no license fee) the ratio is just the first term, $\eta^j_P/\eta^i_P$, which is the familiar basic Ramsey rule (as in Hartwick [1978]). The second term is a deviation from this standard Ramsey rule. It involves the effect of membership and implies that it is possible for a group to have both a lower price elasticity of demand and a lower efficient price at the optimum. If, for example, $\alpha_i = \alpha_j$ and $e_j > 0$, a higher price elasticity of membership for group $i$ makes the deviation negative so that the percentage markup for group $i$ is lower than that implied by the standard Ramsey rule. If the second term happens to be zero $(\alpha_i \epsilon^j_P = \alpha_j \epsilon^i_P)$ then we have the interesting result that the standard Ramsey rule on prices coincides (in form) with the two part tariff rule.

Expression (14) is concerned with the "excess payment" by marginal consumers of each group. $k^i$ is the payment of marginal consumers above marginal cost; dividing by $x^*_i$ yields a per unit measure, and dividing by $P^i$ normalizes by marginal benefit. Marginal consumers of groups with low elasticity of membership make a larger contribution above marginal cost relative to their marginal benefit from consumption.

Price discrimination and two part tariffs require that resale be difficult and that the producer of the good be able to monitor individual purchases. It is common that identification is feasible for a (large) license fee payment but not for subsequent (small) transactions, as with recreational clubs and, in the past, with local telephone service. Thus the efficient discriminating license fee with a common price is of interest. Using Lemma 1 and first order condition (9) yields

(15) $$L^i = \beta P x^*_i / \epsilon^i_P - (P - C')x^*_i$$

where $\beta = \lambda/(1+\lambda)$. Expression (15) is a Ramsey rule for it implies that a group with a higher $\epsilon^i_P$ will have a lower license fee (for equal $x^*_i$). Note that $\beta$, $P$, and $C'$ are the same across groups. For the special case in which $P$ is constrained to be zero the efficient license fees are characterized by another Ramsey rule.

(16) $L^i = C'x_i^* /(1 - \beta/\epsilon_L^i)$

Here, a high license fee (or flat rate since $P^i = 0$) is associated with a low license fee elasticity of membership.

## V. CONCLUDING REMARKS

Efficient many group discriminating two part tariffs may involve cross subsidization among groups. However, for many cases, particularly if marginal cost is nondecreasing, cross subsidies will not occur. At the extreme, if some groups are "captive" so that license fees on them are (locally) nondistorting, other groups will pay only a price equal to marginal cost and no license fee. The cross subsidy results are important because one of the features of public pricing theory is that distributional effects of efficiency pricing may be large compared with actual efficiency gains.

Because cross subsidization is limited, the main result of Ng and Weisser [1974] can be extended to the many group case: the efficient price for a group exceeds, equals, or falls short of marginal cost as average consumption (for the group) exceeds, equals, or falls short of consumption by a marginal consumer.

The efficient prices and license fees are characterized in three expressions that link them to various elasticities. The basic Ramsey insight that high charges are associated with low elasticities is preserved, but in a weakened form.

A second aspect of public pricing theory is that the cost of gathering the relevant information might be high compared to the efficiency gain. The information required here (group specific membership and demand functions) is in principle estimable but not easily obtained.

### NOTES

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1 The term "price discrimination" can be used to refer to any case in which different consumers pay different average prices. Thus a single two part tariff could be considered a type of discrimination (Leland and Meyer [1976]) since high volume users pay a lower average price. We use the term "discriminating two part tariff" for the case in which different groups of consumers face different tariff schedules.

2 This form of the utility function is equivalent to the partial equilibrium assumptions that demand interdependence between $x$ and other goods is negligible and that the marginal utility income is constant for the changes under consideration. This partial equilibrium setting corresponds to our notion of the problem facing the regulator of a single public enterprise. Ng and Weisser [1974] use a 2 good general equilibrium model in which a planner optimally redistributes income behind the scenes. Our results can be derived (with additional algebra) in that setting.

[3] If $L^i = 0$ then the solutions $h_i$ to $s^i(P^i, L^i, h_i) = 0$ may not be unique since with $f_p^i < 0$ then $x_i^* = f^i(P^i, h_i^*) = 0$ and $s^i$ is no longer necessarily strictly increasing in $h_i$. A unique $h_i$ is obtained by defining $h_i^* = sup\{h_i : s(P^i, L^i, h_i) = 0\}$. Hence, taking right hand derivatives at $P^i = 0$ and $L^i = 0$, for $P^i \geqslant 0$ and $L^i \geqslant 0$, $\partial h_i^* / \partial P^i > 0$ and $\partial h_i^* / \partial L^i > 0$. Faulhaber and Panzar [1977] follow this same procedure. (See their footnote 6).

[4] Demand curves would be noncrossing if for example, as in Spence [1980], we made the additional assumption that inverse demand is strictly increasing in $h_i$ for any $x_i$ so that $\partial^2 u_i / \partial x_i \partial h_i > 0$.

[5] A strict welfare theoretic foundation for adopting efficient prices involves making lump sum compensating payments behind the scenes. Since such payments are generally impossible one can adopt one of (at least) two positions: that public prices should incorporate explicit distributional objectives, as in Feldstein [1972], or alternatively, that public prices should be efficient (surplus maximizing) subject to constraints on allowable cross subsidization, hence the interest in cross subsidy.

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**Summary:** *Ramsey Optimal Two Part Tariffs: The Case of Many Heterogeneous Groups.* — This paper examines efficient pricing for a public enterprise that is subject to a minimum profit constraint and that is able to set different two part tariffs for different, internally heterogeneous consumer groups. We examine the extent of cross subsidization implied by efficient pricing. In addition we extend results concerning the relation between price and marginal cost to the multi-group case. Ramsey-like expressions relating efficient prices and license fees to relevant elasticities are derived and interpreted.

**Résumé:** *Des tarifications, sélectives, optimales à la Ramsey: le cas de plusieurs groupes hétérogènes.* — Ce papier étudie la tarification efficiente d'une entreprise publique soumise à une contrainte de profit minimum et qui est en mesure d'établir un tarif sélectif (en deux parties) pour des groupes de consommateurs hétérogènes. Nous examinons le possibilité d'une subsidiation croisée liée à une tarification efficiente. Ensuite nous généralisons les résultats liés à la relation prix et coût marginal pour le cas multi-groupes. Nous dérivons et interprétons les élasticités appropriées relatives aux expressions, à la Ramsey, concernant la tarification efficiente ainsi que les droits de licence.

**Zusammenfassung:** *Ramsey-optimale Zwei-Stufentarife: Der Fall mehrerer heterogener Konsumentengruppen.* — Der Artikel untersucht die effiziente Preisstellung eines öffentlichen Unternehmens, das einer Minimum-Gewinnrestriktion unterliegt und das unterschiedliche Zwei-Stufentarife für verschiedene, intern heterogene Konsumentengruppen festsetzen kann. Dabei wird das Ausmaß einer Kreuz-Subventionierung, die mit effizienter Preisbildung einhergeht, untersucht. Dann werden die Ergebnisse im Hinblick auf das Verhältnis zwischen Preis und Grenzkosten für den Mehr-Gruppen-Fall erweitert und Ramsey-Formeln, die effiziente Preise und Lizenzgebühren mit entsprechenden Elastizitäten verbinden, abgeleitet und interpretiert.