Vertical Foreclosure and International Trade Policy

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International differences in the cost of production of key intermediate product can mean that a domestic firm is dependent on supplies from a foreign vertically integrated firm. This paper considers the incentives for the foreign firm and foreign country to supply the domestic firm when the firms compete in a Cournot or Bertrand market for the final product. The vertical supply decision is significantly affected by domestic supply conditions for the input and a domestic tariff on final product imports. Optimal policy by the exporting country may require a tax on both exports, or a subsidy on both exports.

1. INTRODUCTION

Countries that are dependent on imports of a key intermediate product or raw material from a dominant world supplier are often concerned about the price and the availability of imports. A notable current example involves the computer industry. Japanese suppliers (with the help of the Japanese government) recently restricted the exports of DRAM semiconductors, substantially raising their price. These suppliers control about 80% of the market for semiconductors and the higher prices and shortage in supply have forced U.S. producers of computers to curtail production and increase prices. Vertically integrated Japanese firms such as Toshiba and N.E.C. have benefitted both from increased profits in the export market for semiconductors and from the improvement in their competitive position in the market for final computers.

As this example indicates, the price and availability of imported supplies can depend on both public and private incentives in the exporting country. In this paper, we first examine the private incentives for a vertically integrated firm to export an intermediate product to a higher cost rival, lowering its rival's costs. The exporting firm may choose vertical foreclosure, thus fully cutting off supplies. We then consider the public interest of the exporting country: does the exporting country gain by encouraging the export of the intermediate product, or alternatively, might "government foreclosure" occur? The government is in a position to affect the quantity of exports of both the input and the final product by an appropriate choice of export tax and subsidy policies.

1. This action was facilitated by a U.S. anti-dumping action against certain semiconductors from Japan. However, the price increase for DRAM semiconductors was more than required by the trigger price anti-dumping measure. The U.S. policy encouraging the Japanese to restrict semiconductor exports is hard to defend on the basis of the analysis developed in this paper.
We consider the most extreme form of dependence on a vertically integrated supplier by assuming that a single vertically integrated firm controls the exports of both the input and the final product. Differences in production costs are assumed to arise as a consequence of international differences in endowments and technologies. If the rival firm has access to the input, either through its own production or through imports, then, for most of the paper, we assume that sales of the homogeneous final product are determined by Cournot competition. The implications of Bertrand behaviour with differentiated final products are briefly considered in a later section. If the rival firm in the importing country has no independent source of supply, vertical foreclosure gives the exporting firm a monopoly of the final product market.

The exporting firm is assumed able to act first, by committing to an export strategy (price or quantity) for the input, prior to the decision of the high-cost firm as to its own level of production of the input and to the resolution of the Cournot (or Bertrand) game for the final product. The credibility of a commitment to the quantity of exported supplies is supported by the idea that it takes time to export the input and that these supplies must be available to the rival at the time of production of the final good. In choosing its export strategy, the exporting firm takes into account its rival's reaction including the possibility that the rival will alter its own level of production of the input. This has the advantage that the vertical supply (or foreclosure) decision is made with a full understanding of its consequences.

The profitability of vertical supply as an export strategy is heavily influenced by production conditions for the input in the importing country. Two main aspects of local supply conditions are shown to be important: the absolute quantity of supplies and the response of these supplies to the price charged for the input by the exporting firm. By imposing a tariff on final product imports, the importing country can have a major influence on this decision. In particular, if the rival firm cannot produce the input and there is Cournot competition at the final stage, even a small import tariff will shift the equilibrium from vertical foreclosure to vertical supply.

The role for government policy in the exporting country arises from the inability of the exporting firm to commit to its level of exports of the final product prior to the rival's output decision. As originally shown by Spencer and Brander (1983), the exporting country gains from a subsidy to exports in a simple Cournot duopoly setting: the optimal subsidy increases exports to what would have been the Stackelberg leader level of output in the absence of the subsidy. Consideration of the export market for the input does, however, add some significant new features.

The sign of the difference in profit margins from the export of the input and the final product is shown to play a crucial role in the determination of optimal policy. If this difference in profit margins is positive (as a consequence of an import tariff on the final product) and if there is Cournot competition at the final stage, optimal government policy serves to amplify this difference, thus encouraging an increase in the extent of vertical supply. Perhaps surprisingly, this policy is achieved by a tax, not a subsidy, on the exports of the input, together with a (larger) tax on the exports of the final product. Conversely, if the difference in profit margins is negative, optimal policy calls for a subsidy to both exports, making the difference in profit margins even more negative and discouraging vertical supply.

As might be expected from Bulow, Geanakoplos and Klemperer (1985) and Eaton and Grossman (1986), optimal policy by the exporting country is fundamentally affected by whether the final products produced by each firm are strategic substitutes or complements. Optimal policies tend to have opposite signs if the final products are strategic.
complements (as in our Bertrand analysis) rather than strategic substitutes (as in our Cournot analysis). Nevertheless, whether there is Bertrand or Cournot competition at the final output stage, the underlying conditions in the importing country that determine the profitability of vertical supply are basically unaffected.

Vertical foreclosure has been an important issue in the antitrust literature and in industrial organization. The idea that vertical foreclosure can increase profits is related to the idea, developed by Salop and Scheffman (1983, 1987), that a dominant firm can gain by increasing the costs of its rivals even at some expense to itself. Two very interesting recent papers, Salinger (1988) and Ordover, Saloner and Salop (1990) deal directly with the issue of vertical foreclosure. In both of these papers, upstream producers of the input have identical and constant costs and vertical merger results in a full cutting-off of supplies to downstream firms. In contrast, the present model demonstrates that asymmetries in costs can make vertical supply profitable for a dominant firm. If local supplies of the input in the importing country are sufficiently elastic, then vertical supply by the low-cost firm is always an equilibrium strategy.

This paper is also related to the international trade literature concerning policy towards exports of vertically related products in a perfectly competitive framework (see for example Kemp (1966), Jones (1967) and Jones and Spencer (1989)). Finally, this paper draws on the literature concerning trade policy under imperfect competition. Of special relevance are Dixit (1984), Eaton and Grossman (1986), Grossman and Dixit (1986), Venables (1985) and Brander and Spencer (1985).

The basic model is described in Section 2. Section 3 is concerned with the Cournot equilibrium for the final product and Section 4 with the conditions under which the exporting firm will supply its rival with the input. Section 5 then deals with optimal policy by the exporting country. The implications of Bertrand competition for the final differentiated products are considered in Section 6 and Section 7 contains concluding remarks.

2. THE MODEL

Firm 1, in country 1 exports a final product to country 2 in competition with a higher-cost rival, firm 2, located in country 2. Firm 1 is vertically integrated and also (potentially) exports its lower-cost intermediate product to country 2, reducing its rival's costs. Firm 2 has the option of augmenting these imported supplies by producing the input itself. Although we present the analysis as if firm 2 is vertically integrated, this is not necessary. The input could be produced by a perfectly competitive industry in country 2.

Technological relationships are simplified by assuming that one unit of the intermediate product is required to produce one unit of the final product and that there are no other factors of production. Firm 1 produces the input (and the final good) at a constant marginal cost $c^1$, whereas firm 2 can produce its own supplies of the input only at a higher and increasing marginal cost. This means that $c^2 > c^1$ where $c^2$ denotes firm 2's costs.

2. In Quirmbach (1986), vertical supply (or partial forward vertical integration) by an upstream monopolist can be an equilibrium strategy due to diminishing returns in the monopolist's downstream subsidiary. In Katz (1987), downstream firms can integrate backwards to produce the input. The alternative cost of producing the input affects the price charged for the input by an upstream monopolist, but the monopolist does not produce the final product. Greenhut and Ohta (1979), consider vertical integration by a subset of oligopolists, but vertical supply is not an issue. See Tirole (1988) for a discussion of the literature.

3. With substitutability between inputs, an increase in the price of imported supplies would cause the rival firm to substitute away from the higher priced input. In our model, the rival firm substitutes away from higher priced imports by producing its own supplies so that a relaxation of the fixed proportions assumption should not change the general nature of the results.
marginal cost of production of the first unit of the input. Although brief consideration is given to the possibility that firm 2's marginal cost is constant at \( c_2 \), our general assumption is that firm 2's marginal cost is strictly increasing.

The subgame-perfect equilibrium incorporates two stages of decision. In stage 1, firm 1 commits to the price \( r \) that it will charge its rival for the input. Equivalently, firm 1 could commit to the quantity \( x \) of its exports of the input at this stage.\(^4\) The quantity of exports \( x \) and the price \( r \) charged for these exports are related simply by the requirement that the demand for \( x \) by firm 2 in stage 2 equals the supply at price \( r \). Unless otherwise stated, the stage 2 outputs of the final product are determined by Cournot (quantity-Nash) competition. (We consider Bertrand (price-Nash) competition in Section 6). Firm 2 produces the final product using the cost-minimizing combination of imported supplies and its own production \( x_2 \) of the input in stage 2.

Country 1 is assumed to commit to its policies, a specific subsidy \( s \) to final product exports and a specific tax \( v \) on exports of the input, at stage 0, prior to firm 1's choice of the price and quantity of its exports of the input at stage 1. The subsidy \( s \) and the tax \( v \) may be either positive or negative. Country 2 may commit to a specific tariff \( t \) on imports of the final product at the same stage.\(^5\)

3. THE FINAL GOODS MARKET

In considering the market for the final product in country 2, we abstract from the possibility that the final product is also sold in country 1. If the two markets are segmented, this involves no loss of generality. The price \( p \) of the final good in country 2 is given by the inverse demand curve \( p = p(Y) \) where \( p'(Y) < 0 \) and \( Y = y^1 + y^2 \) represents aggregate output. The quantities \( y^1 \) and \( y^2 \) of the final product are produced by firms 1 and 2 respectively. The total profit of firm 1 from the export of \( y^1 \) and the input \( x \) is,

\[
\pi^1 = (p - (t - s + c^1))y^1 + (r - v - c^1)x.
\] (3.1)

Firm 2 purchases \( x \) at price \( r \) and produces its own supplies, denoted by \( x^2 \), at a total cost \( C^2(x^2) \), so that its profit from the production of \( y^2 \) is,

\[
\pi^2 = py^2 - rx - C^2(x^2).
\] (3.2)

We first consider firm 2's choice between using its own or imported supplies of the input. Firm 2 produces \( x^2 \geq 0 \) so as to maximize (3.2) for given levels of \( y^1 \), \( y^2 \) and \( r \). Substituting \( x = y^2 - x^2 \) into (3.2), at the profit maximum, \( x^2 \) satisfies:

\[
r - C^2_x(x^2) \leq 0 \quad (= 0 \text{ if } x^2 > 0)
\] (3.3)

where \( C^2_x(x^2) \) represents firm 2's marginal cost of production of the input. If \( c^2 \) (the marginal cost of producing the first unit of \( x^2 \) exceeds the import price \( r \), then firm 2 sets \( x^2 = 0 \) and uses imported supplies only. If firm 2 produces the input, (3.3) implicitly defines the supply of the input as an increasing function of \( r \): \( x^2 = x^2(r) \) where \( x^2 = 1/C^2_x(x^2) > 0 \).

At the stage 2 Cournot equilibrium for the final good, firm 1 sets its output \( y^1 \) to maximize (3.1), given \( y^2 \) and the prior committed values of \( r, t, s, \) and \( v \). Similarly, firm

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4. In stage 1, firm 1 sets \( x \) taking into account its effect on the market price \( r \) that will be determined in stage 2. Our results are unaffected provided both firms take \( r \) as given in setting their final outputs. The need to consider the effect of \( x \) on \( r \) lengthens the algebra somewhat.

5. The tariff \( t \) is not chosen optimally. In Spencer and Jones (1989), we use a similar model to consider optimal policy by the importing country.
2 chooses $y^2$ to maximize (3.2), given $y^1, r, t, s$ and $v$. The first-order conditions (using subscripts to represent partial derivatives) are

$$\pi_1^1(y^1, y^2, r, t - s, v) = p + y^1 p' - (t - s + c^1) = 0$$
(3.4)

$$\pi_2^1(y^1, y^2, r) = p + y^2 p' - r = 0.$$  
(3.5)

Solving (3.4) and (3.5) simultaneously, the Cournot equilibrium levels of output can be expressed as functions of $r$ and $t - s$:

$$y^1 = y^1(r, t - s) \text{ and } y^2 = y^2(r, t - s).$$  
(3.6)

Country 1's export tax $v$ does not appear directly in (3.6) because of the definition of $r$ as the price actually paid by firm 2 for imported supplies. However, since the tax $v$ reduces the amount that firm 1 gets for its exports of the input, it affects the equilibrium by changing the price charged for the input.

In considering Cournot competition, we make the assumption that own marginal profit declines with an increase in the output of the other firm:

$$\pi_1 = p' + y^1 p'' < 0 \text{ and } \pi_2 = p' + y^2 p'' < 0.$$  
(3.7)

This implies that reaction functions in output space have negative slopes, or equivalently, that the outputs produced by each firm are strategic substitutes. The second-order conditions for profit maximization hold and the Cournot equilibrium is unique: $\pi_1^1 < 0$, $\pi_2^2 < 0$ and $H = \pi_1^1 \pi_2^2 - \pi_1^2 \pi_2^1 > 0$.

From total differentiation of (3.4) and (3.5), and from (3.7), the comparative static effects of an increase in the input price on $y^1$ and $y^2$ are:

$$y^1_1(r, t - s) = -\pi_1^1 / H > 0 \text{ and } y^2_1(r, t - s) = \pi_2^1 / H < 0.$$  
(3.8)

Similarly, an increase in the subsidy $s$ (set by country 1) or a decrease in the tariff $t$ (set by country 2) serve to increase $y^1$ and decrease $y^2$:

$$y^1_s = -y^1(r, t - s) = -\pi_2^2 / H > 0 \text{ and } y^2_s = -y^2(r, t - s) = -\pi_1^2 / H < 0.$$  
(3.9)

Also, from (3.8) and (3.9), industry output is decreasing in the input price, increasing in $s$ and decreasing in $t$:

$$Y_r(r, t - s) = p' / H < 0 \text{ and } Y_s = -Y_t(r, t - s) = -p' / H > 0.$$  
(3.10)

Of importance in what follows is the sign of $\pi_1^2(y^1, y^2, r)$: the effect of an increase in firm 2's output on firm 1's profits holding $y^1$ and $r$ fixed. Firm 1 gains from the increase in its exports of the input, but the price of its final product exports falls. Let $M(r, t - s, v)$ denote $\pi_1^2(y^1, y^2, r)$ when the outputs are at their Cournot equilibrium levels; thus from (3.1) using (3.4) to substitute for $y^1$,

$$M(r, t - s, v) = r - v - c^1 + y^1 p' = (r - v - c^1) - (p - t + s - c^1).$$  
(3.11)

As (3.11) shows, $M(r, t - s, v)$ is just the difference in profit margins from the export of the intermediate and final products. This difference in profit margins is positive if and only if firm 1's profits are increased by an increase in its rival's output (with $r$ and $y^1$ fixed). Since an increase in the price of the input increases the price of the final product

6. It is possible for the homogeneous outputs to be strategic complements (see Bulow, Geanakoplos and Klemperer (1985)).
by less than one unit,\(^7\) the difference in profit margins is increasing in \(r\): from (3.7) and (3.10),
\[
M_r(r, t-s, v) = 1 - p'Y_r = p'(2p' + Yp')/H > 0.
\] (3.12)

4. THE EXPORT MARKET FOR THE INPUT: VERTICAL FORECLOSURE OR VERTICAL SUPPLY

Firm 2's derived demand for imported supplies is firm 2's output of the final good at the Cournot equilibrium less its own production of the input:
\[
x'(r, t-s) = y^2(r, t-s) - x^2(r)
\] (4.1)
where \(x_r = y^2 - x^2 < 0\) from (3.8) and (3.3). Vertical foreclosure occurs if firm 1 charges a prohibitive price for the input, denoted \(r^p\), at which firm 2's demand for imported supplies is reduced to zero.

Setting \(x(r^p, t-s) = 0\) implicitly defines\(^8\) the foreclosure price as a function of \(t-s\) where
\[
r^p = -r^p(t-s) = -y^2/x_r < 0.
\] (4.2)
An increase in the export subsidy \(s\) and a reduction in the import tariff \(t\) both increase firm 1's exports of the final product, decreasing firm 2's output and the price \(r^p\) at which it ceases to use imported supplies.

We now consider the firm 1's stage 1 choice of the price \(r\) to charge its rival for the input. Taking into account the second-stage relationships, firm 1's profit is a function (denoted \(\pi^E\) where \(E\) stands for the exporting firm) of the input price \(r\) as well as trade taxes and subsidies, \(t, s, v\) set in stage 0:
\[
\pi^1 = \pi^E(r, t-s, v) = (p + t - s - c^1)y^v(r, t-s) + (r - v - c^1)x(r, t-s)
\] (4.3)
In stage 1, firm 1 sets the price \(r\) to maximize profit subject to \(x \geq 0\).

At a vertical supply equilibrium, from (4.3) using (3.4),
\[
\pi^E_r(r, t-s, v) = (r - v - c^1)x_r + x + y^1p'y^2_r = 0
\] (4.4)
Condition (4.4) defines the price charged for the input as a function \(r(t-s, v)\) of the government policies \(t, s, v\) set previously in stage 0. The first two terms of (4.4) represent the direct effect of an increase in \(r\) on firm 1's profits from the export of the input. The third (positive) term captures the "strategic effect" of \(r\) on the profits earned from final product exports.\(^10\) Since \(y^1_r > 0\) and \(Y_r < 0\), an increase in \(r\) increases both the volume and price of final product exports. Vertical foreclosure occurs if this strategic effect is sufficiently large.

7. An increase in \(r\) increases final \(p\) only because of the reduction in \(y^2\). (Imports \(y^1\) increase but there is a net reduction in \(Y\)). For given \(y^1\), the demand curve facing firm 2 is flatter than the marginal revenue curve (from (3.7)) ensuring that \(p\) rises by less than the increase in \(r\).

8. The quantity of exports of the input reduces smoothly to zero as \(r\) increases to \(r^p\) so that there is no discontinuity in \(x(r, t-s)\) at \(r^p\).

9. To obtain the conditions for a maximum, define \(L = \pi^E + \mu x(r, t-s, v)\) where \(\mu\) represents the Lagrange multiplier. The first-order conditions are then: \(L_r = \pi^E_r + \mu x_r = 0\) and \(L_v = x \geq 0\), \(\mu \geq 0\), \(L_\mu = \mu = 0\). We assume that \(\pi^E\) is strictly concave for all \(r \leq r^0\). If demand and supply are linear then \(\pi^E_r = y^v(2 - p'Y_r) - 2x^2_r < 0\) except at \(r = c^2\) where \(\pi^E_r\) is continuous but not differentiable. However, \(\pi^E_r\) remains strictly concave in \(r\) at \(c^2\), since \(x^2_r = 0\) for \(r \leq c^2\) and \(x^2_r < 0\) for \(r \geq c^2\), making the left-hand derivative \(\pi^E_r\) less negative than the right-hand derivative.

10. Bulow, Geanakoplos and Klemperer (1985) have a general determination of the sign of such "strategic" terms based on whether products are strategic substitutes or complements and whether there are joint economies or diseconomies across markets. In our model there are joint diseconomies across markets, since an increase in the level of exports of the input reduces the marginal profitability of exports of the final product.
A main concern is the effects of conditions in the importing country on the vertical supply or foreclosure decision. For this analysis, we assume that \( s = v = 0 \). The results then form a base from which to examine optimal policy by the exporting country. Firm 1 engages in vertical supply if and only if a reduction in \( r \) below the foreclosure price increases its overall profits: i.e. if and only if (from (4.4) using \( x_r = y_r^2 - x_r^2, x = 0 \) and (3.11)),

\[
\pi_v^E(r^o, t, 0) = M(r^o, t, 0)y_r^2 - (r^o - c^i)x_r^2 < 0. \tag{4.5}
\]

From (4.5), firm 1 will always supply its rival if \( M(r^o, t, 0) < 0 \) at the foreclosure price that is if it earns a higher profit margin from the export of the first unit of the input than from its final product exports.

To determine the effects of supply conditions in the importing country it is useful to relate the difference in profit margins to the quantity of local supplies of the input. From (3.5) (setting \( y^2 = x^2(r^o) \)) and from (3.11),

\[
M(r^o, t, 0) = r^o - p + t = x^2(r^o)p' + t. \tag{4.6}
\]

Suppose now that it is prohibitively expensive to produce the input in country 2 (i.e. \( x^2(r^o) = 0 \)). The foreclosure price \( r^o \) is then equal to \( p \); firm 2 will enter as a producer of the final product if and only if the price charged for the input is below the price \( p \) of the final product. With no tariff on final product imports, \( M(r^o, 0, 0) = 0 \) from (4.6) and, from (4.5), firm 1 forecloses. This should not be surprising: with no production of the input in country 2, vertical foreclosure prevents the entry of firm 2, giving firm 1 a monopoly of the market for the final product. Nevertheless, any small import tariff will make the difference in profit margins strictly positive (\( M(r^o, t, 0) = t \) from (4.6)), inducing firm 1 to supply its rival with a small quantity of the input. A tariff on final product imports thus gives the exporting firm an incentive to get "under" the tariff wall by exporting the good produced at a lower stage of production.\(^\text{11}\) These results are reported in Proposition 1.

**Proposition 1** (Assume \( s = v = 0 \)). Suppose that it is prohibitively expensive to produce the input in country 2.

(i) In the absence of a tariff, firm 1 will vertically foreclose.

(ii) A small tariff on imports of the final product will induce firm 1 to supply its rival with the input.

Now consider the possibility that it is profitable to produce the input in the importing country. In addition to the tariff, two aspects of local production conditions prove to be important for the vertical supply decision: the total quantity \( x^2(r^o) \) of supplies available at the foreclosure price and the responsiveness of these supplies.

With the input available in the importing country, vertical foreclosure no longer prevents the rival firm's entry as a competitor in the market for the final product. This consideration might lead one to expect that firm 1 would now have a greater incentive to supply its rival. In fact, the availability of a fixed quantity of local supplies tends to have the reverse effect; as Proposition 2(i) shows, an increase in \( x'(r^o) \) tends to move the equilibrium towards vertical foreclosure. Firm 2 is willing to pay less for the unit of imported supplies and the difference in profit margins \( M(r^o, t, 0) \) falls. Assuming linear demand and supply so as to abstract from secondary changes in the magnitude of responses

\(\text{11. As Spencer and Jones (1989) show, the tariff } t \text{ can also be an effective tool in reducing the price of imported supplies in the region of vertical supply.}\)
and \(x;\), the reduction in \(M(r^o, t, 0)\) translates into a lower profit from vertical supply. Since \(M(r^o, 0, 0)\) is negative when \(x^2(r^o) > 0\) and \(t = 0\), a large tariff may now be required to induce firm 1 to export the input (see Proposition 2(ii)).

**Proposition 2.** (i) Under linear demand and supply conditions, an exogenous increase in firm 2's production of the input at the foreclosure price increases firm 1's incentive for vertical foreclosure.

(ii) A sufficiently large tariff on imports of the final product will induce vertical supply.

**Proof.** (i) Let \(x^2 = x^2(r, \alpha)\) where \(x^2 > 0\) and \(\alpha\) is a shift parameter. Setting \(x(r^o, t, \alpha) = 0\) defines \(r^o = r^o(t, \alpha)\) with \(r^o = x^2/x_r < 0\). From (3.12), \(dM(r^o, t, v)/d\alpha = M_r r^o_\alpha < 0\). If \(p^o(Y) = x^2_r = 0\), then, from (4.5), \(d\pi^E(r^o, t, 0)/d\alpha = (dM/d\alpha)y^2 - x^2 r^o_\alpha > 0\) increasing firm 1's incentive to foreclose. (ii) If \(x^2 = 0\), from Proposition 1(ii), any \(t > 0\) induces vertical supply. If \(x^2(r^o) > 0\), let \(t^*\) denote the prohibitive tariff at which \(y^1(r^o, t^*) = 0\). Then, from (3.4), \(p - t^* - c^1 = 0\) and \(M(r^o, t^*, 0) = r^o - c^1 > 0\) and bounded away from zero (from \(r^o \geq c^2 > c^1\)). Since \(dM(r^o(t), t, 0)/dt = M_r r^o + 1 - p' Y, > 0\) (from (3.10), (3.12) and (4.2)), it follows that \(M(r^o, t, 0) > 0\) at some sufficiently large \(t < t^*\) ensuring vertical supply. \(\|

To examine the implication of the responsiveness of supplies for the vertical supply decision, let \(e_r = x^2/x^2 \geq 0\) represent the elasticity of supply of \(x^2\) in country 2. Greater responsiveness is measured by an exogenous increase in \(e_r\) at \(r^o\), maintaining \(r^o\) constant. There is thus no change in the quantity of firm 2's supply of the input or in final outputs at the foreclosure price. The increase in \(e_r\), can be represented geometrically by a clockwise rotation in the supply curve for the input at the foreclosure price.

**Proposition 3.** An exogenous increase in \(e_r\) at the foreclosure price increases firm 1's incentive for vertical supply. Firm 1 chooses vertical supply if \(e_r\) is sufficiently large.

**Proof.** From (4.5), holding \(r^o\) constant, \(d\pi^E(r^o(t), t, 0)/de_r = -x^2(r^o)(r^o - c^1)/r^o < 0\). A sufficiently large \(e_r\) will make \(\pi^E(r^o, t, 0) < 0\), inducing vertical supply. \(\|

Why does a greater responsiveness of supplies in country 2 tend to move the equilibrium towards vertical supply? When local supplies of the input are responsive to price, a reduction in the price of imported supplies causes firm 2 to cut back on its own production, substituting imports for its own production of the input. Each unit of imports is then associated with a less than one unit increase in firm 2's output of the final product. Although firm 1's profits may be reduced by an increase in firm 2's output holding local supplies fixed, as \(e_r\) is increased, the cut-back in firm 2's own production of the input eventually becomes sufficiently large that vertical supply is profitable. Expressing this in an alternative way: at a higher elasticity of local supply, a given reduction in the input price enables firm 1 to achieve a greater increase in its exports of the input, for the same increase in firm 2's output of the final product.

At the extreme, the elasticity of local supply of the input is infinite when firm 2's marginal cost is constant at the level \(c^2\). Firm 1 can then always gain (relative to vertical foreclosure) by exporting the input at a price \(r\) just below \(c^2\) so as to deter the entry of firm 2 as a producer of the input. These exports do not affect firm 2's marginal cost or its final output. Thus, firm 1 earns positive profits from the export of the input with no
change in its profits from final product exports. When country 2 imposes an import tariff on the final product, it is possible that firm 1 may further reduce the price of the input to the internal solution where the first-order condition (4.4) is satisfied. In either case, vertical supply is the optimal strategy for firm 1.

The underlying conditions behind the vertical supply or foreclosure decision are illustrated in Figure 1. The curve $FF$ illustrates the boundary at which vertical foreclosure just occurs when $\varepsilon$, and the tariff on final product imports are varied. The region of vertical foreclosure is shown by the shaded area on or below $FF$, whereas the area strictly above $FF$ represents the region of vertical supply. Both an increase in $\varepsilon$, and a higher tariff tend to move the equilibrium towards vertical supply. If demand and supply are not too non-linear, a small increase in $\varepsilon$, can be offset by a small decrease in the tariff, making $FF$ negatively sloped.

Along $FF$ in Figure 1, $r^p$ satisfies $\pi^E(r^p, t, 0) = 0$ and, from (4.5),

$$t = (p - r^p) - (r^p - c^1)\varepsilon_r/\eta_r,$$

(4.7)

where $\eta_r = -\gamma y^2/\gamma^2 > 0$ represents the elasticity (with respect to an increase in $r$) of firm 2's derived demand for the input. If the tariff exceeds the price spread $p - r^p$ at which the boundary $FF$ intersects the vertical axis of Figure 1, then $M(r^p, t, 0) = r^p - p + t$ is strictly positive, ensuring that the equilibrium is in the region of vertical supply. Also, an increase in the quantity of supplies $x^2(r^p)$ increases the size of the price spread $p - r^p = -x^2(r^p)p'$. This shifts up the point at which $FF$ intersects the vertical axis, thus increasing the area of vertical foreclosure. If the input is not produced in country 2, then $p - r^p = 0$, and $FF$ reduces to a point at the origin. Firm 1 will then foreclose when the tariff is zero (at the origin), but a small tariff induces vertical supply.

12. This result is related to a Katz and Shapiro (1985) proposition concerning the licensing of a superior technology to a rival Cournot duopolist under constant returns to scale. Licencing occurs because the per unit royalty charge can be set so as to leave the rival's marginal cost unaffected (as in our model).

13. Assume firm 2 preferentially uses its own supplies when $r = c^2$. Thus $x = 0$ at $r = r^p = c^2$. If $r < c^2$, then $x^2 = 0$ and from (4.4) and (3.11), $\pi^E = My^2 + y^2$. If $\pi^E > 0$, then $r < c^2$ is binding and firm 1 sets $r = c^2 - \delta$ where $\delta > 0$ is small. If $t$ is large enough to make $M > 0$ and $\pi^E < 0$ at $r = c^2 - \delta$, then $\pi^E = 0$ in equilibrium.
We have shown that the sign of the profit margin condition $M$ evaluated at the foreclosure price is important for firm 1's decision to supply its rival. At a vertical supply equilibrium, the difference in profit margins is given by $M(r(t,0), t, 0) = r(t,0) - p + t$. Proposition 4 considers the sign of $M(r(t,0), t, 0)$, which subsequently proves to be significant for optimal policy by the exporting country.

**Proposition 4.** (i) The difference in profit margins at a vertical supply equilibrium is strictly positive if $e_i$ is sufficiently small: $M(r(t,0), t, 0) > 0$ if $e_i < rx/(r - c_1)x_e$. Conversely, $M(r(t,0), t, 0) < 0$ if $e_i \geq rx/(r - c_1)x_e$.

(ii) If $M(r^n, t, 0) \leq 0$ at foreclosure and $e_i$ is sufficiently large to induce vertical supply, then $M(r(t,0), t, 0) < 0$.

**Proof.** (i) Rearranging (4.4) using (3.4) and (4.1), at a vertical supply equilibrium,

$$\pi^E(r, t, 0) = M(r(t,0), t, 0)y^2 + x - (r - c_1)x^2 = 0. \quad (4.8)$$

From (4.8), $M(r(t,0), t, 0) > 0$ if and only if $x - (r(t,0) - c_1)x_e > 0$. Rearrangement of this last expression yields the stated elasticity conditions. (ii) Since $M_i > 0$ from (3.12) and $r(t,0) < r^o$ when there is vertical supply, it follows that $M(r(t,0), t, 0) < M(r^n, t, 0) \leq 0$. ||

Proposition 4(i) implies that the exporting firm earns a higher profit margin from the export of the input than the final product at a vertical supply equilibrium if the quantity of independent supplies of the input is not very responsive to price. This equilibrium can arise only if the difference in profit margins $M(r^n, t, 0)$ is strictly positive at the foreclosure price (as a consequence of a positive tariff on final product imports). If $M(r^n, t, 0)$ is negative, Proposition 4(ii) implies that $M(r(t,0), t, 0) < 0$ when there is vertical supply.

### 5. OPTIMAL EXPORT POLICY

This section is concerned with policy incentives in the exporting country. We consider a subsidy $s$ on final good exports as well as a tax $v$ on exports of the input. The welfare or objective function in country 1 is:

$$W(t - s, v) = \pi^E(r, t - s, v) - sy^1(r, t - s) + vx(r, t - s). \quad (5.1)$$

Thus the objective of country 1 is the same as the objective of firm 1 in the absence of tax and subsidy payments. The role for government in this situation arises from the inability of the firm to commit to its level of exports of the final product in stage 1 so as to be a "Stackelberg leader" in both export markets. By credibly committing to export policies in stage zero, the government can do what the firm itself is unable to do.

**Proposition 5.** Optimal policy by the exporting country adjusts exports to the levels that would have occurred at $s = v = 0$ if the exporting firm could commit to the quantity of its exports of the final product as well as to the export price of the input at stage 1.

**Proof.** See Appendix. ||

To understand optimal policy for the exporting country, we first examine the actions of a Stackelberg leader able to commit to both exports. In choosing its strategy towards
both exports, the Stackelberg leader is constrained by the requirement that \( x \geq 0 \). If this constraint is binding, the equilibrium reduces to the familiar one in which the leader expands its exports of the final product above the Cournot level and the input is not exported.

The export market for the input becomes significant for the leader's behaviour if the constraint \( x \geq 0 \) is not binding. From (A1) in the Appendix, exports of the final product then satisfy,

\[
d\pi^1/dy^1 = (p + y^1 p' - t - c') + (r - c^1 + y^1 p')dy^2/dy^1 = 0,
\]

where \( dy^2/dy^1 \) is the slope of firm 2's reaction function and the input price \( r \) is set optimally. From the second term of (5.2), the leader firm recognizes that a reduction in its rival's output increases its profits from final product exports (as in the standard Stackelberg model), but that, in addition, its profits from the export of the input fall.

From (3.4) and (5.2), at the Cournot equilibrium with \( s = v = 0 \),

\[
d\pi^1/dy^1 = M(r, t, 0)dy^2/dy^1 = 0
\]

where \( dy^2/dy^1 \) is set from our assumption that the products are strategic substitutes. If the difference in profit margins \( M(r, t, 0) \) is positive, the leader firm reduces its final product exports below the Cournot equilibrium level. From (3.11), firm 1's profits are then increased by an increase in firm 2's output: the additional profits from the export of the input more than offset the lower profits from final product exports. As we shall show, these are just the conditions under which country 1 has an incentive to tax, not to subsidize, final product exports. Conversely, if \( M(r, t, 0) < 0 \), the leader firm expands its exports above the Cournot equilibrium level. In setting its Cournot equilibrium level of exports, firm 1 takes the rival's output \( y^2 \) as given. Since \( y^2 \) falls in response to an increase in \( y^1 \), firm 1 is "too aggressive" in increasing \( y^1 \) when \( M(r, t, 0) > 0 \), and "insufficiently aggressive" when \( M(r, t, 0) < 0 \).

### Export taxes or export subsidies

The exporting country sets \( s \) and \( v \) to maximize welfare as in (5.1). As shown in Appendix A, the optimal values of \( s \) and \( v \) satisfy the conditions (5.3), (5.4) and (5.5) that follow.

As in the previous Stackelberg analysis, if the constraint \( x \geq 0 \) is binding, then exports of the input are zero and the equilibrium reduces to the familiar single market case. The subsidy on final product exports then satisfies,

\[
W_s = y^1 p' dy^2 / ds - sdy^1 / ds = 0
\]

where \( dy^1 / ds > 0 \). If firm 2 enters as a producer of the final product, then \( dy^2 / ds < 0 \) and, from (5.3), the optimal subsidy is positive. This is just the Spencer and Brander (1983) result that an export subsidy increases domestic welfare when there is Cournot competition between a foreign and a domestic firm. With no exports of the input, a change in the tax \( v \) applied to exports of the input has no effect.

In the region of vertical supply, the optimal export tax \( v \) on the input satisfies,

\[
W_v = (v x_r - s y_r^1) r_v = 0.
\]

Thus the optimal export tax depends on \( s \):

\[
v(s) = sy^1 / x_r.
\]

From (5.4), if \( s \) equals zero, then \( v \) equals zero. Since the exporting firm is able to commit to the price charged for the input in stage 1, the gain from government intervention in the export market for the input arises only as a consequence of intervention in the export market for the final product.

When \( v \) is set optimally at \( v(s) \), the optimal value of \( s \) satisfies

\[
dW(s, v(s))/ds = (r(t - s, v) - p + t)y^2_t - sY_s = 0.
\]
where the "social difference" in profit margins \( r(t-s, v(s)) - p + t \) reduces to the private difference in profit margins \( M(r(t, 0), t, 0) \) prior to policy by the exporting country. It follows that whether a subsidy or tax on final goods exports increases welfare depends on the sign of \( M(r(t, 0), t, 0) \). Proposition 6 sets out country 1's jointly optimal policy towards both exports if there is initial vertical supply.

**Proposition 6.** It is optimal for the exporting country:

(i) to tax the exports of both products if \( M(r(t, 0), t, 0) > 0 \).
(ii) to subsidize the exports of both products if \( M(r(t, 0), t, 0) < 0 \).
(iii) not to intervene if \( M(r(t, 0), t, 0) = 0 \).

**Proof.** From (5.5) at \( s = v = 0 \), \( dW/ds < 0 \) if \( M(r(t, 0), t, 0) > 0 \) and vice versa. \( dW/ds = 0 \) if \( M(r(t, 0), t, 0) = 0 \). From (5.4), we have \( v > 0 \) if \( s < 0 \) and vice versa.

As Proposition 6 shows, active commercial policy requires a subsidy to both exports or a tax to both exports. It is never optimal to subsidize exports of the final product and to tax exports of the input or vice versa. We know from Proposition 4(i) that \( M(r(t, 0), t, 0) > 0 \) if \( \epsilon \) is not too large and there is vertical supply as a consequence of a tariff set by country 2. From (5.2), a Stackelberg leader would then restrict its exports of the final product below the Cournot equilibrium level so as to gain additional profits from the export of the input. From Proposition 6(i), country 1 gains by taxing both exports in this case. High values of \( \epsilon \) are associated with the subsidization of both exports.

To explain the need to tax (or subsidize) both exports, first consider that if firm 1 initially earns a higher profit margin from the export of the input than the final product, then, at any given price for the input, a tax on final product exports serves to switch sales to the more profitable export market for the input. Such a tax, however, creates a wedge between the exporting firm's objective function and welfare in country 1. Firm 1 has an incentive to set the input price below the optimal level so as to reduce its exports of the final product and the total tax paid. This distortion can be corrected if country 1 also imposes a tax on exports of the input; the tax \( v \) is set so that changes in \( r \) do not affect the net tax, \( wx - sy \), that firm 1 pays to the government (see (5.4)). A similar but opposite argument applies if the difference in profit margins from the export of the intermediate and final products is initially positive.

Since \( s \) is larger than \( v \) in absolute value (from (5.4) and \(|y_1^1/x_1^1| < 1\)), the direction in which policy by the exporting country aims to switch trade flows is generally indicated by the sign of \( s \). There is some ambiguity in the response of exports to a change in \( s \) when \( v \) is set optimally, but under linear demand and supply conditions, an increase in \( s \), maintaining \( v = v(s) \), causes a net expansion in exports of the final product and a reduction in exports of the input.\textsuperscript{14} If \( s \) is positive, then both exports are subsidized, but sales of the final good are given relatively more encouragement. Since exports of the final good are subsidized if and only if firm 1 earns a lower profit margin on intermediate than final product exports, it follows that the net effect of government intervention is to widen or amplify the initial difference in profit margins. With linear demand and supply,

\textsuperscript{14} The effects of \( s \) on \( y_1^1 \) and \( x \) depend on \( dr/ds = r_x + r_y \psi(s) \). From (4.4), \( r_x = x/\pi_1^y > 0 \), but the signs of \( r_y \) and \( dr/ds \) are ambiguous. From \( \pi_1^y = My_1^y + y - (r - v - \epsilon')x_1^y = 0 \) (as in (4.8)), we obtain \( r_y = -[My_1^y + y^2 - My_1^y]/\pi_1^y > 0 \). If demand is linear, then \( r_x > 0 \). Thus if country 1 is restricted to trade policy towards the final product alone (i.e. \( v = 0 \)), the subsidy \( s \) can reduce \( x \) both by raising firm 2's costs and by directly increasing \( y_1^1 \). Allowing for changes in \( v \), if \( p^* = x_1^y = 0 \), then \( dr/ds < 0 \), but the signs, \( dy_1^1/ds > 0 \) and \( dx/ds < 0 \), are unchanged.
optimal policy then serves to expand the export of the good with the higher profit margin and to contract the export of the good with the lower profit margin.

**Government policy and vertical supply**

What are the implications of policy by country 1 for the availability of imported supplies in country 2? If firm 1 supplies the input in the absence of export policy and \( M(r, t, 0) < 0 \), then Proposition 6(ii) implies that it is optimal for country 1 to subsidize both exports. As previously explained, the subsidy to final product exports dominates and, under linear demand and supply conditions, firm 1 reduces its exports of the input. Vertical supply becomes less profitable because the subsidy to final product exports reduces the final output of firm 2 and its demand for imports of the input. If initially very little of the input is exported, government foreclosure may occur; the subsidy policy may shift the equilibrium from vertical supply to vertical foreclosure. Conversely, if \( M(r, t, 0) > 0 \) at a vertical supply equilibrium, there is a tendency for policy by the exporting government to increase the extent of vertical supply.

In considering the implications of policy by country 1, it must be remembered that country 2 can critically affect the vertical supply decision by setting a tariff on final product imports. If country 1's government does not intervene, country 2 can induce vertical supply by setting the tariff so as to make \( M(r, t, 0) \) strictly positive (see Propositions 1 and 2). Furthermore, the tariff can be set at a level which makes \( M(r(t, 0), t, 0) > 0 \) at the vertical supply equilibrium; such a tariff moves the equilibrium into the region in which country 1 has an incentive to tax both exports. Thus policy by the importing country can serve to change the conditions underlying the vertical supply decision so that both the exporting firm and country have an incentive to increase the extent of vertical supply.

6. **BERTRAND BEHAVIOUR**

In considering Bertrand behaviour, we assume that each firm produces a differentiated final product. Firm 2 can then earn positive profits despite a higher marginal cost of production. The demand functions for the final substitute products are represented by \( y^1 = q^1(p^1, p^2) \) and \( y^2 = q^2(p^1, p^2) \) where the outputs \( y^1 \) and \( y^2 \) have prices \( p^1 \) and \( p^2 \) respectively. Own-price effects \( q^1 \) and \( q^2 \) are assumed to be negative and dominate the positive cross-price effects \( q^1 \) and \( q^2 \). Firm 1's profit is given by (3.1) where \( p \) is replaced by \( p^1 \) and \( x = q^2(p^1, p^2) - x^2(r) \). The first-order condition for the second-stage choice of \( p^1 \) by firm 1 given \( p^2 \) and \( r, t-s, v \) (where the superscript \( B \) stands for Bertrand) is

\[
\Pi^B_1(p^1, p^2, r, t-s, v) = y^1 + (p^1 - t + s - c^1)q^1_1 + (r - v - e^1)q^2_2 = 0. \tag{6.1}
\]

The presence of an export market for the input has a fundamental effect on the price \( p^1 \). As shown by the positive third term of (6.1), firm 1 recognizes that an increase in \( p^1 \) (for a given value of \( p^2 \)) increases its profits from the export of the input by raising its rival's level of production of the final good.\(^{17} \)

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15. In an earlier working paper (Rochester Center for Economic Research, 1989, No. 194), we show that government foreclosure can occur in the linear case if \( M(r^*, t, 0) < 0 \) and the subsidy required to induce foreclosure is sufficiently small.

16. At the prohibitive tariff \( r^* \), \( y^1(r, r^*) = 0 \) and \( M(r, r^*, 0) = r(r^*, 0) - c^1 > 0 \) is bounded away from zero. Since \( M \) is continuous, \( M(r(t, 0), t, 0) > 0 \) at some \( t < r^* \).

17. If firm 1 commits to \( x \) (rather than \( r \)) in stage 1, we require that the market price of the input (determined in stage 2) is taken as given by both firms.
The second-stage choice of \( p^2 \) by firm 2 satisfies the standard Bertrand first-order condition: 
\[
\pi^{2B}_2 = y^2 + (p^2 - r)q^2_2 = 0.
\]
The products are assumed to be strategic complements, \( \pi^{1H}_{12} > 0 \) and \( \pi^{2H}_{21} > 0 \), so that the reaction functions in price space have positive slopes. We assume the second-order and uniqueness conditions hold.

To relate the Bertrand results to those that would be obtained with Cournot competition for the final differentiated products, it is useful to define the inverse demand functions 
\[
p_i = p_i(y^1, y^2) \quad \text{and} \quad p^2 = p^2(y^1, y^2).
\]
These functions have partial derivatives (from total differentiation of the direct demand functions),
\[
\frac{\partial p_i}{\partial q_i} = -\frac{q_i}{Q} < 0, \quad \frac{\partial p^2}{\partial q^2_i} = -\frac{q_i}{Q} < 0 \tag{6.2}
\]
where \( Q = q^1_i q^2_i - q^2_i q^2_i > 0 \). By totally differentiating \( \pi^i(y^1, y^2, r) \) with respect to \( p^1 \), holding \( p^2 \) fixed, (6.1) can be written in the form,
\[
\pi^{1B}_1(p^1, p^2, r, t - s, v) = \pi^{1B}_1(y^1, y^2, r, t - s, v)q^1_i + Mq^2_i = 0, \tag{6.3}
\]
where \( M = \pi^{1B}_1(y^1, y^2, r) = r - v - c^1 + y^1 p^1_2 \) is the effect of an increase in the rival’s output on firm 1’s profit holding \( y^1 \) and \( r \) fixed. As before, \( M \) can be expressed in terms of the “difference in profit margins”. Substituting for \( y^1 \) from (6.1) and using (6.2), at the Bertrand equilibrium, \( M = M^B(r, t - s, v) = q^1_i p^1_2 [(r - v - c^1) - (p^1_i - t + s - c^1) p^2_i / p_i] \).

Similarly, at the Cournot equilibrium, obtaining \( y^1 \) from \( \pi^1(y^1, y^2) = 0 \), it follows that 
\[
M = M^C(r, t - s, v) = (r - v - c^1) - (p^1_i - t + s - c^1) p^2_i / p_i. \tag{6.4}
\]
If the products are homogeneous, \( M^C \) reduces to the actual difference in profit margins. Since they are evaluated at different prices and outputs, \( M^B \) and \( M^C \) may differ in sign. 18

We next examine the incentives for vertical supply when there is Bertrand competition at the final stage. Taking the total derivative of \( \pi^1(y^1, y^2, r) \) with respect to \( r \) and imposing (6.3), firm 1 sets \( x > 0 \) when \( s = v = 0 \) if, at \( r^p \),
\[
d\pi^{1B}/dr = M^B(r^p, t, 0)[y^i_1 - (q^2_i / q^1_i)y^i_1] - (r^p - c^1)x^2_i < 0. \tag{6.4}
\]
where \( y^i_1 = dq^i(p^1, p^2)/dr \) for \( i = 1, 2 \). From (6.4), there is no change in the general factors underlying the vertical supply decision. Since \( y^2_i - (q^2_i / q^1_i)y^i_1 = (dp^2 / dr)/p^2_i < 0 \) (from \( p^2_i / p^2 = -q^2_i / q^1_i \) and (6.2)), the condition \( M^B(r^p, t, 0) > 0 \) is sufficient (but not necessary) for vertical supply. By similar reasoning to that in the Cournot case, a sufficiently large import tariff \( t \) or responsiveness of supplies \( e, \) in the importing country will induce vertical supply. Moreover, if demand and supply are linear, an increase in \( x^2_i(r^p) \) again moves the equilibrium towards foreclosure. 19

The policy incentives for the exporting country can again be seen most easily by examining the decisions that would be made by the exporting firm if it were a Stackelberg leader in both markets. Assuming vertical supply (from (6.3) and (5.2) with \( p' \) replaced by \( p^1_2 \)), \( y^1 \) satisfies,
\[
d\pi^{1S}/dy^1 = \pi^{1B}(p^1, p^2, r, t, 0)/q^1_i + (r - c^1 + y^1 p^1_2)(dy^2/dy^1 - q^2_i / q^1_i) = 0. \tag{6.5}
\]
The term \( q^2_i / q^1_i \) represents firm 1’s Bertrand conjectural variation in output space: it is

18. At the Cournot equilibrium outputs, \( \pi^{1B} = M^C q^2_i \) and \( \pi^{2B} = \pi^{2} q^2_i / q^1_i < 0 \) (from \( \pi^1 = \pi^2 = 0 \), (6.3) and (6.4)). Thus, under Bertrand competition, firm 2 reduces \( p^2 \) below its Cournot level (holding \( r \) fixed).
19. Price overshifting may make \( dM^B/dr < 0 \) for a range of values of \( t \). Nevertheless, vertical supply occurs at some \( t < t^* \) where \( y^i(r^p, t^*) = 0 \) (since \( M^B > 0 \) and bounded away from zero at \( t^* \)). When demand is linear, \( M^B > 0 \). The effect of an increase in \( x^2(r^p) \) then follows as in the proof of Proposition 2(i).
the ratio of the change in outputs from an increase in $p^1$ holding $p^2$ fixed. Since the products are assumed to be strategic complements, the "conjectured" reduction in firm 2's output from an increase in $y^1$ exceeds any actual reduction in output making $dy^2/dy^1 - q^2_i/q^1_i > 0.20$

At the Bertrand equilibrium at $s = v = 0$, from (6.1) and (6.5), $d\pi^{1s}/dy^1 = M^B(r, t, 0)(dy^2/dy^1 - q^2_i/q^1_i)$. If $M^B(r, t, 0) > 0$ then $d\pi^{1s}/dy^1 > 0$: profits increase if final product exports are increased above the Bertrand equilibrium level. Conversely, if $M^B(r, t, 0) < 0$, the exporting firm gains if exports of the final product are restricted below the Bertrand level. In setting its Bertrand equilibrium price $p^1$, firm 1 takes the rival's price $p^2$ as given. If $M^B(r, t, 0) > 0$, an increase in $y^2$, or, equivalently, a reduction in $p^2$, increases firm 1's profits from exports of intermediates by more than it reduces profits from final product exports. Since $p^2$ falls in response to a reduction in $p^1$, firm 1 is then "not sufficiently aggressive" in reducing $p^1$. Conversely, when $M^B(r, t, 0) > 0$, firm 1 is "too aggressive" in reducing $p^1$.

At a vertical supply equilibrium (as shown in the Appendix), the optimal subsidy $s$ on final product exports and the optimal export tax $v$ on the input is

$$s = (r - c^1 + y^1p^1_i)(dy^2/dy^1 - q^2_i/q^1_i)/z$$
and
$$v(s) = sy^1_i/x_r,$$  

(6.6)

where $z = [1 - (q^2_i/q^1_i)(y^1_i/x_r)] > 0$. Optimal export policy in the Cournot case can be seen from (6.6) by setting the conjectural variation $q^2_i/q^1_i$ equal to zero.

Although the function $v(s)$ is unaffected, the form of the final-stage competition can change the relationship between the two export policies. With Bertrand competition an increase in $r$ may reduce firm 1's final product exports. It then follows from (6.6) that if country 1 subsidizes exports of the final product, then it should tax exports of the input and vice versa. Firm 1's vertically integrated structure is important for this result. In the standard Bertrand model, both $p^1$ and $p^2$ increase with $r$ but, when demand is linear, the increase in $p^2$ dominates, making $y^1_i > 0$. In our setting, firm 1 has an additional motive to raise $p^1$ (and reduce $y^1$); an increase in $p^1$ increases firm 2's output and firm 1's sales of the higher priced input (see (6.1)). This effect is sufficient to make $y^1_i < 0$ in the linear demand case (see the Appendix).

As might be expected from Eaton and Grossman (1986), the direction of optimal policy towards exports of the final product is reversed if the final products are strategic substitutes (in the Bertrand case $dy^2/dy^1 - q^2_i/q^1_i < 0$ in (6.6)) rather than strategic complements. Moreover, a move from Bertrand competition (with strategic complements) to Cournot competition (with strategic substitutes) also reverses the sign of $s$ when $M^B$ and $M^C$, evaluated at $s = v = 0$, have the same sign (see (6.5) and (5.2)). In the single-market case examined by Eaton and Grossman (1986), $M^B$ and $M^C$ are both negative. More generally, the signs of $M^B$ and $M^C$ may differ. The simple reversal of optimal policies does not fully extend to the case of two vertically related markets.

7. CONCLUSION

Many large manufacturing firms have secured their access to important intermediate inputs by integrating backwards so as to produce the input within the corporation. If the input can be produced more cheaply in one country than another, then vertical

20. From total differentiation of the direct demand functions, using (6.2), we obtain $dy^2/dy^1 - q^2_i/q^1_i = Q(dp^2/dp^1)/(q^2_i(dy^1/dp^1)) > 0$ when $dp^2/dp^1 > 0$.
integration can give firms in one country a cost advantage relative to foreign rivals. This leads to the question as to whether high-cost manufacturers need be concerned about dependence on imports of a key input from a country that is also a major exporter of the final manufactured product.

This paper first addresses this issue by examining the conditions under which a low-cost vertically integrated manufacturer will export an intermediate product, lowering the costs of a rival producer of the final product. We show that, for either Cournot or Bertrand competition at the final output stage, the incentive for vertical supply is increased both by a greater responsiveness of supplies in the importing country and (more surprisingly) by a reduction in the quantity of supplies available at foreclosure. An importing country can induce vertical supply by imposing a sufficiently large tariff on final product imports.

Secondly, we consider the implications of optimal policy by the exporting country. When there is Cournot competition for the homogeneous final products, policy by the exporting country tends to reinforce existing private incentives as measured by the difference in profit margins from the export of the input and the final product. If the difference in profit margins is initially positive, then optimal policy tends to increase the extent of vertical supply. In the reverse case, optimal policy tends to move the equilibrium towards foreclosure.

It is a familiar result from the analysis of duopoly behaviour for a single product that a Cournot firm sets output levels too low, and a Bertrand firm too high, relative to a firm that can behave as a Stackelberg leader. In our model, optimal policy by the exporting country encourages the exporting firm to expand sales in the market with the higher profit margin in the Cournot case and to de-emphasize sales in such a market in the Bertrand case. Since the specific nature of optimal policy depends on the nature of competition at the final stage, we would not expect these results to be directly useful for practical policy by the exporting country and our analysis is not intended for this purpose. In a world of imperfect information and imperfect governments, activist policy often provides an avenue for socially wasteful rent-seeking. It nevertheless seems a worthwhile objective to gain some understanding of the factors that are important determinants of vertical supply.

Overall, our results indicate that taking into account both private and public incentives, in a broad class of cases a high-cost firm need not be concerned about full vertical foreclosure. Our analysis is based on the assumption that there is only one potential supplier of the input. If there were more than one exporting firm, it seems reasonable to conjecture that competition between exporters would result in a further increase in vertical supply. Nevertheless, the price at which supplies can be imported is likely to be high. Generally, producers of the final product will not overcome an initial cost disadvantage in the production of the input by importing supplies from low-cost foreign rivals.

We have abstracted from the possibility that the firms might bargain over the price charged for the input. If the importing country imposes a tariff on final product imports, the joint profit maximizing solution is for the low-cost firm to export the input only, giving the rival firm a monopoly of the market for the final product. However, this solution would require non-linear pricing and may be difficult to enforce. Merger between the two firms would seem to be a better means of achieving this outcome, but it may be ruled out by antitrust policy. Also, if we allow for more than one rival firm in the importing country, both the bargaining and (full) merger solutions become more difficult to achieve. The arms-length price or quantity setting behaviour that we have analysed may well be an attractive strategy for the low-cost firm.
APPENDIX

We first set out the conditions for the "Stackelberg" equilibrium and then derive the conditions (5.3), (5.4) and (5.5) of the text characterizing optimal policy by the exporting country. We next combine the two sets of conditions to prove Proposition 5. Turning to Bertrand competition, we derive condition (6.6) concerning optimal policy towards exports and the comparative static effects of r.

Stackelberg equilibrium

Suppose that the exporting firm is able to commit to both \( y^1 \) and \( r \) in stage 1, and firm 2 is a follower, setting its output in stage 2. From (3.5), \( y^2 \) is defined as a function \( f^2(y^1, r) \) with partial derivatives \( f^2_r = dy^2/dr^1 = -\pi_5/\pi_{12}<0 \) and \( f^2_s = 1/\pi_{12}<0 \). The values of \( y^1 \) and \( r \) are chosen to maximize profits subject to \( x = f^2(y^1, r) - x^2(r) \geq 0 \). Let \( L(y^1, r, \theta = \pi^1 + \theta x \) where \( \theta \geq 0 \) is the Lagrange multiplier. At \( s = v = 0 \), \( y^1 \) and \( r \) respectively satisfy:

\[
L_1(y^1, r, \theta) = p + y^1p' - t - c^1 + (r - c^1 + y^1p' + \theta) f^1_s = 0 \quad (A1)
\]

\[
L_4(y^1, r, \theta) = (r - c^1 + y^1p' + \theta) f^1_s - (r - c^1 + \theta) x^2 + x = 0 \quad (A2)
\]

where \( x = 0 \) if \( \theta > 0 \).

Optimal policy by the exporting country

The exporting country sets \( s \) and \( v \) to maximize (5.1), subject to the constraint \( x(r, t-s) \geq 0 \). Let \( L^* = W(t-s, v) + \mu(x(r, t-s)) \) where \( \mu \geq 0 \) represents the Lagrange multiplier. At the maximum, \( s \) and \( v \) satisfy the first-order conditions:

\[
L_i^* = W_i + \mu(x_i) = 0 \quad (A3)
\]

If \( \mu > 0 \), then \( x = 0 \) and \( dx_i = x_i r^0 + y^2_i = 0 \), so that \( L_i^* = W_i \) for \( \mu \geq 0 \). We assume that the second-order conditions, \( W_{ss^2} < 0 \), \( W_{ss} < 0 \), and \( W_{sv} + W_{sv} = 0 \) are satisfied. These conditions hold if \( p''(Y) = 0 \) and \( x^0 = 0 \).

From (A3) and (5.1), the optimal values of \( s \) and \( v \) satisfy,

\[
W_i(s, v) = \pi^E_i dr_i / ds + \pi^V_i - y^1 - sy^1 + vy^2_1 - (sy_1 - vx_1) dr_i / ds = 0 \quad (A4)
\]

\[
W_v(s, v) = \pi^E_v + x + (vx_1 - sy_1) dr_i / dv = 0 \quad (A5)
\]

where \( dx_i / dv = r_i (t-s, v) \) and \( dy_i / dv = r_i (t-s, v) \) if \( r < r^0 \). At \( r^0 \), \( dr_i / dv = r_i \) and \( dy_i / dv = 0 \). Since \( \pi^E_s = 0 \) when \( r_i > 0 \), the term \( \pi^E_s dr_i / dv = 0 \). From (4.3) and (3.4),

\[
\pi^E_i(r, t-s, v) = (r - c^1 y^1 + y^1 + y^2 p_2 y^2_1) \quad \text{and} \quad \pi^E_i(r, t-s, v) = -x_i < 0 \quad (A6)
\]

If \( \mu > 0 \), then \( r = r^0 \) and using (A6), \( \pi^E_i(r, t-s, v) = (r - c^1) y^1 + y^1 + y^2 p^2 y^2_1 + x r^0 \) and \( dx_i / dv = 0 \), (A4) becomes

\[
W_i = y^1 p^2 dy^2 / ds - sy^1 - vy^2_1 = 0 \quad \text{which is (5.3) of the text. Since} \ dx_i / dv = d y_i / d s - x^2 r^0 / ds \Rightarrow d y_i / d s = x^2 r^0 / ds < 0 \ \text{if} \ x^2 (r^0) > 0 \ . \text{If} \ \mu = 0 \ , \text{then} \ x \geq 0 \ \text{which includes the region of vertical supply and}, \ \text{using (A6), (A5) reduces to} \ W_i = (ux_1 - sy_1) r_i = 0 \ \text{so that} \ x(s) = sy_1 / x, \text{as in (5.4) of the text}. \text{Using} \ \pi^E_s = 0, (A4), (A5) \text{and (A6),}
\]

\[
dW_i(s, v(s)) / ds = (r - c^1 + y^1 p^2) y^2_1 - sy_1 = 0, \text{which, applying (3.4) and (5.4), reduces to (5.5) of the text.}
\]

Proof of Proposition 5. Suppose that the constraint \( x \geq 0 \) is not binding so that \( \theta = 0 \). Substituting \( s = (r - c^1 + y^1 p^2) y^2_1 / y^1 \) into (3.4), and using \( y^2_1 / y^1 = f^1_s \), it follows that (3.4) is equal to (A1) and that \( y^1 \) is the same in both cases. Similarly, from (5.4) and (5.5), and using \( y^2_1 = f^1_s y^1_2 + f^2_s \), (4.4) reduces to (A2): the price \( r \) is the same in both cases. If the constraint \( x \geq 0 \) is binding, then \( \theta > 0 \) and from (A2) and (A1), \( L_1(y^1, r, \theta = \pi^1 + \theta x \) and \( f^1_s = 0 \). From \( dx_i / ds = f^2_s dy_i / ds + (f^2_r - x^2) r^0 \) and \( dy_i / ds = x^2 r^0 \), it follows that (A1) is equal to (3.4) where \( s \) is given by (5.3).

Bertrand behaviour: derivation of \( s \) and \( v \) as in (6.6)

From (6.1), \( \pi^E_i(r, p^1, p^2, r, t-s, v) = \pi^V_i(p^1, p^2, r, t, 0) + sq^1 - vq^1 = 0 \). Assuming vertical supply, \( \theta = 0 \), and from (6.5), the optimal values of \( s \) and \( v \) satisfy

\[
d\pi^E / dy^1 = -(s - v(q^1_1 / q^1_1)) + (r - c^1 + y^1 p^1_2)[dy^1 / dy^1 - q^1_1 / q^1_1] = 0 \quad (A8)
\]
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