ELSEVIER

Journal of International Economics 48 (1999) 199–232

Journal of INTERNATIONAL ECONOMICS

Rent-shifting export subsidies with an imported intermediate product

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Received 26 January 1996; received in revised form 6 February 1998; accepted 20 April 1998

Abstract

This paper examines the implications of foreign or domestic imperfect competition in intermediate-goods supply for strategic trade policy. Assuming Cournot competition, an export subsidy aimed at shifting rents from foreign to domestic final-good producers may also shift rents to foreign suppliers, weakening the incentive for the subsidy. However, the incentive for a subsidy tends to increase if the intermediate-good industry is purely domestic or if the industry is purely foreign, but the subsidy reduces the price of the imported input. Alternative rent-shifting policies (a production subsidy and an import tariff) applied to the input are also considered.

Keywords: Export subsidies; Strategic trade policy; Vertically related markets; Vertical oligopolies; Trade in intermediate goods

JEL classification: F12; F13

1. Introduction

Export subsidies, such as attractive terms of credit for export sales,[1] have often been promoted on the basis that they might allow domestic exporters to gain

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[1]Low interest loans to finance exports and tax incentive systems such as deductions for export earnings and accelerated depreciation for exporting firms are common forms of export subsidies.

PII: S0022-1996(98)00042-7

market share and presumably profit at the expense of foreign rivals. Indeed, as the Brander and Spencer (1985) model of strategic-trade policy has shown, there are conditions under which such a subsidy might actually raise domestic welfare. However, given the importance of world trade in intermediate products, firms producing manufactured goods for export would also commonly import parts or components from foreign suppliers. This suggests that if the foreign suppliers are oligopolistic, they could share in some of the rents resulting from, say, a domestic subsidy to final-good exports, opening the question as to the ultimate recipients of the rents shifted by strategic trade policies. For example, Samsung Electronics Inc. is a major exporter of electronic products from Korea, but has also imported significant components such as flat panel displays (for computers), glass bulbs and electronic guns (for televisions), and magnetrons (for microwave ovens) from Japanese firms such as Toshiba Corp. and Sharp Corp.[2] If Korea chose to provide tax or financing incentives to promote electronics exports, would the rents go to Korean firms such as Samsung or would they be further shifted so as to mostly benefit the Japanese parts suppliers?

This paper explores the strategic-trade policy implications of having foreign rather than domestic firms supply an imperfectly competitive intermediate good.[3] So as to clarify the additional effects arising from the intermediate-good market, we assume, as in the original Brander and Spencer (1985) model, that final-good producers in both the domestic and foreign countries act as Cournot competitors and that all the final product is exported to a third market. Cournot competition is also assumed in the intermediate-good market, with some consideration given to the special cases of domestic or foreign monopoly. The main policy considered is an export subsidy applied to the final good, but attention is also given to the rent-shifting implications of a production subsidy and an import tariff applied to the intermediate good.

With the addition of the intermediate-good market, strategic-trade policy involves consideration of three kinds of rent-shifting; between foreign and domestic final-good producers, between foreign and domestic intermediate-good producers and between final-good producers and intermediate-good producers. In support of the basic theme, we are able to show that if an export subsidy shifts

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[2]Other examples include LG Electronics Inc. which imports compressors for air conditioners from Matsushita Electric Industrial Co. Ltd of Japan. Also, Hyundai Motor Company, a major Korean exporter of autos, has used Mitsubishi Motors Corp. of Japan to supply engines. This phenomenon of the export of a good incorporating an imported intermediate product involves *vertical specialization*. Ishii and Yi (1997) provide evidence that *vertical specialization* has become highly significant in the growth of world trade.

[3]Other papers considering strategic trade policy in vertically related markets include Spencer and Jones (1991), (1992), Rodrik and Yoon (1989), Ishikawa and Lee (1997) and Chang and Chen (1994). Independently of an earlier version of this paper (see Ishikawa and Spencer, 1996), Bernhofen (1997) considers the effect of foreign monopoly supply of an input with linear demand, but the focus differs and there is very little overlap.

rents to intermediate-good producers, then the desirability of the policy is reduced to the extent that these producers are foreign. What is perhaps more surprising, at least initially, is the contrasting result that imperfect competition in a purely domestic intermediate-good industry tends to strengthen the argument for an export subsidy. By raising output, the subsidy reduces the efficiency loss arising from 'double-marginalization' in vertical oligopolies. Moreover, under a fairly broad set of demand conditions, imperfect competition in even an entirely foreign intermediate-good industry can increase the incentive to subsidize exports. The only requirement for this last result is that the subsidy reduce the price of the imported input, improving the terms of trade.

Although our analysis is limited to the Cournot case, outputs may be either strategic substitutes or complements because of general demand conditions. The model is also general with respect to the numbers of domestic and foreign firms in both the intermediate and final-good markets. Interestingly the Eaton and Grossman (1986) result that the optimal export policy is a tax in the strategic complements case proves to be robust to this more complex environment, provided that at least two domestic firms produce the final good.[4] For the case of only one domestic firm, we demonstrate a counterexample. The conditions determining the sign of export policy are particularly simple if the input is supplied by a monopolist and demand is linear. If the monopoly is domestic, the optimal policy is always a subsidy, regardless of the relative numbers of domestic and foreign final-good producers, whereas for a foreign monopoly supplier, the policy switches to an export tax.

Assuming segmented markets for the intermediate good, the domestic price for the input can differ from the foreign price even in the absence of trade restrictions. However, for simplicity, our main analysis concerns the case in which the foreign price is an exogenously given constant, perhaps because the foreign final-good firms are vertically integrated.[5] This provides a better focus on main effects, but also, when we later relax this assumption, there is surprisingly little impact on results. If demand is linear, the results are identical since the foreign price of the intermediate good is actually unaffected by domestic export policy.

The paper is organized as follows. Section 2 provides an overview of the model of vertical Cournot oligopolies. Section 3 then sets out the market equilibrium conditions and Section 4 develops the domestic welfare effects of a subsidy to final-good exports with a focus on the additional effects caused by the foreign or

[4]Maggi (1996) shows that similar results also hold in a duopoly model involving capacity and then price competition. As the rigidity of the capacity constraint is relaxed, the model outcome varies from Cournot to Bertrand and optimal export policy shifts from a subsidy to a tax. However, capacity subsidies (weakly) raise home welfare under both Cournot and Bertrand outcomes.

[5]Spencer and Raubitschek (1996) make a similar assumption. They also suggest that the input could be produced by a foreign competitive industry but be exported through a government mandated export cartel. Alternatively, regulation could constrain the price within the foreign country, but not exports.

domestic intermediate-good industry. Next, Section 5 examines the conditions under which the policy is a subsidy or a tax. The assumption that the input price paid by foreign final-good firms is exogenous is relaxed in Section 6 and Section 7 concerns the welfare effects of a domestic production subsidy and import tariff applied to the intermediate good. Finally, Section 8 provides concluding remarks.

2. Model structure

As illustrated by Fig. 1, there are two vertically related activities in two countries, country *D* (for domestic) and country *F* (for foreign). In the upstream stage, a homogeneous intermediate good is produced at a constant marginal cost from labor alone, whereas in the downstream stage, the intermediate good and labor are combined to produce a homogeneous final good, also at constant marginal cost. Both the markets for the upstream and downstream goods involve

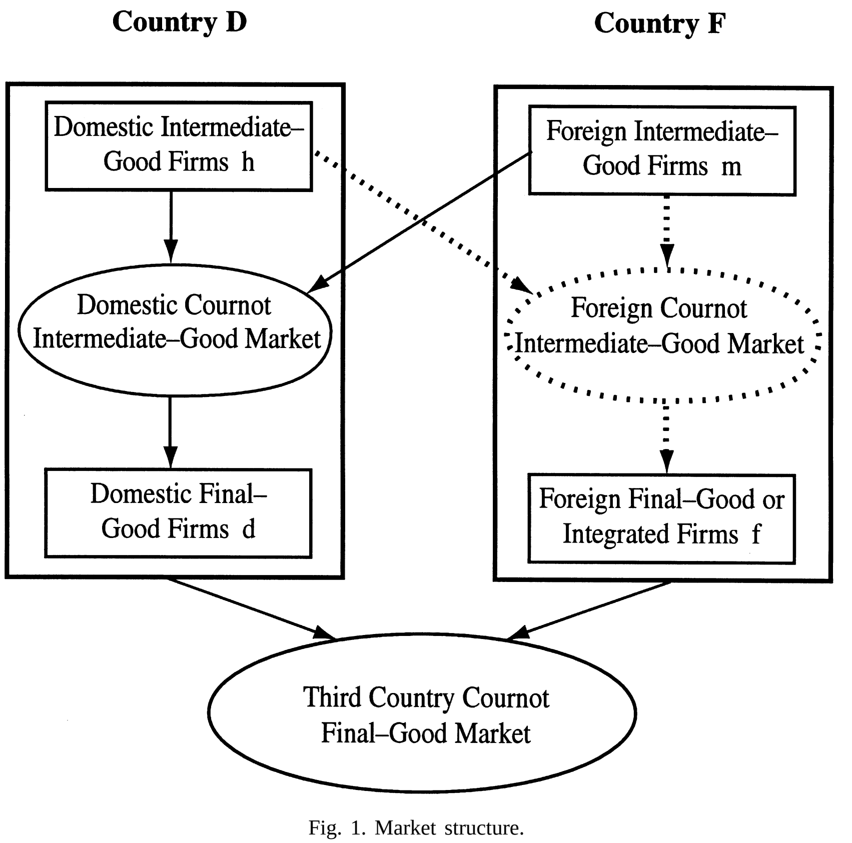

Fig. 1. Market structure.

Cournot competition between general numbers of domestic and foreign firms. Typical domestic and foreign final-good producers are referred to as firm d (d for domestic) and f (f for foreign), respectively, and typical domestic and foreign intermediate-good producers as firms h (h for home) and m (m for imports), respectively. There are n^i firms of type i for $i = d, f, h$, and m respectively.

As shown by the solid arrows to the oval shaped field at the bottom of Fig. 1, firms d in country D and firms f in country F, export all of their output of the final good to a third country market. In our main model, as illustrated by the solid arrows from firms h and m to the domestic intermediate-good market (shown as the oval within country D), we focus on the domestic trade policy implications of the source of the intermediate good, whether domestic or foreign. This market determines the domestic price, denoted r^D, at which domestic firms d purchase the input, whereas the foreign price, denoted r^F, paid by firms f in the foreign country is assumed to be exogenous. However, we later relax this assumption (in Section 6) to incorporate the endogenous determination of intermediate-good prices in both countries. As illustrated by the dotted (in addition to the solid) arrows in Fig. 1, the same group of identical firms h or m then provide the input to the segmented markets in both countries.

The model involves three stages of decision. In stage 0, the domestic government commits to the values of its trade policy instruments. Next, stage 1 in our main model involves a Cournot-Nash equilibrium in which firms h and m commit to the quantities of the intermediate good supplied to country D so as to maximize profits taking rival's outputs as given. When r^F is made endogenous, there are segmented Cournot markets for the intermediate good in each country. Each firm h or m then commits to the quantities supplied to each country, taking the quantities supplied by other firms to each country as given. The market for the final good in stage 2 also involves a Cournot-Nash equilibrium in which firms d and f set their exports to the third country market so as to maximize profits taking rival's exports and also the input prices r^D and r^F as given. Hence the input price r^D is simply the domestic market-clearing price, which equates the demand by firms d at the price r^D to the total amount of the input supplied to country D in stage 1. Similarly, when r^F is endogenous, r^F is the market-clearing price in country F.

Although the model assumed for the vertical Cournot oligopolies is well established in the antitrust literature,[6] the game-theoretic structure is open to the criticism that downstream firms recognize their market power as sellers of the

[6] See, for example, Greenhut and Ohta (1979), Abiru (1988) and Salinger (1988). Trade papers using this structure include Spencer and Raubitschek (1996), Ishikawa and Lee (1997) and Ishikawa (1998). The model provides a convenient way to incorporate a homogeneous input with general numbers of upstream as well as downstream firms earning above normal profits. Since strategic-trade policy results are known to be sensitive to the numbers of domestic and foreign firms, this generality is important in our context.

final-good, but act non-strategically, taking price as given, as buyers of the input. One possibility might be to try to relax the assumption of price-taking behavior so as to incorporate monopsony power by downstream firms. However, it is not obvious how to do this in a vertical Cournot setting in which upstream firms move first to produce the input.[7] A more promising approach is to try to justify this price-taking behavior as arising from a reduced form version of a Kreps and Scheinkman (1983) type game in which price setting under capacity constraints by upstream producers leads to Cournot outcomes.[8] Nevertheless, the development of such a model is likely to be quite technically difficult.[9]

An alternative and simpler approach, which we favor in this context, arises from the observation that any monopsony power of downstream firms becomes vanishingly small in the limit as the number of these firms increase. Hence, with large numbers of downstream firms, a natural simplifying assumption is that monopsony power is absent. This directly solves the problem for the domestic input market when n^d is large (and for the foreign market when n^f is large), but, although the large numbers case can be of interest when considering the effects of the intermediate-good market, the most relevant cases for strategic trade policy involve small numbers of firms. To handle the small numbers case, imagine there is some integer number K of identical downstream industries all demanding the same input from the single upstream industry and all subject to the same government policy interventions.[10] Since there is no change in the number of

[7]In a typical monopsony model, a buyer can reduce price by demanding less of an input that is supplied on the basis of a positively sloped supply curve. However, in the vertical Cournot setting, the supply curve for the input is vertical once the input is produced. If buyers reduce demand below the supplied quantity, the price would drop to zero, causing problems of existence of equilibrium.

[8]We would like to thank an anonymous referee for this very helpful suggestion.

[9]Consider the following game. In stage 1, firms h and m produce the quantities or ‘capacity’ of the input available to country D on the basis of Cournot competition. Marginal costs, which include the costs of any tariffs or subsidies applied to the intermediate good (from stage 0) are positive, but once ‘capacity’ is produced, no further costs are incurred (as in Kreps and Scheinkman, 1983). In stage 2, the input price r^D is determined on the basis of Bertrand price competition between firms h and m. In stage 3, taking r^D as given, firms d and f buy the input and produce the final good under Cournot competition. As explained by Tirole (Tirole, 1988; Ch. 5), Cournot outcomes are achieved if there is a pure (i.e. non-randomized) strategy equilibrium in stage 2, with capacities binding on each firm so that all available supplies are sold at a positive price. Profit maximization by Cournot producers (in stage 1) would lead to this pure strategy equilibrium under some assumptions. One well known key assumption is efficient rationing. Another assumption is concave demand, but Kreps and Scheinkman (1983) point out that this assumption is stronger than necessary. In addition, the literature assumes two firms with identical marginal costs of capacity, whereas, in our setting, the ‘capacity’ costs of producing the input can differ across general numbers of foreign and domestic firms. Analysis of these issues is beyond the scope of this paper, but it has been suggested that the Cournot result is more likely to obtain if marginal costs of capacity are high (see Tirole, 1988, p. 217 and Maggi, 1996, p. 246, footnote 18).

[10]In terms of Fig. 1, one could imagine the bottom-two rectangles and the oval being replicated across many different final-good industries, all using the same input. We are extremely grateful to Robert W. Staiger for the idea of justifying the model in this way and for his detailed descriptions of the method.

upstream firms, the quantity of the intermediate good produced by each firm would rise by the factor K, but results are otherwise unaffected because the price of the input and the response of downstream firms to variations in the price of the input are unchanged. Downstream firms continue to produce the same firm-level outputs and all firms have the same responses to policy. Nevertheless, since there are actually Kn^d of domestic firms demanding the input, K can be made sufficiently large to remove the potential for monopsony power by firms d even if n^d is small. Similarly, if r^F is endogenous, both Kn^d and Kn^f can be made large. For simplicity, it is assumed for the subsequent analysis that there is only one downstream industry purchasing the input. Nevertheless, since the results do not depend on the size of K, a more general interpretation is that this single industry is in fact a 'representative downstream industry' with K identical replications.

As a final point, whether or not the above justifications are fully convincing to the reader, we feel that the model is worthwhile because of its role as an interesting polar case in which the absence of monopsony power favors upstream relative to downstream profits.[11] Comparisons can then be made with the other polar extreme in which downstream firms have monopsony power and upstream firms are price takers. Given a horizontal supply curve (as in our model), this would imply that upstream firms earn no above normal profits or infra-marginal rents. Indeed, this is the natural base for comparison since the strategic-trade policy results are then the same as in the existing literature without an intermediate-good sector.

3. Market equilibrium and comparative statics

This section sets out the equilibrium conditions for our main model and also develops the effects of a subsidy to final-good exports on production and profits in both the intermediate and final-good markets. The model is presented as partial equilibrium analysis, but, implicitly, we are assuming that, behind the scenes, trade balance is achieved by the existence of another traded final good, produced by a purely competitive constant cost industry, using labor alone.[12]

3.1. The final-good market

Firms d and f produce outputs y^d and y^f, respectively, of the final good giving rise to aggregate output $Y \equiv Y^d + Y^f$, where $Y^d \equiv n^d y^d$ represents total domestic output and $Y^f \equiv n^f y^f$ represents total foreign output. The price, denoted p, of the final product is given by the inverse demand function $p = p(Y)$ where $p'(Y) < 0$. As in the previous literature in the area, the analysis is simplified by assuming that the final good is produced using fixed proportions of the intermediate good and

[11] Demand is assumed to be sufficiently large relative to costs that the overall industry is viable.
[12] See Brander and Spencer (1985) for a simple general equilibrium approach.

another factor, labor. By an appropriate choice of units, there is then no loss of generality in assuming that just one unit of the intermediate good is required for each unit of the final good. Labor is supplied to the industry in each country at a constant wage, with unit wage costs given by w^D in country D and w^F in country F.

Letting s^d denote a specific subsidy applied to firm d's exports and $\rho \equiv r^D - s^d$, the net price of the intermediate good after subtracting the subsidy, firms d and f, respectively, earn profits

$$\pi^d = [p - (\rho + w^D)]y^d \text{ for } \rho \equiv r^D - s^d \text{ and } \pi^f = [p - (r^F + w^F)]y^f. \qquad (1)$$

Since each firm maximizes profit, taking the outputs of rivals as given, the first order conditions are:

$$\begin{aligned} \partial\pi^d/\partial y^d &= p + y^d p' - (\rho + w^D) = 0, \\ \partial\pi^f/\partial y^f &= p + y^f p' - (r^F + w^F) = 0. \end{aligned} \qquad (2)$$

The following second order and stability conditions are assumed to hold globally:[13] i.e.

$$2p' + y^i p'' < 0, \gamma^i \equiv n^i + 1 - \sigma^i E > 0 \ (i = d, f) \text{ and } \psi \equiv N + 1 - E > 0, \qquad (3)$$

where $\sigma^i \equiv Y^i/Y$ (the share of firms i in the final-good market), $E \equiv -Yp''/p'$ (the elasticity of the slope of the inverse demand curve of the final good), and $N \equiv n^d + n^f$ (the total number of final-good firms).

The first order conditions (2) define the Cournot equilibrium output levels $y^d(\rho)$ and $y^f(\rho)$ for firms d and f, respectively, as functions of the net-price $\rho \equiv r^D - s^d$ of the intermediate good (the constants w^D and w^F are omitted and r^F is omitted until it becomes variable in Section 6). Now, using subscripts to denote partial derivatives, from (A.3) (equations numbered as (A.) are all in Appendix A) we obtain

$$y^d_\rho(\rho) = \gamma^f/p'\psi < 0 \text{ and } y^f_\rho(\rho) = -(n^d/n^f)(n^f - \sigma^f E)/p'\psi, \qquad (4)$$

where $\gamma^f > 0$ and $\psi > 0$ from (3). Conditions (4) show the standard results that an increase in domestic marginal cost always reduces domestic output and that foreign output rises if and only if foreign firms view outputs as strategic substitutes (i.e. iff $p' + y^f p'' < 0$, since $n^f - \sigma^f E = n^f(p' + y^f p'')/p'$). Nevertheless, total output, given by $Y = Y(\rho) \equiv Y^d(\rho) + Y^f(\rho)$ where $Y^i(\rho) \equiv n^i y^i(\rho)$ for $i = d$, f, always falls: from (4),

[13] These conditions are more commonly expressed as: $(n^d + 1)p' + Y^d p'' < 0$, $(n^f + 1)p' + Y^f p'' < 0$ and $(N + 1)p' + Yp'' < 0$. The first two conditions are used to sign the comparative statics and the last is needed for uniqueness and stability of equilibrium (see Seade, 1980, 1985).

$$Y_\rho(\rho) = n^d/p'\psi < 0. \quad (5)$$

3.2. The domestic market for the intermediate good

Now considering the intermediate-good market, domestic firms h and foreign firms m produce outputs x^h and x^m, respectively, of the input for sale in country D. Aggregate supplies in country D are given by $X^D \equiv X^h + X^m$ where $X^h \equiv n^h x^h$ represents total domestic production and $X^m \equiv n^m x^m$ represents total foreign production. When foreign firms m supply the intermediate good, the natural interpretation is that these firms are located in a foreign country. However, under some circumstances, it may be possible to interpret firms m as being foreign-owned plants located in the domestic country through foreign direct investment. This interpretation would require that foreign profit is not captured back through domestic taxes or rents to domestic workers. A similar point would apply to the location of domestic firms h.

In setting output in stage 1, firms h and m fully anticipate the derived demand $Y^d(\rho)$ for the input arising from the second stage Cournot equilibrium for the final good. Equating demand with supply (i.e. $Y^d(\rho) = X^D$) and taking the inverse, defines the inverse demand curve $\rho = \rho(X^D)$ for the input, where

$$\rho'(X^D) = 1/Y^d_\rho < 0 \text{ and } \rho''(X^D) = -Y^d_{\rho\rho}/(Y^d_\rho)^3. \quad (6)$$

Domestic policy towards the intermediate good potentially involves a specific production subsidy s^h and a specific import tariff, t^m. Letting c^h for firm h and c^m for firm m represent the (constant) marginal costs of production, and using $r^D = \rho(X^D) + s^d$, it follows that firms h and m, respectively, earn profits,

$$\begin{aligned} \pi^h &= [r^D - (c^h - s^h)]x^h = [\rho(X^D) - \upsilon^h]x^h \text{ and} \\ \pi^m &= [r^D - (c^m + t^m)]x^m = [\rho(X^D) - \upsilon^m]x^m, \end{aligned} \quad (7)$$

where $\upsilon^h \equiv c^h - s^h - s^d$ and $\upsilon^m \equiv c^m + t^m - s^d$ are constants. Cournot competition in intermediate-good production then gives rise to the following first order conditions: from (7), for $k = h, m$,

$$\partial\pi^k/\partial x^k = \rho(X^D) + x^k\rho'(X^D) - \upsilon^k = 0. \quad (8)$$

Letting $E^u \equiv -X^D\rho''/\rho'$ (superscript u for upstream) represent the elasticity of the slope of the inverse demand curve for the intermediate good, then, analogously to the market for the final good, the following second order and stability conditions are assumed to hold globally: i.e. for $k = h, m$

$$2\rho' + x^k\rho'' < 0, \gamma^{uk} \equiv n^k + 1 - \sigma^{uk}E^u > 0 \text{ and } \psi^u \equiv N^u + 1 - E^u > 0, \quad (9)$$

where $\sigma^{uk} \equiv X^k/X^D$ is the share of firms k in the domestic intermediate-good market and $N^u \equiv n^h + n^m$ is the total number of intermediate-good firms.

3.3. A subsidy to final-good exports: comparative statics

Considering first the effects of the export subsidy s^d on the production of the input by firms h and m, it can be shown (see (A.6)) that

$$\mathrm{d}x^h/\mathrm{d}s^d = -[1 + n^m\delta^u E^u/X^D]/\rho'\psi^u \text{ and}$$
$$\mathrm{d}x^m/\mathrm{d}s^d = -[1 - n^h\delta^u E^u/X^D]/\rho'\psi^u, \tag{10}$$

where $\delta^u \equiv x^h - x^m$ represents the difference between the outputs of firms h and m. A useful case arises when firms h and m face the same marginal costs, making $\delta^u = 0$. As can be seen from (10), if all suppliers produce the same level of output (which holds if they are all domestic, all foreign or if $\delta^u = 0$), or if demand is linear (i.e. if $E^u = 0$),[14] then the second term of each expression vanishes, ensuring that firm-level outputs increase. However with asymmetric costs and non-linear demand, x^h or x^m may fall. Nevertheless the subsidy s^d always raises the total supply X^D of the input and, since $Y^d = X^D$, domestic exports of the final product must also rise:

$$\mathrm{d}X^D/\mathrm{d}s^d = \mathrm{d}Y^d/\mathrm{d}s^d = -N^u/\rho'\psi^u > 0. \tag{11}$$

As for the effect of s^d on the price r^D, Proposition 1 shows that if $E^u < 1$, which holds if demand for the input is not too convex (including linear and concave demand), an increase in s^d is partially offset by an increase in the price r^D. Thus part of the gain from the subsidy is shifted from firms d in the final-good sector to firms h and m in the intermediate-good sector. Nevertheless, firms d experience a reduction in marginal cost, since the net-price $\rho \equiv r^D - s^d$ always falls. By contrast, if $E^u > 1$, an increase in s^d reduces the price r^D, magnifying the reduction in ρ. To understand these results, it is useful to recognize that since s^d shifts out the inverse demand curve $r^D = \rho(X^D) + s^d$ by a constant, it has the same effect on the net-price ρ as would a uniform reduction in the marginal costs c^h and c^m (see (7)) for the intermediate-good industry. In the case $E^u > 1$, ρ is over-shifted in the sense that it falls by more than the increase in s^d. Hence the condition $E^u > 1$ is analogous to the Seade (1985) condition, $E > 1$, under which price is over-shifted by industry-wide changes in marginal cost.

Proposition 1. *An increase in the subsidy s^d applied to final-good exports raises*

[14] If $p'' = 0$, then $\rho'' = -Y^d_{\rho\rho}/(Y^d_\rho)^3 = 0$, which implies $E^u = E = 0$. See (A.11) for the relationship between E^u and E in the constant elasticity case and Spencer and Raubitschek (1996) for general demand.

the price r^D of the intermediate good (respectively lowers the price) if and only if $E^u<1$ (resp. $E^u>1$). Nevertheless, $\rho=r^D-s^d$ always falls, causing an overall reduction in the marginal cost faced by domestic firms d.

Proof. From $r^D=\rho(X^D)+s^d$, using (11) and $\psi^u\equiv N^u+1-E^u>0$, we obtain

$$\begin{aligned} &\mathrm{d}r^D/\mathrm{d}s^d=\rho'(\mathrm{d}X^D/\mathrm{d}s^d)+1=(1-E^u)/\psi^u \text{ and}\\ &\mathrm{d}\rho/\mathrm{d}s^d=\mathrm{d}r^D/\mathrm{d}s^d-1=-N^u/\psi^u<0. \end{aligned} \tag{12}$$

Q.E.D.

If the net-price ρ is over-shifted (i.e. if r^D falls), it is also possible that intermediate-good profits are over-shifted in the sense that ρ falls sufficiently in response to an increase in the subsidy s^d that the profits of firms h and m fall. Since $\mathrm{d}\pi^k/\mathrm{d}s^d=x^k+x^k\rho'[(\mathrm{d}X^D/\mathrm{d}s^d)-(\mathrm{d}x^h/\mathrm{d}s^d)]$ for $k=h$, m from (7) and (8), it follows using (12) and (10), that

$$\begin{aligned} \mathrm{d}\pi^h/\mathrm{d}s^d &= x^h[(\mathrm{d}r^D/\mathrm{d}s^d)-\rho'(\mathrm{d}x^h/\mathrm{d}s^d)]\\ &= x^h[2-E^u+n^m\delta^uE^u/X^D]/\psi^u \text{ and } \mathrm{d}\pi^m/\mathrm{d}s^d\\ &= x^m[2-E^u-n^h\delta^uE^u/X^D]/\psi^u. \end{aligned} \tag{13}$$

Hence from (13), if the firms h and m are identical (i.e. if $\delta^u=x^h-x^m=0$), then their profits rise in response to the subsidy (profit is not over-shifted) if and only if $E^u<2$.[15] If $1<E^u<2$, the price r^D falls (recall Proposition 1), but profits nevertheless rise because of higher output. Allowing for $\delta^u\neq 0$, intermediate-good profits always increase if demand is linear ($E^u=0$) or if the supplier is a monopolist (since $\psi^u=2-E^u>0$ from (9) for $N^u=1$). However, for non-linear demand, if firms h and m are very asymmetric in size, it is possible that the higher cost firms lose from the subsidy even if $E^u<2$.

Now considering the profits of domestic final-good producers, the presence of foreign final-good producers not receiving the subsidy affects the conditions under which the subsidy raises domestic profits. Spencer and Raubitschek (1996) have extended the Seade (1985) analysis to consider profit over-shifting when only a subset of the industry experiences the change in marginal cost. Following their approach, we first define $\beta\equiv-p'(Y_\rho-y^d_\rho)$ to represent the effect of a change in ρ on the price of the final product through changes in the outputs of all other firms, but one's own. The term β is positive if a reduction in ρ would cause the aggregate output of other firms to fall and is negative otherwise. Letting $\alpha\equiv\gamma^f-n^d$ and using (3), (4) and (5), β can be expressed as:

[15] The condition $E^u<2$ corresponds to the Seade (1985) condition $E<2$, ruling out profit over-shifting for identical Cournot final-good firms.

$$\beta \equiv -p'(Y_\rho - y^d_\rho) = \alpha/\psi \text{ for } \alpha \equiv n^f + 1 - \sigma^f E - n^d. \tag{14}$$

Next, from (1) and (2), we obtain $d\pi^d/d\rho = -y^d + y^d p'(Y_\rho - y^d_\rho)$, which, using (14), becomes:

$$\begin{aligned} &d\pi^d/d\rho = -y^d(\rho)(1+\beta) \\ &\text{for } 1+\beta = (\psi+\alpha)/\psi = [2(1+n^f) - E(1+\sigma^f)]/\psi. \end{aligned} \tag{15}$$

As (15) shows, a reduction in the net-price ρ increases firm d's profits if and only if $1+\beta>0$. Since $p''\leq 0$ implies $E\leq 0$ and hence $1+\beta>0$ from (15), firm d's profits rise if the inverse demand curve for the final good is concave or linear. More generally, as in Seade (1985), $E\leq 2$ is sufficient to rule out profit over-shifting, but even greater convexity in demand can be accommodated if $n^f>0$. Now, extending this analysis to the subsidy s^d, from (12) and (15), an increase in s^d increases firm d's profits if and only if

$$d\pi^d/ds^d = (d\pi^d/d\rho)(d\rho/ds^d) = y^d(1+\beta)N^u/\psi^u > 0. \tag{16}$$

The following alternative expression for $d\pi^d/ds^d$ also proves useful: from (1), (2) and (12),

$$d\pi^d/ds^d = y^d p'[(dY^f/ds^d) + (n^d - 1)(dy^d/ds^d)] - y^d[(dr^D/ds^d) - 1]. \tag{17}$$

In summary, with respect to the effects of s^d on profits, the following proposition is established.

Proposition 2. *An increase in the subsidy s^d increases the profits of (i)domestic final-good producers if and only if $1+\beta>0$. (ii) Domestic and foreign intermediate-good producers (a) if and only if $E^u<2$ provided $\delta\equiv x^h - x^m = 0$ or (b) if demand is linear or (c) if there is only one intermediate-good producer.*

4. The intermediate good and the welfare effects of export policy

In this section we develop a general formula for rent-shifting export policy, expressing it in such a way as to separate out the effects of the market for the intermediate good. We then use this formulation to examine the additional policy considerations arising from the intermediate-good sector.

4.1. A general formula

Since all of the final good is exported, country D's welfare, denoted W^D, is just the total domestic profit earned from final and intermediate good production, less the cost of the subsidy:

$$W^D = n^d \pi^d + n^h \pi^h - s^d Y^d - s^h X^h + t^m X^m. \tag{18}$$

(The subsidy s^h and the tariff t^m are included in (18) for completeness.) From (18), assuming $\mathrm{d}^2 W^D/(\mathrm{d}s^d)^2 < 0$, the optimal rent-shifting export subsidy, denoted $\hat{s}^d$, with $s^h = t^m = 0$ satisfies

$$\mathrm{d}W^D/\mathrm{d}s^d = n^d(\mathrm{d}\pi^d/\mathrm{d}s^d) + n^h(\mathrm{d}\pi^h/\mathrm{d}s^d) - Y^d - \hat{s}^d(\mathrm{d}Y^d/\mathrm{d}s^d) = 0. \tag{19}$$

Rearranging (19) using (17), $\hat{s}^d$ can usefully be expressed as

$$\hat{s}^d = Y^d p' \left[\frac{\mathrm{d}Y^f/\mathrm{d}s^d}{\mathrm{d}Y^d/\mathrm{d}s^d} + \frac{(n^d - 1)}{n^d} \right] + \frac{Y^d \Omega}{\mathrm{d}Y^d/\mathrm{d}s^d} \tag{20}$$

where $\Omega \equiv [n^h(\mathrm{d}\pi^h/\mathrm{d}s^d) - Y^d(\mathrm{d}r^D/\mathrm{d}s^d)]/Y^d$. The first two terms of (20) (in square brackets) capture the standard 'strategic' and 'terms of trade' effects of an export subsidy arising from the final-good market in a form very similar to an expression derived by Krishna and Thursby (1991, p. 307). The last term of (20) arises because of the existence of the Cournot market for the intermediate good.

To briefly explain the standard analysis, the 'strategic' effect, as represented by the first term of (20), involves the effect of the subsidy on foreign output. If firms f view outputs as strategic substitutes then their output falls in response to the subsidy (i.e. $\mathrm{d}Y^f/\mathrm{d}s^d = Y^f_\rho(\mathrm{d}\rho/\mathrm{d}s^d) < 0$ from (4) and (12)). This tends to make $\hat{s}^d > 0$ as in Brander and Spencer (1985). However, if outputs are strategic complements, then $\mathrm{d}Y^f/\mathrm{d}s^d > 0$ and an export tax raises welfare. The 'terms of trade' effect, as represented by the second term of (20), arises when there is more than one domestic exporter. Since firms do not take into account the effect of their exports on the exports of other firms, each firm d produces beyond the joint profit maximizing level for domestic firms taking foreign output as given. This lowers the export price, worsening the terms of trade. Correction of this distortion alone would require an export tax.[16]

Further insight is obtained by combining the 'strategic' and 'terms of trade' effects to express the optimal export subsidy in the form[17]

$$\hat{s}^d = Y^d[-\beta(\mathrm{d}\rho/\mathrm{d}s^d) + \Omega]/(\mathrm{d}Y^d/\mathrm{d}s^d), \tag{21}$$

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[16] See Eaton and Grossman (1986) and Krishna and Thursby (1991).

[17] Follows from (20) using $p'[(\mathrm{d}Y^f/\mathrm{d}s^d) + (n^d - 1)(\mathrm{d}y^d/\mathrm{d}s^d)] = p'(Y^f_\rho + (n^d - 1)y^d_\rho)(\mathrm{d}\rho/\mathrm{d}s^d) = -\beta(\mathrm{d}\rho/\mathrm{d}s^d)$.

where, from (14), $\beta = \alpha/\psi$. Since $d\rho/ds^d < 0$ from Proposition 1, it follows from (21) that if there is no effect of the input market (i.e. if $\Omega = 0$), then $\beta > 0$, or equivalently $\alpha \equiv n^f + 1 - \sigma^f E - n^d > 0$, is both necessary and sufficient for a positive subsidy. Expressing α in the form $\alpha = n^f(p' + y^f p'')/p' + 1 - n^d$, this implies the familiar result that $\hat{s}^d$ can be positive only if foreign firms view outputs as strategic substitutes (i.e. only if $p' + y^f p'' < 0$). An additional requirement is that the number of domestic firms be sufficiently small to prevent the 'terms of trade' effect from dominating. For example, if demand is linear, $\alpha = n^f + 1 - n^d > 0$ is required.

4.2. The influence of the intermediate-good market

To explore the additional effects caused by the intermediate-good market, we examine the sign of Ω, where a positive value of Ω favors an export subsidy. From (20) and (13) we obtain

$$\begin{aligned}\Omega &\equiv [n^h(d\pi^h/ds^d) - Y^d(dr^D/ds^d)]/Y^d \\ &= -[\sigma^{uh}\rho'(dx^h/ds^d) + \sigma^{um}(dr^D/ds^d)]. \end{aligned} \tag{22}$$

From the first expression of (22), Ω tends to be positive if the subsidy would raise the profits of domestic firms h, but this effect is reduced to the extent that the price r^D of the intermediate good also rises. If the intermediate-good industry is 100% domestic, then we show in Proposition 3(i) that the first effect dominates, making $\Omega > 0$. By contrast, if the intermediate-good industry is 100% foreign, the direction of change in the price r^D becomes critical. From Proposition 3(ii), an export tax is then favored (Ω is negative) if and only if use of a subsidy would raise the price r^D paid to the foreign suppliers. We have already shown (see Proposition 1), that a subsidy would raise r^D if $E^u < 1$, which holds if demand is not too convex, including linear demand. Hence if $E^u < 1$, a switch from a purely domestic to a foreign intermediate-good industry would tend to shift policy from an export subsidy to an export tax. However, since a subsidy would reduce r^D if $E^u > 1$, we also have the perhaps surprising implication that even a 100% foreign intermediate-good industry can push optimal export policy towards a subsidy.

Proposition 3. *If the intermediate-good industry is (i) 100% domestic then* $\Omega > 0$*; (ii) 100% foreign then* $\Omega < 0$ *if and only if* $dr^D/ds^d > 0$.

Proof. (i) If $n^m = 0$, then $\sigma^{uh} = 1$ and it follows from (22) using (10) that $\Omega = -\rho'(dx^h/ds^d) = 1/\psi^u > 0$. (ii) If $n^h = 0$, then $\sigma^{um} = 1$ and the result follows since (22) implies $\Omega = -(dr^D/ds^d)$. Q.E.D.

In understanding why the presence of a domestic intermediate-good industry favors an export subsidy, it is useful to recognize that Proposition 3(i) holds even if there is no role for strategic trade policy because there are no foreign firms in either industry (i.e. $n^f = n^m = 0$). Since all of the final good is exported, the motive for trade policy is then simply to maximize the joint profits of the two industries net of the subsidy. Considering the final-good industry alone, this would imply an export tax if there is more than one domestic firm, and no intervention in the case of a domestic monopoly.[18] However, imperfect competition in the intermediate-good industry creates a wedge between the price of the input and its marginal cost, the familiar 'double-marginalization' effect of vertical oligopolies.[19] Since the wedge tends to reduce output below the joint profit-maximizing level for the two goods, this favors a subsidy. Hence, if a domestic monopolist exports the final-good, but independent domestic firms *h* produce the input, a subsidy raises domestic welfare.

By contrast, for a 100% foreign intermediate-good industry, only the profits of firms *d* count for domestic welfare. Since the intermediate-good market affects these profits only through changes in the input price r^D, this gives rise to a different domestic incentive, namely a desire to reduce r^D so as to improve the terms of trade at which the intermediate good is imported. This explains the critical importance for policy of the direction of change in the input price (see Proposition 3(ii)). Thus if a domestic monopolist exports the final good (which is neutral for government policy), then an export subsidy would raise domestic welfare if and only if it would reduce the price r^D paid to the foreign suppliers.[20]

Despite the fact that an export subsidy can raise domestic welfare even when intermediate-good producers are 100% foreign, we are able to confirm the central point that having profits leak to foreign intermediate-good producers reduces the incentive to use an export subsidy for rent-shifting purposes. Proposition 4 shows that if the subsidy s^d would raise the profits of the foreign suppliers, then replacing a domestic firm *h* with an otherwise identical foreign firm *m*, holding the total number N^u of intermediate-good firms fixed,[21] must lower any welfare gain from a subsidy.

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[18] This can be seen from (21) since $\beta = (1 - n^d)/\psi$ for $n^f = 0$.

[19] This wedge would not exist under vertical integration and also could be removed through contractual relationships or two-part or other pricing schemes. Such schemes are common, but the use of a single price is also common, particularly in an international context where firms are more likely to be arms length.

[20] This result follows since $\beta = 0$ (for $n^d = 1$ and $n^f = 0$) and (21), (22) and $n^h = 0$ then imply $\hat{s}^d = -Y^d(\mathrm{d}r^D/\mathrm{d}s^d)/(\mathrm{d}Y^d/\mathrm{d}s^d)$. Since from Propositions 1 and 2(ii), intermediate-good profits rise when r^D falls for $1 < E^u < 2$, this implies the somewhat counterintuitive result that the subsidy can raise domestic welfare, at the same time transferring rents to foreign firms. However, all firms can gain because of the above mentioned effect of the subsidy in reducing the inefficiency arising from double-marginalization.

[21] Assuming identical firms allows a better focus on the ownership issue and also greatly simplifies the analysis, since, with no efficiency effects, a change in ownership has no effect on prices or outputs.

Proposition 4. *Assume* $\delta^u \equiv x^h - x^m = 0$ *and* $N^u = n^h + n^m$ *is fixed. An increase in the proportion of foreign relative to domestic suppliers of the intermediate product reduces the incentive to subsidize final-good exports if and only if the subsidy shifts profits to intermediate-good producers.*

Proof. Holding N^u fixed, $\mathrm{d}n^h = -\mathrm{d}n^m$, but with $\delta^u = 0$, all other variables are unchanged ensuring that Y^d, r^D, $\mathrm{d}\pi^d/\mathrm{d}s^d$ and $\mathrm{d}\pi^h/\mathrm{d}s^d$ are unaffected. From (19), this implies $\mathrm{d}^2W/\mathrm{d}s^d\mathrm{d}n^m = -\mathrm{d}\pi^h/\mathrm{d}s^d$ evaluated at $\hat{s}^d$. Hence $\hat{s}^d$ is reduced iff $\mathrm{d}\pi^h/\mathrm{d}s^d > 0$. Since $\mathrm{d}\pi^h/\mathrm{d}s^d = \mathrm{d}\pi^m/\mathrm{d}s^d$, the result follows. Q.E.D.

5. Export policy towards the final-good: subsidy or tax

This section explores the effects of the imperfectly competitive intermediate-good industry on the overall conditions determining whether final-good exports should be subsidized or taxed. For this analysis, it proves useful to express $\hat{s}^d$ in the following form: from (21), $\mathrm{d}\rho/\mathrm{d}s^d = -N^u/\psi^u$, (22) and (10),

$$\begin{aligned} &\hat{s}^d = Y^d[\beta N^u + \psi^u \Omega]/\psi^u(\mathrm{d}Y^d/\mathrm{d}s^d) \\ &\text{for } \Omega = \sigma^{uh}(1 + n^m \delta^u E^u/X^D)/\psi^u - \sigma^{um}(\mathrm{d}r^D/\mathrm{d}s^d) \end{aligned} \tag{23}$$

where $\beta = \alpha/\psi$ and $\alpha \equiv n^f + 1 - \sigma^f E - n^d$. We initially focus on implications for the necessary and sufficient conditions for a subsidy, including the effects of strategic complements, before briefly considering the more global effects arising from a shift from perfect to imperfect competition at the intermediate-good stage.

5.1. Necessary and sufficient conditions for a subsidy

Recalling that in the absence of an intermediate-good market, the export subsidy $\hat{s}^d$ is positive if and only if α is positive, Proposition 5 examines the robustness of this condition in the presence of the imperfectly competitive intermediate-good sector.

Proposition 5. *If the Cournot industry supplying the intermediate good is (i) (a) 100% domestic or (b) 100% foreign with* $\mathrm{d}r^D/\mathrm{d}s^d < 0$*, then* $\alpha = n^f + 1 - \sigma^f E - n^d > 0$ *is sufficient but not necessary for* $\hat{s}^d > 0$*; (ii) 100% foreign and* $\mathrm{d}r^D/\mathrm{d}s^d > 0$*, then* $\alpha > 0$ *is necessary but not sufficient for* $\hat{s}^d > 0$*; (iii) if demand is linear and* $\sigma^{uh} \geq 1/2$*, then* $\alpha = n^f + 1 - n^d > 0$ *is sufficient for* $\hat{s}^d > 0$*.*

Proof. From (23), $\Omega = 1/\psi^u$ and $\hat{s}^d = Y^d[\beta N^u + 1]/\psi^u(\mathrm{d}Y^d/\mathrm{d}s^d)$ for $\sigma^{uh} = 1$. Also, $\Omega = -(\mathrm{d}r^D/\mathrm{d}s^d)$ and $\hat{s}^d = Y^d[\beta N^u - \psi^u(\mathrm{d}r^D/\mathrm{d}s^d)]/\psi^u(\mathrm{d}Y^d/\mathrm{d}s^d)$ for $\sigma^{um} = 1$. Since $\alpha = \beta\psi$, results (i) and (ii) follow. To prove (iii), since $p'' = 0$ implies $E = E^u = 0$, it follows from (23), using (12) and $\sigma^{um}=1-\sigma^{uh}$ that $\Omega=(2\sigma^{uh}-1)/\psi^u$ and $\hat{s}^d=Y^d[\beta N^u+2\sigma^{uh}-1]/\psi^u(\mathrm{d}Y^d/\mathrm{d}s^d)$ for $\alpha=\beta\psi=n^f+1-n^d$. Q.E.D.

If the intermediate-good industry is 100% domestic or alternatively 100% foreign, but with $\mathrm{d}r^D/\mathrm{d}s^d<0$, Proposition 5(i) shows that $\alpha>0$ remains sufficient for a positive export subsidy, but it is now possible that $\hat{s}^d>0$ even if $\alpha<0$. As explained in the last section, under these conditions, Ω is positive, favoring an export subsidy. Conversely, from Proposition 5(ii), if the intermediate-good industry is 100% foreign and if $\mathrm{d}r^D/\mathrm{d}s^d>0$ (the subsidy worsens the terms of trade for the input), then $\alpha>0$ continues to be necessary, but is not sufficient for $\hat{s}^d>0$. Not surprisingly, the proportion of foreign versus domestic intermediate-good firms matters. As shown in Proposition 5(iii) for linear demand, if domestic firms control at least half of the input market, the condition $\alpha>0$ remains sufficient for an export subsidy.

Proposition 6 further explores the conditions giving rise to an export subsidy. Whatever the ownership of the intermediate-good industry, Proposition 6(i) shows that when intermediate-good firms face the same marginal cost (i.e. if $\sigma^{uh}=1$, $\sigma^{um}=1$ or $\delta^u=0$), a requirement for $\hat{s}^d>0$ is that domestic final-good profits increase. More surprisingly, as set out in Proposition 6(ii), this condition is both necessary and sufficient for an export subsidy when a domestic monopoly supplies the input. The result follows because the monopolist's profits rise by the amount of the subsidy payment[22] just offsetting the cost of the subsidy to taxpayers. For linear demand, since the profits of firms *d* always rise, it follows (see Proposition 6(iii)) that exports should be subsidized whenever a domestic monopolist supplies the input. By contrast, with foreign monopoly supply and linear demand, the policy switches to a tax. These last results are worth emphasizing because they hold regardless of the relative numbers of final-good firms *d* and *f*.

Proposition 6. *(i) If intermediate-good firms face the same marginal costs, a necessary condition for* $\hat{s}^d>0$ *is* $\mathrm{d}\pi^d/\mathrm{d}s^d>0$ *(i.e. that the profits of firms d* ***not*** *be over-shifted), which holds iff* $1+\beta>0$*. (ii) If* $N^u=n^h=1$ *(domestic monopoly), then* $\hat{s}^d>0$ *iff* $\mathrm{d}\pi^d/\mathrm{d}s^d>0$*. (iii) Assume linear demand: if (a)* $N^u=n^h=1$ *(domestic monopoly) then* $\hat{s}^d>0$ *and if (b)* $N^u=n^m=1$ *(foreign monopoly) then* $\hat{s}^d<0$*.*

Proof. From (23), (12), and (16), we obtain $\hat{s}^d=[\psi^u n^d(\mathrm{d}\pi^d/\mathrm{d}s^d)-Y^d(N^u-\psi^u\Omega)]/\psi^u(\mathrm{d}Y^d/\mathrm{d}s^d)$, where

$$N^u-\psi^u\Omega=N^u-\sigma^{uh}(1+n^m\delta^u E^u/X^D)+\sigma^{um}(1-E^u). \tag{24}$$

[22] This follows since $\mathrm{d}\pi^h/\mathrm{d}s^d=x^h=Y^d$ from (13) for $n^h=N^u=1$.

(i) Since $d\pi^d/ds^d>0$ iff $1+\beta>0$ from (16), it remains to show that $N^u-\psi^u\Omega\geq 0$. From (24), $N^u-\psi^u\Omega=N^u-1\geq 0$ for $\sigma^{uh}=1$ and $N^u-\psi^u\Omega=N^u+1-E^u>0$ from (9) for $\sigma^{um}=1$. If $0<\sigma^{um}<1$ and $\delta^u=0$, then, using $\gamma^{um}\equiv n^m+1-\sigma^{um}E^u>0$ from (9), it follows from (24) that $N^u-\psi^u\Omega=\gamma^{um}+n^h-2\sigma^{uh}>0$ for $n^h\geq 2$ and also for $n^h=1$, since $n^m\geq 1$ then implies $\sigma^{uh}\leq 1/2$. (ii) If $N^u=n^h=1$, then, from (24), $N^u-\psi^u\Omega=0$ and $\hat{s}^d=n^d(d\pi^d/ds^d)/(dY^d/ds^d)$. (iii) If $p''=0$, then, using $\psi^u\Omega=\sigma^{uh}-\sigma^{um}$ and $\beta=(n^f+1-n^d)/(N+1)$, the results (a) and (b) follow from (23), since $\beta N^u+\psi^u\Omega=2(n^f+1)/(N+1)>0$ if $N^u=n^h=1$ and $\beta N^u+\psi^u\Omega=-2n^d/(N+1)<0$ if $N^u=n^m=1$. Q.E.D.

5.2. Strategic complements

In a Cournot setting without an intermediate-good sector, it is well known that the rent-shifting motive gives rise to an export subsidy only if outputs are strategic substitutes. As Proposition 7(i) shows, this condition remains a requirement if there are at least two domestic final-good producers (i.e. if $n^d\geq 2$) regardless of whether the input is supplied by domestic or foreign firms. However, if there is only one domestic firm *d* (which favors an export subsidy because of the absence of the 'terms of trade' effect), then there are some limited conditions under which an export subsidy raises domestic welfare in the strategic complements case. As demonstrated in Proposition 7(ii), these conditions involve supply of the intermediate good by a domestic monopolist, the existence of at least some foreign competition in the final-good market (we assume $n^f=1$) and highly special demand conditions.

Proposition 7. *Suppose final-good outputs are strategic complements. (i) If $n^d\geq 2$ and intermediate-good firms face the same marginal cost, then $\hat{s}^d<0$. (ii) If $n^d=n^f=1$ and if $N^u=n^h=1$ (domestic monopoly), then counterexamples exist with $\hat{s}^d>0$.*

Proof. (i) We first demonstrate a result due to Spencer and Raubitschek (1996) that if $n^d\geq 2$, then profit is over-shifted in the strategic complements case. From (15), profit is over-shifted iff

$$\begin{aligned}1+\beta &= [2(1+n^f)-E(1+\sigma^f)]/\psi \\ &= [2(n^f-\sigma^f E)+(n^d-\sigma^d E)+(2-n^d)]/\psi<0.\end{aligned}$$

Since $p'+y^i p''>0$ in the strategic complements case, this implies $n^i-\sigma^i E=n^i(p'+y^i p'')/p'<0$ for $i=d,\ f$, and hence that $1+\beta<0$ when $n^d\geq 2$. It then follows from Proposition 6(i) that $\hat{s}^d<0$.

(ii) To demonstrate an example with $\hat{s}^d>0$ for the strategic complements case, we assume $y^d=y^f$, which is convenient, since $n^d=n^f=1$ implies equal market shares, i.e. $\sigma^i = 1/2$ for $i = d$, f. However, following the approach below, other examples can be found for which $y^d \neq y^f$. We also assume that the elasticity of demand, denoted $\epsilon \equiv -p/Yp'$, is constant, but this is also not necessary.

For $N^u = n^h = 1$, it follows from (23) that $\psi^u \Omega = 1$ and hence $\hat{s}^d > 0$ iff $\beta + 1 > 0$. Using $n^i = 1$ and $\sigma^i = 1/2$ for $i = d$, f to obtain $\beta = \alpha/\psi = (1 - E/2)/(3 - E)$, this implies $\hat{s}^d > 0$ iff $E < 8/3$. Since the requirement $n^i - \sigma^i E = n^i(p' + y^i p'')/p' < 0$ for strategic complements implies $E > 2$, E is restricted to the range $2 < E < 8/3$. The second order and stability conditions (3) given by $2p' + y^i p'' = p'(2 - E/2) < 0$ and $\psi \equiv 3 - E > 0$ for our example are then both satisfied. However, since the analogous conditions (9) for the intermediate-good market imply $E^u < 2$, to check that this condition is satisfied, we first relate E^u to E. Assuming ϵ is constant, then, as shown in Appendix A, $E = 1 + 1/\epsilon$ is constant and from (A.11):

$$E^u \equiv -X^D \rho''/\rho' = Y^d Y^d_{\rho\rho}/(Y^d_\rho)^2 = \sigma^d E[2\gamma^f - (1 - \sigma^f)]/(\gamma^f)^2, \tag{25}$$

where $\gamma^f \equiv n^f + 1 - \sigma^f E$. This implies for our example that $E^u = E(7 - 2E)/(4 - E)^2 < 2$ iff $4E^2 - 23E + 32 > 0$. Finding the roots of this quadratic, we obtain $E < 2.3596$ or $E > 3.3904$. Hence, combining all the conditions, it follows that $\hat{s}^d > 0$ for this example if $2 < E < 2.3596$. Q.E.D.

5.3. Comparison with perfect competition in the intermediate-good industry

The comparison with perfect competition at the intermediate-good stage is not straightforward, because, with the input priced at marginal cost rather than at the higher oligopoly price r^D, there are global changes in prices and production levels. Denoting the base case in which the input is produced under perfect competition by a superscript 0, and letting $\rho \equiv c^h - s^d$ for a domestic industry and $\rho^0 \equiv c^m - s^d$ for a foreign industry, it follows from (21) using $\Omega = 0$ and $d\rho^0/ds^d = -1$ that

$$\hat{s}^{d0} = -Y^d(\rho^0)\beta^0/Y^d_\rho(\rho^0), \tag{26}$$

where $\beta^0 \equiv (n^f + 1 - \sigma^{f0}E^0 - n^d)/\psi$. By comparison, when Cournot firms h and m produce the input, it follows from (23), using $dY^d/ds^d = -(N^u/\psi^u)Y^d_\rho$ from (11) and (6) that

$$\hat{s}^d = -Y^d(\rho)[\beta + \psi^u \Omega/N^u]/Y^d_\rho(\rho). \tag{27}$$

Hence $\hat{s}^d$ is similar in form to $\hat{s}^{d0}$ except that it has an additional term, namely $\psi^u \Omega/N^u$. Although, as previously discussed, the direction of policy implied by this additional term is relatively clear cut, ambiguity is created in the comparison because the higher input price under imperfect competition reduces exports, making $Y^d(\rho) < Y^d(\rho^0)$ for any given subsidy level. Apart from changes in the magnitude[23] of $\hat{s}^d$, this can cause a difference in sign, since with $\sigma^f \equiv Y^f(\rho)/Y(\rho) > \sigma^{f0}$, β differs from β^0. Nevertheless, if demand is linear or constant elastic (but not too convex), Proposition 8 provides support for our previous results. Under these demand conditions, imperfect competition in a 100% domestic intermediate-good industry increases the range of cases for which the export subsidy is positive, but the range of cases is reduced if the industry is 100% foreign and a subsidy would raise the price of the imported input.

Proposition 8. *Cournot competition (in contrast to perfect competition) in intermediate-good supply: (a) increases the range of cases in which $\hat{s}^d > 0$ for a 100% domestic industry if demand is linear or constant elastic with $E < (N+1)/(1+n^h(\sigma^f - \sigma^{f0}))$. (b) decreases the range of cases in which $\hat{s}^d > 0$ for a 100% foreign industry if demand is linear or constant elastic with $E^u \leq 1$ (i.e. if $\mathrm{d}r^D/\mathrm{d}s^d > 0$).*

Proof. Since $\hat{s}^{d0} > 0$ iff $\mathrm{d}W^{D0}/\mathrm{d}s^d = Y^{d0}\beta^0 + s^d Y^d_\rho > 0$ at $s^d = 0$, it follows that $\hat{s}^{d0} > 0$ iff $\beta^0 > 0$ and similarly (from (27)) that $\hat{s}^d > 0$ iff $\beta + \psi^u \Omega / N^u > 0$. Hence, $\hat{s}^d > 0$ for a greater range of cases iff $Z \equiv \beta + \psi^u \Omega / N^u - \beta^0 > 0$ or equivalently, using $\beta = \beta^0 - (\sigma^f E - \sigma^{f0} E^0)/\psi$, iff $Z \equiv \psi^u \Omega / N^u - (\sigma^f E - \sigma^{f0} E^0)/\psi \geq 0$ evaluated at $s^d = 0$. Since $r^d > c^k$ for $k = h$, m (see (8)) implies $\rho > \rho^0$, we have that $Y^d < Y^{d0}$ and $\sigma^f > \sigma^{f0}$ for any given subsidy s^d. (a) If $N^u = n^h$, then $\Omega = 1/\psi^u$ and hence $Z = 1/n^h > 0$ for linear demand. For $\epsilon \equiv -p/Yp'$ constant, using $E = 1 + 1/\epsilon > 0$, $\sigma^f - \sigma^{f0} > 0$ and $\psi \equiv N + 1 - E$, we obtain $Z = 1/n^h - (\sigma^f - \sigma^{f0})E/\psi > 0$ under the stated condition. (b) If $N^u = n^m$, then $\Omega = -(1 - E^u)/\psi^u$. For linear demand, $E^u = E = 0$ implies $Z = -1/n^m < 0$. For ϵ constant, since $E > 0$ and $\sigma^f - \sigma^{f0} > 0$, it follows that $Z = -(1 - E^u)/n^m - (\sigma^f - \sigma^{f0})E/\psi < 0$ if $1 - E^u \geq 0$, i.e. if $\mathrm{d}r^D/\mathrm{d}s^d \geq 0$ (recall Proposition 1). Q.E.D.

6. Endogenous foreign price for the input

This section relaxes the assumption that the price r^F of the input in the foreign country is exogenous by assuming that the same intermediate-good firms *h* and *m* act as Cournot competitors in supplying the input to the segmented markets in countries *D* and *F*. For simplicity, it is assumed that firms *h* and *m* face the same

[23] For linear demand, a higher domestic marginal cost makes an export subsidy less effective at shifting rent from foreign final-good firms and, considering the final-good market alone, $\hat{s}^d$ falls (see Neary, 1994). However, with the input market included, the magnitude of $\hat{s}^d$ relative to $\hat{s}^{d0}$ becomes ambiguous.

marginal cost,[24] denoted c, of supplying the input to either country. Nevertheless, the prices r^D and r^F can differ because of differing levels of demand by final-good firms in each country. When the intermediate-good firms are located in both countries (there are both firms h and m), trade in the input takes place because price discrimination across the segmented markets makes cross-hauling profitable.

6.1. Extending the model

To derive the demand for the input in each country, the first order conditions (2) are initially used to express the outputs of firms d and f as functions $y^d = y^d(\rho, r^F)$ and $y^f = y^f(\rho, r^F)$ respectively, with aggregate final-good output given by $Y(\rho, r^F) = n^d y^d + n^f y^f$. Analogously to (4) and (5), these functions have the following partial derivatives with respect to r^F:

$$y^f_{r^F} = \gamma^d / p'\psi < 0,\ y^d_{r^F} = -(n^f/n^d)(n^d - \sigma^d E)/p'\psi,$$
$$Y_{r^F} = n^f / p'\psi < 0. \tag{28}$$

Next to equate demand with supply in the (segmented) domestic and foreign markets for the input, we set $Y^d = n^d y^d(\rho, r^F) = X^D$ and $Y^f = n^f y^f(\rho, r^F) = X^F$. Solving these equations simultaneously, then defines the inverse demand curves $r^D = \phi^D(X^D, X^F) + s^d$ and $r^F = \phi^F(X^D, X^F)$ for the input in countries D and F, respectively. As shown in (A.12), ϕ^D and ϕ^F have partial derivatives:

$$\phi^D_D = p'\gamma^d / n^d < 0,\ \phi^F_F = p'\gamma^f / n^f < 0,$$
$$\phi^D_F = p' + y^d p'',\ \phi^F_D = p' + y^f p''. \tag{29}$$

As might be expected, (29) implies that an increase in the availability of the intermediate good in any one country always reduces price in that country and that the price in the other country also falls if final-good outputs are strategic substitutes, but rises if they are strategic complements.

Using the superscripts D and F to distinguish the destinations of the intermediate good, the profit earned by a typical producer k, where $k = h$ if domestic and $k = m$ if foreign, is then given by:

$$\pi^k = (r^D - c)x^{kD} + (r^F - c)x^{kF}, \tag{30}$$

[24] The important simplification is that all firms h and m be identical in the quantity of the input they supply to country D and in the (generally different) quantity they supply to country F (making $\delta^u \equiv x^h - x^m = 0$ in each country). If the firms are all domestic ($n^m = 0$) or all foreign ($n^h = 0$), it is relatively easy to allow for marginal costs that vary by the country of destination, perhaps due to transport costs.

where $r^D = \phi^D(X^D, X^F) + s^d$ and $r^F = \phi^F(X^F, X^D)$. At the Cournot equilibrium in each market, each firm k sets the quantities x^{kD} and x^{kF} to maximize profit as in (30), taking the respective quantities supplied by the other firms as given. Hence, x^{kD} and x^{kF} for the N^u identical firms satisfy the first order conditions:

$$\begin{aligned} \partial\pi^k/\partial x^{kD} &\equiv \pi^k_D = \phi^D + x^{kD}\phi^D_D + x^{kF}\phi^F_D - c + s^d = 0; \\ \partial\pi^k/\partial x^{kF} &\equiv \pi^k_F = \phi^F + x^{kD}\phi^D_F + x^{kF}\phi^F_F - c = 0. \end{aligned} \tag{31}$$

The second order conditions are also assumed to be satisfied: i.e.

$$\pi^k_{DD} < 0,\ \pi^k_{FF} < 0 \text{ and } \pi^k_{DD}\pi^k_{FF} - (\pi^k_{DF})^2 > 0 \tag{32}$$

where, from (31), $\pi^k_{ii} = 2\phi^i_i + x^{ki}\phi^i_{ii} + x^{kj}\phi^j_{ii}$ and $\pi^k_{ij} = \phi^i_j + \phi^j_i + x^{ki}\phi^i_{ij} + x^{kj}\phi^j_{ij}$ for $i \neq j$ and $i,j = D,F$. Next, letting $\pi^k_{ii0} \equiv \phi^i_i + x^{ki}\phi^i_{ii} + x^{kj}\phi^j_{ii}$ and $\pi^k_{ij0} \equiv \phi^i_j + x^{ki}\phi^i_{ij} + x^{kj}\phi^j_{ij}$ represent the cross partials with respect to changes in the outputs, x^{oD} and x^{oF}, of other firms in each market, we impose the stability conditions:

$$A^k_{DD} < 0,\ A^k_{FF} < 0 \text{ and } H^u \equiv A^k_{DD}A^k_{FF} - A^k_{DF}A^k_{FD} > 0, \tag{33}$$

where[25] $A^k_{ii} \equiv \pi^k_{ii} + (N^u - 1)\ \pi^k_{ii0}$ and $A^k_{ij} \equiv \pi^k_{ij} + (N^u - 1)\ \pi^k_{ij0}$ for $i \neq j$ and $i,j = D,F$.

As can be seen from (31), the quantity of the intermediate good destined for any one country is determined taking account of its effect on both the prices r^D and r^F. Thus, although the markets for the intermediate good are segmented, they are nevertheless connected through the decisions of the firms k as to how much to supply. The effect of the subsidy s^d on the quantities supplied is found by totally differentiating (31) for the N^u firms, then using (33), $Y^d = X^D = N^u x^{kD}$ and $Y^f = X^F = N^u x^{kF}$ to obtain

$$\begin{aligned} &\mathrm{d}Y^d/\mathrm{d}s^d = \mathrm{d}X^D/\mathrm{d}s^d = -N^u A^k_{FF}/H^u > 0 \text{ and} \\ &\mathrm{d}Y^f/\mathrm{d}s^d = \mathrm{d}X^F/\mathrm{d}s^d = N^u A^k_{FD}/H^u. \end{aligned} \tag{34}$$

As (34) shows, an increase in the subsidy always causes more of the intermediate good to be supplied to the domestic market and hence domestic final-good exports must rise. Also foreign final-good exports fall if $A^k_{FD} < 0$ (each firm k reduces its output x^{kF} destined for country F) and rise if $A^k_{FD} > 0$.

6.2. Implications for results

With respect to results, the first point to make is that the domestic welfare function (18), the first order condition (19) for the choice of $\hat{s}^d$ and the overall expression (20) for $\hat{s}^d$ are all unchanged from the previous analysis. Since,

[25] In general $A^k_{DF} \neq A^k_{FD}$, since $A^k_{DF} = A^k_{FD} + (N^u - 1)(\phi^D_F - \phi^F_D)$ where $\phi^D_F - \phi^F_D = p''(y^d - y^f)$.

Proposition 4, one of our central propositions, follows directly from (19), it can be immediately extended to this expanded model. Hence, allowing r^F as well as r^D to vary, an increase in the foreign ownership of the intermediate-good industry reduces the incentive to subsidize final-good exports if and only if intermediate-good producers would benefit from the subsidy.

However, the conditions under which intermediate-good producers gain from the subsidy now differ since they depend on effects through r^F as well as through $\rho \equiv r^D - s^d$. From (A.17), we obtain

$$\mathrm{d}\pi^k/\mathrm{d}s^d = x^{kD} + [(N^u - 1)/N^u][x^{kD}(\mathrm{d}\rho/\mathrm{d}s^d) + x^{kF}(\mathrm{d}r^F/\mathrm{d}s^d)], \tag{35}$$

whereas, comparing with (13) for the domestic market alone, the effect of the subsidy was $\mathrm{d}\pi^k/\mathrm{d}s^d = x^k$ for $k = h$, m given that firms h and m face the same marginal costs ($\delta^u = 0$). Hence if $N^u > 1$ (firm k is not a monopolist), then effects of the subsidy on firm k's profits from sales in country F matter. In particular, if r^F falls, this tends to make an increase in s^d less profitable and vice versa if r^F rises.

By contrast, setting $N^u = 1$ in (35), it follows that for a monopoly firm k, sales of the input have no effect on the response of profits π^k to the domestic subsidy. Moreover, if the monopoly is domestic (i.e. if $k = h$), then, from (19) and (35), we obtain

$$\hat{s}^d = n^d(\mathrm{d}\pi^d/\mathrm{d}s^d)/(\mathrm{d}Y^d/\mathrm{d}s^d), \tag{36}$$

which implies that optimal export policy towards the final good involves a subsidy if and only if $\mathrm{d}\pi^d/\mathrm{d}s^d > 0$. This extends Proposition 6(ii) to this more general case with r^F endogenous, but, since $\mathrm{d}\pi^d/\mathrm{d}s^d$ depends on changes in r^F, the sign and magnitude of $\hat{s}^d$ can differ. From (1) and (15), we obtain

$$\mathrm{d}\pi^d/\mathrm{d}s^d = -y^d(1 + \beta)(\mathrm{d}\rho/\mathrm{d}s^d) + (\partial\pi^d/\partial r^F)(\mathrm{d}r^F/\mathrm{d}s^d), \tag{37}$$

where, from (1), (2), (28), and $2n^d - \sigma^d E = n^d(2p' + y^d p'')/p' > 0$, it can be shown that

$$\partial\pi^d/\partial r^F = y^d p'[Y^f_{r^F} + (n^d - 1)y^d_{r^F}] = y^d n^f(2n^d - \sigma^d E)/n^d\psi > 0. \tag{38}$$

Comparing (37) with (16), $\mathrm{d}\pi^d/\mathrm{d}s^d$ has an additional term which is positive, favoring a subsidy, if and only if $\mathrm{d}r^F/\mathrm{d}s^d > 0$.

Next, to examine the changes in the foreign price r^F, letting $\delta \equiv y^d - y^f$, from (A.18) we obtain

$$\mathrm{d}r^F/\mathrm{d}s^d = N^u\{[p'/n^f + 2\phi^F_D]\delta p'' + (p'/n^f)[X^F\phi^F_{FD} + X^D\phi^D_{DF}]\}/H^u, \tag{39}$$

where $\phi^F_{FD} = p''(n^f + 1)/n^f + y^f p'''$ and $\phi^D_{DF} = p''(n^d + 1)/n^d + y^d p'''$ from (A.14). Supposing that p'''/p'' is small, then ϕ^F_{FD} and ϕ^D_{DF} have the same sign as p''. Proposition 9 follows.

Proposition 9. *(i) If $p''=0$, then* $\mathrm{d}r^F/\mathrm{d}s^d=0$. *(ii) If $p''<0$ with p'''/p'' small and* $\delta\equiv y^d-y^f\geq 0$ *then* $\mathrm{d}r^F/\mathrm{d}s^d>0$.

Proof. (i) If $p''=0$, then (39) vanishes. (ii) If $p''<0$ with p'''/p'' small, then, from (29) and (A.14), $\phi^i_j<0$ and $\phi^i_{ij}<0$ for $i,j=D,F$. From (39), this implies $\mathrm{d}r^F/\mathrm{d}s^d>0$ for $\delta\geq 0$. Q.E.D.

Proposition 9 is interesting partly because it shows that the active involvement in both countries of the same input suppliers can significantly change the reaction of r^F to an export subsidy. For example, suppose that there are two firms, one domestic and one foreign, each acting independently to supply only its own country market. Then, for linear demand, the downward shift in foreign output and foreign demand for the input due to $s^d>0$ would cause the price r^F charged by the single supplier in country F to fall. This makes the subsidy less effective in raising domestic exports. By contrast, from Proposition 9(i), if $p''=0$ and both intermediate-good firms supply both countries, then the firms respond to the subsidy by supplying less of the input to country F and more to country D, so as to leave r^F unchanged.[26]

The response of $\rho\equiv r^D-s^d$ to the export subsidy s^d is also affected by changes in r^F. From (A.19),

$$\mathrm{d}\rho/\mathrm{d}s^d = -N^u\{(p'/n^d)A^k_{FF} + \phi^D_F(A^k_{FF} - A^k_{FD})\}/H^u, \tag{40}$$

where $A^k_{FF}-A^k_{FD}=(N^u+1)p'/n^f-2\delta p''$ for $\delta\equiv y^d-y^f$ from (A.15). Although there is now some ambiguity as to the sign of $\mathrm{d}\rho/\mathrm{d}s^d$, if final-good outputs are strategic substitutes (making $\phi^D_F<0$) and if $A^k_{FF}-A^k_{FD}<0$,[27] then (40) implies that the subsidy would reduce the marginal cost of firms d. For linear demand, since r^F is unchanged, the result is the same as if r^F were exogenous (see (12) with $E^u=0$): from (A.20),

$$\begin{aligned} &\mathrm{d}\rho/\mathrm{d}s^d = -N^u/(N^u+1) < 0 \text{ and} \\ &\mathrm{d}r^D/\mathrm{d}s^d = \mathrm{d}\rho/\mathrm{d}s^d + 1 = 1/(N^u+1) > 0. \end{aligned} \tag{41}$$

The overall effect of having r^F endogenous on $\hat{s}^d$ is best seen by expressing the optimal subsidy in a form analogous to (21): from (19) using (37) and (22), we obtain

$$\hat{s}^d = \{Y^d[-\beta(\mathrm{d}\rho/\mathrm{d}s^d) + \Omega] + n^d(\partial\pi^d/\partial r^F)(\mathrm{d}r^F/\mathrm{d}s^d)\}/(\mathrm{d}Y^d/\mathrm{d}s^d), \tag{42}$$

[26] For $p''=0$, we obtain $A^k_{FD}=(N^u+1)p'<0$ and hence $\mathrm{d}X^F/\mathrm{d}s^d<0$ and $\mathrm{d}X^D/\mathrm{d}s^d>0$ from (34).

[27] If marginal costs differ significantly across countries so as to make $p''\delta$ large and negative, it is possible $A^k_{FF}-A^k_{FD}>0$. This need not violate the stability conditions (33) provided $A^k_{DD}-A^k_{DF}=p'(n^k+1)/n^d+2p''\delta$ is sufficiently negative.

where Ω is given by (22). Expression (42) is similar in form to (21), but has an additional term, which is positive, tending to raise $\hat{s}^d$, if $\mathrm{d}r^F/\mathrm{d}s^d > 0$ and is negative if $\mathrm{d}r^F/\mathrm{d}s^d < 0$. If demand is linear, since, from (41) and $\mathrm{d}r^F/\mathrm{d}s^d = 0$, (42) reduces to (21), the outcome is again the same as if r^F were set exogenously. Hence, we have the rather remarkable implication that all the previous results concerning the sign of $\hat{s}^d$ when demand is linear, including Proposition 5(iii), Proposition 6(iii) and Proposition 8 for $p'' = 0$, also apply when r^F is set endogenously.

7. Domestic policy toward the intermediate good

Returning to the model in which r^F is exogenous, this section concerns policy applied to the intermediate good, namely an import tariff t^m and a subsidy s^h applied to domestic production. We first consider the joint application of these policies before briefly examining the role of s^h as a sole policy.

The central insight is that if s^h is combined with an equal subsidy to imports of the intermediate good, i.e. if $s^h = -t^m$, then, since $\upsilon^h = c^h - s^h$ and $\upsilon^m = c^m + t^m$ at $s^d = 0$, the first order conditions (8), determining the levels of output of firms h and m, are identical to those that would occur from an export subsidy s^d alone, set at the same level $s^d = s^h = -t^m$. Since, there is also no change in the marginal costs and output levels of final-good producers, Proposition 10 follows.

Proposition 10. *If domestic production and imports of the intermediate good are jointly subsidized at the same level, i.e. if $s^h = -t^m$, this gives rise to the same output and welfare effects as an equal subsidy to final-good exports.*

The following corollary is immediate from this proposition.

Corollary 1. *(i) If all the intermediate good is imported, an import subsidy to the intermediate good and an export subsidy to the final good set at the same levels are equivalent. (ii) If all the intermediate good is domestic, a production subsidy to the intermediate good and an export subsidy to the final good set at the same levels are equivalent.*

Although the policy combination of $s^h = -t^m$ has the same effect on output, profit and domestic welfare as an export subsidy alone set at the same level, it is not optimal to combine t^m and s^h so as to mimic an export subsidy. Letting s^{h*} and t^{m*} represent the jointly optimal policies, Proposition 11 follows.

Proposition 11. *Suppose $n^h > 0$ and $n^m > 0$. The policy combination of the subsidy s^{h*} to domestic intermediate-good production and the tariff t^{m*} on intermediate-*

good imports raises domestic welfare by more than the export subsidy $\hat{s}^d$ alone. The optimal policy combination always requires $s^h* > -t^m*$.

Proof. From (A.29), the jointly optimal values of s^h and t^m are respectively

$$s^h* = -[Y^d\beta + x^h + x^m\sigma^{um}E^u]/Y^d_\rho \text{ and}$$
$$t^m* = [Y^d\beta - x^m(1 - \sigma^{um}E^u)]/Y^d_\rho, \tag{43}$$

where $s^h* + t^m* = -(x^m + x^h)/Y^d_\rho > 0$ from (A.28). Hence it is not the case that $s^h* = -t^m*$. Q.E.D.

Not surprisingly Proposition 11 shows that the optimal policy combination involves a higher subsidy (or lower tax) on domestic production of the intermediate good than on imports. Insight into the conditions determining the signs of the joint policies is obtained by relating t^m* and s^h* to the export subsidy $\hat{s}^d$. Supposing firms h and m have identical costs (i.e. $\delta^u = 0$), (A.32) and (A.33) imply that

$$s^h* = \hat{s}^d - n^m(x^h + x^m)/N^uY^d_\rho \text{ and } t^m* = -s^d - n^h(x^m + x^h)/N^uY^d_\rho \tag{44}$$

and hence that $s^h* > \hat{s}^d > -t^m*$. Thus if $\hat{s}^d$, used as a sole policy, happens to be zero, then for $\delta^u = 0$, the optimal policy combination involves both a strictly positive subsidy s^h* and tariff t^m*. However if $\hat{s}^d$ would be positive, it is possible that $t^m* < 0$, which would imply that imports should be subsidized. Even if such import promotion directly shift profits to foreign intermediate-good producers, nevertheless the domestic country can gain if the policy promotes final-good exports so as to raise domestic profits.[28]

Finally, it is instructive to analyze the effects of a subsidy to domestic production of the intermediate-good as a sole policy.[29] Denoting the optimal value of this subsidy by $\hat{s}^h$, (A.34) shows that

$$\hat{s}^h = X^h\rho'\left[\frac{(\mathrm{d}X^m/\mathrm{d}s^h)}{(\mathrm{d}X^h/\mathrm{d}s^h)} + \frac{n^h - 1}{n^h}\right] + \frac{n^d(\mathrm{d}\pi^d/\mathrm{d}s^h)}{(\mathrm{d}X^h/\mathrm{d}s^h)}. \tag{45}$$

Analogously to (20) for the subsidy $\hat{s}^d$, the first and second terms (in square brackets) of (45) respectively represent the 'strategic' and the 'terms of trade' effects of the production subsidy. Supposing $n^h > 1$, the (negative) 'terms of trade'

[28] This motive for an import subsidy differs fundamentally from the motive in Brander and Spencer (1984). In their case, a foreign monopolist supplied a consumption good and a subsidy was called for only if the import price would be over-shifted, falling by more than the amount of the subsidy.

[29] Correspondingly, from (A.25b) and (A.24) with $s^h = 0$, the optimal tariff alone is $\hat{t}^m = -[n^h(\mathrm{d}\pi^h/\mathrm{d}t^m) + X^m - Y^d(1+\beta)(\mathrm{d}r^D/\mathrm{d}t^m)]/(\mathrm{d}X^m/\mathrm{d}t^m)$. The effect of t^m in raising π^h and tariff revenue tends to make $t^m > 0$, but it is possible $t^m < 0$ since t^m raises r^D, causing π^d to fall if $1 + \beta > 0$.

effect arises from expansion of domestic input production beyond the joint profit-maximizing level for a given level of imports. The third term of (45) reflects the effect of the subsidy in shifting profits to domestic final-good producers.

Letting $\alpha^u \equiv \gamma^{um} - n^h = n^m + 1 - \sigma^{um} E^u - n^h$, (45) reduces to (see (A.35))

$$\hat{s}^h = -[x^h \alpha^u + Y^d(1+\beta)]/\gamma^{um} Y^d_\rho, \tag{46}$$

where $\gamma^{um} > 0$ from (9). Proposition 12 follows.

Proposition 12. *Suppose the subsidy s^h to domestic intermediate-good production is imposed as a sole policy. If $1+\beta>0$, then $\alpha^u \equiv n^m + 1 - \sigma^{um}E^u - n^h > 0$ is sufficient for the optimal subsidy $\hat{s}^h$ to be strictly positive. If $1+\beta=0$, then $\alpha^u>0$ is necessary and sufficient for $\hat{s}^h>0$.*

Proposition 12 can be understood by considering two profit-shifting games, one with respect to the foreign producers of the final good and the other with respect to the foreign exporters of the input to the domestic country. If $1+\beta=0$, then the cost of the intermediate good has no effect on final-good profits (see (15)), so s^h affects domestic welfare only through profit-shifting in the intermediate-good market. The requirement $\alpha^u \equiv n^m + 1 - \sigma^{um}E^u - n^h > 0$ for $\hat{s}^h > 0$ is then fully analogous to the requirement $\alpha = n^f + 1 - \sigma^f E - n^d > 0$ for $\hat{s}^d > 0$ when there is no effect of the intermediate-good market (i.e. when $\Omega = 0$). However if $1+\beta>0$ then the conditions under which the subsidy s^h raises domestic welfare become less stringent since profits are also shifted to domestic final-good producers.

8. Concluding remarks

This paper examines the implications of Cournot imperfect competition in intermediate-good production for strategic trade policy with particular attention to the effects of foreign versus domestic supply. Since a subsidy to final-good exports raises the demand for the intermediate-good, typically some of the profit gained from an export subsidy is further shifted to the intermediate-good producers. To the extent that these producers are foreign rather than domestic, a main result is to show that this weakens the domestic incentive to subsidize exports. This provides an additional argument undermining the use of export subsidies for rent-shifting purposes.

We also show the contrasting result that the extra layer of Cournot competition arising from the presence of a purely domestic intermediate-good industry tends to strengthen the argument for an export subsidy because of a reduction in the inefficiency associated with ‘double marginalization’ in vertical oligopolies. Under certain demand conditions, including linear demand, the range of cases involving a subsidy is increased, even allowing for the global price changes that occur in moving from pure competition to oligopoly. Under very limited circumstances, it is even possible for an export subsidy to raise welfare when final goods are strategic complements. Finally, imperfect competition in a purely foreign intermediate-good industry can raise the incentive for a subsidy when demand is sufficiently convex to cause the price of the input to fall, improving the terms of trade.

With respect to rent-shifting policies applied directly to the input, we show that a combination of a domestic production subsidy and an import subsidy of the same amount is equivalent to an equal subsidy applied to final-good exports. However, higher domestic welfare can be obtained from an optimal combination of the two policies at the intermediate-good stage than from the optimal export subsidy alone.

As a final remark, we would like to emphasize that this paper in no way advocates the use of strategic trade policy. The paper helps to provide insight as to some of the conditions under which governments might be motivated to use rent-shifting policies, but the hope is that such understanding will aid in designing international agreements in which mutual gains are achieved through cooperation rather than through beggar thy neighbor policies.

Acknowledgements

We would like to thank Amy Glass, Kenzo Abe and the participants of seminars at a number of universities and conferences, including Osaka University, TCER, the Australian National University (RSSS), the meetings of the Japan Association of Economics and Econometrics, the meetings of the Japan Society of International Economics and the ASSA meetings of the International Economics and Finance Society. The research was initiated while Jota Ishikawa was visiting the Faculty of Commerce at UBC, which he thanks for their support and hospitality. Jota Ishikawa would also like to thank the Kajima Foundation and the Government of Canada Awards for financial support. Barbara Spencer is grateful for support from C.I.B.S. at the University of British Columbia.

Appendix A

Comparative static effects of s^d, t^m and s^h on output

Effect of $\rho \equiv r^D - s^d$ on final-good output

Taking the total differential of the first order conditions (2), we obtain:

$$[(n^d + 1)p' + Y^d p'']\mathrm{d}y^d + n^f(p' + y^d p'')\mathrm{d}y^f = -\mathrm{d}\rho \tag{A.1}$$

$$n^d(p' + y^f p'')\mathrm{d}y^d + [(n^f + 1)p' + Y^f p'']\mathrm{d}y^f = 0. \tag{A.2}$$

Solving (A.1) and (A.2) using Cramer's rule and $Y^f p'' = -p'\sigma^f E$, the effect of an increase in $\rho \equiv r^D - s^d$ on final-good output (shown as (4) of the text) is given by

$$y^d_\rho(\rho) = \gamma^f / p'\psi < 0,\ y^f_\rho(\rho) = -(n^d/n^f)(n^f - \sigma^f E)/p'\psi \text{ and}$$
$$Y_\rho = n^d / p'\psi \tag{A.3}$$

where $\gamma^f \equiv n^f + 1 - \sigma^f E > 0$ and $\psi \equiv N + 1 - E > 0$ from (3).

Effects of s^d, s^h and t^m on intermediate-good output

Taking the total differential of the first order conditions (8), we obtain:

$$[(n^h + 1)\rho' + X^h \rho'']\mathrm{d}x^h + n^m(\rho' + x^h\rho'')\mathrm{d}x^m = -\mathrm{d}s^h - \mathrm{d}s^d \tag{A.4}$$

$$n^h(\rho' + x^m\rho'')\mathrm{d}x^h + [(n^m + 1)\rho' + X^m\rho'']\mathrm{d}x^m = \mathrm{d}t^m - \mathrm{d}s^d. \tag{A.5}$$

From (A.4) and (A.5), using Cramer's rule, $E^u \equiv -X^D\rho''/\rho'$, $\delta^u \equiv x^h - x^m$ and $\psi^u \equiv N^u + 1 - E^u$, we then obtain (10) of the text:

$$\mathrm{d}x^h/\mathrm{d}s^d = -[1 + n^m\delta^u E^u/X^D]/\rho'\psi^u \text{ and}$$
$$\mathrm{d}x^m/\mathrm{d}s^d = -[1 - n^h\delta^u E^u/X^D]/\rho'\psi^u. \tag{A.6}$$

Similarly, using $X^k = n^k x^k$ and $n^k(\rho' + x^k\rho'') = \rho'(n^k - \sigma^{uk}E^u)$ for $k = h,\ m$,

$$\mathrm{d}X^h/\mathrm{d}s^h = -n^h\gamma^{um}/\rho'\psi^u > 0,\ \mathrm{d}X^m/\mathrm{d}s^h = n^h(n^m - \sigma^{um}E^u)/\rho'\psi^u, \tag{A.7}$$

$$\mathrm{d}X^m/\mathrm{d}t^m = n^m\gamma^{uh}/\rho'\psi^u < 0 \text{ and}$$
$$\mathrm{d}X^h/\mathrm{d}t^m = -n^m(n^h - \sigma^{uh}E^u)/\rho'\psi^u, \tag{A.8}$$

where $\gamma^{uk} \equiv n^k + 1 - \sigma^{uk}E^u > 0$ for $k = h,\ m$ from (9). Also, from (A.6), (A.7) and (A.8),

$$\mathrm{d}X^D/\mathrm{d}s^d = -N^u/\rho'\psi^u > 0,\ \mathrm{d}X^D/\mathrm{d}s^h = -n^h/\rho'\psi^u > 0 \text{ and}$$
$$\mathrm{d}X^D/\mathrm{d}t^m = n^m/\rho'\psi^u < 0. \tag{A.9}$$

Expression for E^u

Linear demand

From (6), we obtain $E^u \equiv -X^D \rho''/\rho' = Y^d Y^d_{\rho\rho}/(Y^d_\rho)^2$ which is zero for $p''=0$.

Constant elasticity demand

For $\epsilon \equiv -p/Yp'$ constant, $d\epsilon/dY = -[1+\epsilon(E-1)]/Y = 0$ implies $E \equiv -Yp''/p' = 1+1/\epsilon > 0$. Using $Y^d_\rho = \gamma^f Y_\rho$ for $Y_\rho = n^d/p'\psi$, $\gamma^f = n^f + 1 - \sigma^f E$ and E constant, we then obtain:

$$Y^d_{\rho\rho} = \gamma^f Y_{\rho\rho} - Y_\rho E(d\sigma^f/d\rho) \text{ where } Y_{\rho\rho} = (Y_\rho)^2 E/Y. \tag{A.10}$$

Also, from $d\sigma^f/d\rho = (Y^f_\rho - (Y^f/Y)Y_\rho)/Y$ using $Y^f_\rho = -(n^f - \sigma^f E)Y_\rho$ (see (A.3)) and (3), we obtain $d\sigma^f/d\rho = -[\gamma^f - (1-\sigma^f)]Y_\rho/Y$. It then follows from (A.10) that $Y^d_{\rho\rho} = (Y_\rho)^2 E[2\gamma^f - (1-\sigma^f)]/Y$ and hence that

$$E^u = Y^d Y^d_{\rho\rho}/(Y^d_\rho)^2 = \sigma^d E[2\gamma^f - (1-\sigma^f)]/(\gamma^f)^2. \tag{A.11}$$

Endogenous foreign price r^F

Totally differentiating $Y^d(\rho, r^F) = X^D$ and $Y^f(\rho, r^F) = X^F$, using (28) and $J = Y^d_p Y^f_{r^F} - Y^d_{r^F} Y^f_p = n^d n^f/(p')^2\psi$, $\rho = \phi^D(X^D, X^F)$ and $r^F = \phi^F(X^D, X^F)$ have partial derivatives:

$$\begin{aligned} \phi^D_D &= Y^f_{r^F}/J = p'\gamma^d/n^d < 0, \quad \phi^F_F = Y^d_\rho/J = p'\gamma^f/n^f < 0, \\ \phi^D_F &= -Y^d_{r^F}/J = p' + y^d p'', \quad \phi^F_D = -Y^f_\rho/J = p' + y^f p''. \end{aligned} \tag{A.12}$$

From (A.12) and $\delta \equiv y^d - y^f$, we obtain the following useful relationships:

$$\phi^F_F - \phi^F_D = p'/n^f,\ \phi^D_D - \phi^D_F = p'/n^d \text{ and } \phi^F_D - \phi^D_F = -\delta p''. \tag{A.13}$$

Differentiating (A.13) using $p = p(Y) = p(X^D + X^F)$, it then follows that:

$$\begin{aligned} &\phi^F_{FF} - \phi^F_{DF} = \phi^F_{FD} - \phi^F_{DD} = p''/n^f \text{ and} \\ &\phi^D_{DD} - \phi^D_{FD} = \phi^D_{DF} - \phi^D_{FF} = p''/n^d, \end{aligned} \tag{A.14}$$

where $\phi^F_{DF} = p''(n^f+1)/n^f + y^f p'''$ and $\phi^D_{DF} = p''(n^d+1)/n^d + y^d p'''$ from (A.12) and (3). Using (A.13), (A.14) since $A^k_{ii} \equiv (N^u+1)\phi^i_i + X^i\phi^i_{ii} + X^j\phi^j_{ii}$ and $A^k_{ij} \equiv N^u\phi^i_j + \phi^j_i + X^i\phi^i_{ij} + X^j\phi^j_{ij}$ from (33), we then obtain

$$\begin{aligned} A^k_{FF} - A^k_{FD} &= (N^u+1)p'/n^f - 2\delta p'' \text{ and} \\ A^k_{DD} - A^k_{DF} &= (N^u+1)p'/n^d + 2\delta p''. \end{aligned} \tag{A.15}$$

Derivation of $d\pi^k/ds^d$, dr^F/ds^d and $d\rho/ds^d$

Since $\rho=\phi^D(X^D,X^F)$ and $r^F=\phi^F(X^D,X^F)$, we obtain

$$\mathrm{d}\rho/\mathrm{d}s^d = N^u[\phi^D_D(\mathrm{d}x^{kD}/\mathrm{d}s^d) + \phi^D_F(\mathrm{d}x^{kF}/\mathrm{d}s^d)]$$
$$\text{and } \mathrm{d}r^F/\mathrm{d}s^d = N^u[\phi^F_F(\mathrm{d}x^{kF}/\mathrm{d}s^d) + \phi^F_D(\mathrm{d}x^{kD}/\mathrm{d}s^d)]. \tag{A.16}$$

Using (A.16) and imposing $\partial\pi^k/\partial x^{kD}=\partial\pi^k/\partial x^{kF}=0$ from (31), this implies (35) of the text:

$$\mathrm{d}\pi^k/\mathrm{d}s^d = x^{kD} + [(N^u-1)/N^u][x^{kD}(\mathrm{d}\rho/\mathrm{d}s^d) + x^{kF}(\mathrm{d}r^F/\mathrm{d}s^d)]. \tag{A.17}$$

Also, from (A.16) and (34), we obtain $\mathrm{d}r^F/\mathrm{d}s^d=N^u[\phi^F_F A^k_{FD}-\phi^F_D A^k_{FF}]/H^u$, which, using (A.13) implies $\mathrm{d}r^F/\mathrm{d}s^d=N^u\{(p'/n^f)A^k_{FD}-\phi^F_D(A^k_{FF}-A^k_{FD})\}/H^u$. Hence, using (A.15) and $\phi^D_F=\phi^F_D+\delta p''$ from (A.13), we obtain (39) of the text:

$$\mathrm{d}r^F/\mathrm{d}s^d = N^u\{[p'/n^f + 2\phi^F_D]\delta p'' + (p'/n^f)[X^F\phi^F_{FD} + X^D\phi^D_{DF}]\}/H^u. \tag{A.18}$$

Similarly, since $\mathrm{d}\rho/\mathrm{d}s^d=-N^u(\phi^D_D A^k_{FF}-\phi^D_F A^k_{FD})/H^u$ (from (A.16) and (34)), this implies (40) of the text:

$$\mathrm{d}\rho/\mathrm{d}s^d = -N^u\{(p'/n^d)A^k_{FF} + \phi^D_F(A^k_{FF} - A^k_{FD})\}/H^u. \tag{A.19}$$

For $p''=0$, using $A^k_{FF}=(N^u+1)\phi^F_F=(N^u+1)p'(n^f+1)/n^f$ and $H^u=(N^u+1)^2(p')^2(N+1)/n^d n^f$ from (33) in (A.19), we obtain (41) of the text:

$$\mathrm{d}\rho/\mathrm{d}s^d = -N^u/(N^u+1) < 0 \text{ and}$$
$$\mathrm{d}r^D/\mathrm{d}s^d = \mathrm{d}\rho/\mathrm{d}s^d + 1 = 1/(N^u+1) > 0. \tag{A.20}$$

Policies applied to the intermediate good

Effects of the production subsidy, s^h and the import tariff, t^m, on profits

From (7) using (8) and (A.7), the effect of s^h on firm h's profit is

$$\mathrm{d}\pi^h/\mathrm{d}s^h = x^h\{1 + \rho'[(\mathrm{d}X^m/\mathrm{d}s^h) + (n^h-1)(\mathrm{d}x^h/\mathrm{d}s^h)]\}$$
$$= x^h(1+\beta^u), \tag{A.21}$$

where $\beta^u\equiv\alpha^u/\psi^u$ for $\alpha^u\equiv n^m+1-\sigma^{um}E^u-n^h$. Also, noting that $\mathrm{d}\pi^h/\mathrm{d}t^m=x^h\rho'[(\mathrm{d}X^D/\mathrm{d}t^m)-(\mathrm{d}x^h/\mathrm{d}t^m)]$ and $2n^k-\sigma^{uk}E^u=n^k(2\rho'+x^k\rho'')/\rho'>0$ for $k=h$, m from (9), we have

$$\mathrm{d}\pi^h/\mathrm{d}t^m = n^m x^h(2n^h - \sigma^{uh}E^u)/n^h\psi^u > 0. \tag{A.22}$$

Similarly, the effects of s^h and t^m on the profit of a foreign firm m are given by

$$\mathrm{d}\pi^m/\mathrm{d}s^h = -n^h x^m(2n^m - \sigma^{um}E^u)/n^m\psi^u < 0 \text{ and}$$
$$\mathrm{d}\pi^m/\mathrm{d}t^m = -x^m(1+\beta^u). \tag{A.23}$$

Finally, it follows from (15) and $r^D = \rho(X^D) - s^d$ that

$$\mathrm{d}\pi^d/\mathrm{d}s^h = -y^d(1+\beta)(\mathrm{d}r^d/\mathrm{d}s^h) \text{ and}$$
$$\mathrm{d}\pi^d/\mathrm{d}t^m = -y^d(1+\beta)(\mathrm{d}r^D/\mathrm{d}t^m), \tag{A.24}$$

where $\mathrm{d}r^D/\mathrm{d}s^h = \rho'(\mathrm{d}X^D/\mathrm{d}s^h) = -n^h/\psi^u < 0$ and $\mathrm{d}r^D/\mathrm{d}t^m = n^m/\psi^u > 0$ from (A.9).

Effects of s^h and t^m on welfare

Setting $s^d = 0$ in W^D as given by (18), s^{h*} and t^{m*} satisfy

$$\mathrm{d}W^D/\mathrm{d}s^h = n^d(\mathrm{d}\pi^d/\mathrm{d}s^h) + n^h(\mathrm{d}\pi^h/\mathrm{d}s^h) - X^h - s^h(\mathrm{d}X^h/\mathrm{d}s^h) + t^m(\mathrm{d}X^m/\mathrm{d}s^h) = 0 \tag{A.25a}$$

and

$$\mathrm{d}W^D/\mathrm{d}t^m = n^d(\mathrm{d}\pi^d/\mathrm{d}t^m) + n^h(\mathrm{d}\pi^h/\mathrm{d}t^m) + X^m + t^m(\mathrm{d}X^m/\mathrm{d}t^m) - s^h(\mathrm{d}X^h/\mathrm{d}t^m) = 0. \tag{A.25b}$$

Now using (A.7), (A.8), (A.21), (A.22) and (A.24), expressions (A.25a and b) become

$$\mathrm{d}W^D/\mathrm{d}s^h = (n^h/\psi^u)\{Y^d(1+\beta) + x^h\alpha^u + (s^h + t^m)(n^m - \sigma^{um}E^u)/\rho' + s^h/\rho'\} = 0$$

$$\mathrm{d}W^D/\mathrm{d}t^m = (n^m/\psi^u)\{-Y^d(1+\beta) + x^h(2n^h - \sigma^{uh}E^u) + x^m\psi^u + (s^h + t^m)(n^h - \sigma^{uh}E^u)/\rho' + t^m/\rho'\} = 0. \tag{A.26}$$

From (A.26) using $2n^h - \sigma^{uh}E^u = \psi^u - \alpha^u$, it follows that

$$s^{h*} = -\rho'[Y^d(1+\beta) + x^h\alpha^u] - (s^{h*} + t^{m*})(n^m - \sigma^{um}E^u) \text{ and } t^{m*} = -\rho'[-Y^d(1+\beta) + (x^h + x^m)\psi^u - x^h\alpha^u] - (s^{h*} + t^{m*})(n^h - \sigma^{uh}E^u), \tag{A.27}$$

which, summing the two expressions, implies,

$$s^{h*} + t^{m*} = -\rho'(x^h + x^m) > 0. \tag{A.28}$$

Substituting (A.28) back into (A.27) and using $\alpha^u \equiv (n^m - \sigma^{um}E^u) + 1 - n^h$, $\psi^u = n^m + n^h + 1 - E^u$ and $\rho' = 1/Y_\rho^d$, we then obtain (43) of the text:

$$s^h* = -[Y^d\beta + x^h + x^m\sigma^{um}E^u]/Y^d_\rho \text{ and}$$
$$t^m* = [Y^d\beta - x^m(1-\sigma^{um}E^u)]/Y^d_\rho. \tag{A.29}$$

Relationship between $\hat{s}^d$ and the joint policies s^h* and t^m*

From (A.29), (21) and $\sigma^{um} = X^m/X^D$,

$$s^h* = \hat{s}^d - [(x^h + X^m x^m E^u/X^D)N^u - Y^d\Omega\psi^u]/N^u Y^d_\rho \tag{A.30}$$

Using $\sigma^{um} = 1 - \sigma^{uh}$, $\sigma^{uh} = X^h/X^D$ and $X^D = Y^d$ in $\Omega \equiv [\sigma^{uh}(1 + n^m\delta^u E^u/X^D) - \sigma^{um}(1-E^u)]/\psi^u$ from (22), we obtain $Y^d\Omega = \{X^h - X^m + (E^u/X^D)[X^mY^d + n^hn^mx^h\delta^u]\}/\psi^u$, which using (A.30) implies,

$$s^h* = \hat{s}^d - \{n^m(x^h + x^m) - (E^u/X^D)[X^mX^h + n^hn^mx^h\delta^u - X^mx^mn^h]\}/N^uY^d_\rho. \tag{A.31}$$

Since $[X^mX^h + n^hn^mx^h\delta^u - X^mx^mn^h] = n^hn^m\delta^u(x^h + x^m)$, (A.31) reduces to

$$s^h* = \hat{s}^d - n^m(x^h + x^m)[1 - n^h\delta^uE^u/X^D]/N^uY^d_\rho. \tag{A.32}$$

Also from (A.32) using $t^m* = -s^h* - (x^h + x^m)/Y^d_\rho$ from (A.28), it follows that

$$t^m* = -\hat{s}^d - n^h(x^m + x^h)[1 + n^m\delta^uE^u/X^D]/N^uY^d_\rho \tag{A.33}$$

A production subsidy $\hat{s}^h$ to the intermediate good alone

Setting $t^m = 0$ in (A.25a) and using the first expression of (A.21), $\hat{s}^h$ is given by

$$\hat{s}^h = X^h\rho'\left[\frac{(\mathrm{d}X^m/\mathrm{d}s^h)}{(\mathrm{d}X^h/\mathrm{d}s^h)} + \frac{n^h - 1}{n^h}\right] + \frac{n^d(\mathrm{d}\pi^d/\mathrm{d}s^h)}{(\mathrm{d}X^h/\mathrm{d}s^h)}, \tag{A.34}$$

which is (45) of the text. Now, using (A.7), (A.24), $\mathrm{d}r^D/\mathrm{d}s^h = -n^h/\psi^u$ and $\rho' = 1/Y^d_\rho$ in (A.34),

$$\hat{s}^h = [X^h\beta^u - Y^d(1+\beta)(\mathrm{d}r^D/\mathrm{d}s^h)]/(\mathrm{d}X^h/\mathrm{d}s^h)$$
$$= -[x^h\alpha^u + Y^d(1+\beta)]/\gamma^{um}Y^d_\rho, \tag{A.35}$$

where $\gamma^{um} \equiv n^m + 1 - \sigma^{um}E^u > 0$.

References

Abiru, M., 1988. Vertical integration, variable proportions and successive oligopolies. Journal of Industrial Economics 36, 315–325.

Bernhofen, D.M., 1997. Strategic trade policy in a vertically related industry. Review of International Economics 5, 429–433.

Brander, J.A., Spencer, B.J., 1984. Trade warfare: tariffs and cartels. Journal of International Economics 16, 227–242.
Brander, J.A., Spencer, B.J., 1985. Export subsidies and international market share rivalry. Journal of International Economics 18, 83–100.
Chang, W.W., Chen, F., 1994. Vertically related markets: export rivalry between DC and LDC firms. Review of International Economics 2, 131–142.
Eaton, J., Grossman, G.M., 1986. Optimal trade and industrial policy under oligopoly. Quarterly Journal of Economics 101, 383–406.
Greenhut, M.L., Ohta, H., 1979. Vertical integration of successive oligopolies. American Economic Review 69, 137–141.
Ishikawa, J., 1998. Expanding the purchase of a foreign intermediate good: an analysis of VIEs and content protection under oligopoly, in: Sato, R., Ramachandran, R. (Eds.), Global Integration and Competition. Kluwer Academic Publishers, Boston, in press.
Ishikawa, J., Lee, K., 1997. Backfiring tariffs in vertically related markets. Journal of International Economics 42, 395–423.
Ishikawa, J., Spencer, B.J., 1996. Rent-shifting export subsidies with an imported intermediate product. NBER working paper (February) ≠5458.
Ishii, J., Yi, K., 1997. The growth of world trade. Federal Reserve Bank of New York, research paper ≠9718.
Kreps, D.M., Scheinkman, J.A., 1983. Quantity precommitment and Bertrand competition yield Cournot outcomes. Bell Journal of Economics 14, 326–337.
Krishna, K., Thursby, M., 1991. Optimal policies with strategic distortions. Journal of International Economics 31, 291–308.
Maggi, G., 1996. Strategic trade policies with endogenous mode of competition. American Economic Review 86(1), 237–258.
Neary, J.P., 1994. Cost asymmetries in international subsidy games: should governments help winners or losers? Journal of International Economics 32, 197–218.
Rodrik, D., Yoon, C., 1989. Strategic trade policy when domestic firms compete against vertically integrated rivals. NBER Working Paper No. 2919 (revised, June 1992).
Salinger, M.A., 1988. Vertical mergers and market foreclosure. Quarterly Journal of Economics 103, 345–356.
Seade, J., 1980. The stability of Cournot revisited. Journal of Economic Theory 23, 15–27.
Seade, J., 1985. Profitable cost increases and the shifting of taxation: equilibrium responses of markets in oligopoly. Warwick University manuscript.
Spencer, B.J., Jones, R.W., 1991. Vertical foreclosure and international trade policy. Review of Economic Studies 58, 153–170.
Spencer, B.J., Jones, R.W., 1992. Trade and protection in vertically related markets. Journal of International Economics 32, 31–55.
Spencer, B.J., Raubitschek, R.S., 1996. High-cost domestic joint ventures and international competition: do domestic firms gain? International Economic Review, 37 (2), 315–340.
Tirole, J., 1988. The Theory of Industrial Organization. MIT Press, Cambridge MA, pp. 211–234.