Strategic trade policy with endogenous choice of quality and asymmetric costs

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Abstract

This paper examines strategic trade and joint welfare maximizing incentives towards investment in the quality of exports by an LDC and a developed country. Firms first compete in qualities and then export to an imperfectly competitive, third country market. Under Bertrand competition, unilateral policy involves an investment subsidy by the low-quality LDC and an investment tax by the developed country, whereas jointly optimal policy calls for the reverse so as to reduce price competition by increasing product differentiation. Under Cournot competition, unilateral policy is also reversed from the Bertrand outcome, but jointly optimal policy involves a tax in both countries. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The availability of a greater variety of products with increasing levels of world trade has emphasised the importance of non-price competition for success in...

exporting. At one extreme, there is Japan with its demanding consumers and quality oriented production culture and, at the other, there is the emergence of lower quality, but cost competitive producers among the newly industrialized countries (NICs). Thus success for a company can often involve the careful positioning of products in the quality spectrum taking into account the qualities chosen by foreign rivals. The importance of this strategy is particularly evident in the rapidly expanding, knowledge intensive industries, such as pharmaceuticals and computer software. First, these industries often exhibit high up-front costs of product development with subsequent low variable costs of production. Also, firms tend to be oligopolistic because of limitations on entry due to this cost structure and an ability to patent. In such an environment, the particular features that differentiate products are the main determinants of success and a major focus of competition is at the product development stage.\(^1\)

There are a number of possible motives for government policy targeted at product quality. In particular, regulations affecting quality, such as minimum quality standards, may simply be a response to the need for consumer protection due to asymmetric information about product quality. Such policies may also be a means to protect domestic industry from import competition.\(^2\) Other motives, however, are needed to explain the existence of policies targeted at the quality of exports.\(^3\) Taiwan, for example, has a long standing policy to influence the quality of exports through compulsory inspection of certain export items and the subsidization of quality control associations in some sectors (e.g., machine tools, heavy electrical machinery, umbrellas and toys, see Wade, 1990, 144). Korea has also encouraged product quality improvement in some sectors, while, as part of the so called ‘Northern strategy’, it has also subsidized the marketing of certain low quality products, thus eliminating incentives to improve product quality (Ursacki and Vertinsky, 1994). In Finland, the government subsidized product oriented R&D in paper production, offering incentives for climbing the product quality scale in an industry which was already a world leader in the production of high quality papers (Wilson et al., 1998). Subsidies for product quality improvement in the newsprint industry have also been recommended in Canada, despite Canadian leadership in quality (see Binkley, 1993).

There are various arguments as to why governments might want to raise the quality of exports when quality levels are low. For example, Taiwan may have imposed quality controls to avoid damage to the reputation of all its exports from

\(^1\)These features can be broadly interpreted as any attributes, including attributes of the production process (e.g. impacts of production on the environment) that consumers care about (see Inglehart, 1990).

\(^2\)For example, the U.S. has long complained that Japanese regulations specifying detailed characteristics that particular products must satisfy are discriminatory against imports.

\(^3\)Quality upgrading of exports could be an indirect consequence of growth policies that generally target investment and R&D. Our concern is with policies that specifically target the quality of exports.
the export of shoddy goods. There may also be a motive to improve the quality of exports so as to satisfy minimum quality standards in importing countries. However, these arguments do not explain why governments would subsidize quality improvements for firms that are already industry leaders in quality or even discourage the development of quality for their low-quality exporters.

This paper explores the implications of a strategic-trade policy or rent-shifting motive for subsidy or tax policy applied to investments in quality improvements for exported products. There are two countries, a developed country and an LDC (less developed country), each with one firm producing a quality differentiated good. To focus on strategic trade policy effects, we assume that the entire production is exported to a third country market on the basis of either Bertrand or Cournot competition. A feature of the model is asymmetry of investment costs across countries, reflecting the reduced opportunities for investment in the LDC relative to the developed country. This cost difference is assumed to be sufficiently large that the firm in the LDC will produce a lower quality product than does its developed country rival. However, even if investment costs are identical, policies differ sharply conditional on whether a country produces the high or low quality. As we show, under Bertrand competition, domestic welfare in the low-quality country is increased by a subsidy to investment, whereas the high-quality country gains from an investment tax. These policies are reversed under Cournot competition, with policy switching to a tax in the low-quality country and a subsidy in the high-quality country. As these results indicate, strategic trade policy can explain why a country might intervene to raise the quality of low-quality exports, but it also shows that there are circumstances in which there is a motive for less obvious policies, such as a subsidy to a high-quality producer or a tax on quality development by a low-quality producer.

The model involves a three stage (full information) game in which governments act first to maximize domestic welfare by committing to subsidy or tax policy. If both countries intervene, there is a Nash equilibrium in subsidy and tax levels. Firms then commit to their levels of investment in quality and subsequently compete in quantities or prices. The structure follows Spencer and Brander (1983),

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4Strategic trade policy was originally developed by Spencer and Brander (1983) and Brander and Spencer (1985). Eaton and Grossman (1986) show the importance of Bertrand versus Cournot competition.

5For some of the technical development including the effects of asymmetry, we refer to Zhou et al. (2000), which is a working paper version of the current paper. See De Meza (1986) and Neary (1994) for strategic trade models with asymmetric production costs.

6As shown in Zhou et al. (2000), for a sufficiently large difference in investment costs and for both Bertrand and Cournot competition, there exists a unique pure strategy equilibrium in which the LDC exports the low-quality product and the developed country exports the high-quality product. This result overcomes the problem that in the symmetric model, allocation of qualities to firms is indeterminate. It also rules out the possibility that a low-quality producer can ‘leapfrog’ its quality above that of its rival (see, for example, Motta et al., 1997).
except that government policy affects positioning in product space, rather than levels of cost-reducing investment or R&D for products that are fixed in nature. Since, in the current application, firms are constrained by the Nash assumption that they take the rival’s quality as given, the strategic trade policy incentive is to set subsidy (or tax) policy towards domestic investment in quality so as to manipulate quality choices in such a way as to raise the domestic rents (profits less the cost of any subsidy) earned from exports.

We also explore the implications of coordinated policy choices by the two producing nations so as to maximize their joint welfare. With the elimination of the motive for rent-extraction from the rival firm, the aim is to increase the total profits extracted from third country consumers. Nevertheless, the appropriate policy direction for each country is not immediately obvious. For Bertrand competition, a move from the Nash policies to the jointly optimal policy causes a switch in policies for both countries, namely the LDC should tax rather than subsidize quality and the developed country should subsidize rather than tax quality. For Cournot competition, the jointly optimal policy involves taxes by both countries.

We use a standard model of vertical quality differentiation in which consumers purchase at most one unit of the differentiated product. The assumption that the costs of quality development are sunk prior to the determination of prices and output is also well established in the literature. However, international trade theory has mostly concentrated on an alternative model, in which quality affects variable production costs and there are no up-front investment or R&D costs. This international literature also differs because of its main focus on the effects of domestic import restrictions on quality upgrading or downgrading. Following the approach of Ronnen (1991), we use analytical methods to develop our results for a significantly more general formulation of investment costs than is typically found in the literature.

The paper is organized as follows: Section 2 sets out the structure of the game and the basic consumer preferences and costs underlying the model of quality choice. Section 3 investigates investment policy and quality choice under Bertrand competition.

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1 Policy towards cost-reducing investment differs, since it involves a subsidy under both Bertrand and Cournot competition (see Spencer and Brander, 1983; Bagwell and Staiger, 1994). We refer to firms as investing in quality, but, this investment could also be interpreted as R&D expenditure aimed at improving quality.


3 Papers include Krishna (1987), Bond (1988), Das and Donnenfeld (1987, 1989) and Ries (1993). This model is often termed a ‘variable cost of quality model’, whereas the model with up-front investment costs is termed a ‘fixed cost of quality model’.

4 There is also empirical work on quality upgrading (see, for example, Feenstra, 1988).

5 Previous analysis has mostly modeled investment costs as simply depending on the square of quality and, particularly for the Cournot case, numerical values have been used to characterize equilibrium (see, for example, Motta, 1993; Herguera et al., 2000).
competition whereas Section 4 develops and contrasts the results for Cournot competition. Finally, Section 5 contains concluding remarks.

2. The basic model: consumer demand and costs

There are two firms, firm $L$, located in an LDC and firm $H$, in a developed country. Each firm produces a quality differentiated product, all of which is exported to a third country market. The game between firms involves a sub-game perfect equilibrium with two stages of decision. In stage 1, the quality of each product is determined at a Nash equilibrium in which each firm sets its investment in quality so as to maximize profit, taking the quality of the other firm as given. In stage 2, the products are sold on the basis of a Bertrand–Nash equilibrium if price is the decision variable or a Cournot–Nash equilibrium if quantity is the decision variable. This two-stage structure reflects the idea that price (or quantity) can be changed more easily than product quality, which is a longer term decision. Governments commit to policy towards investment at stage 0, prior to the game played by firms.

The asymmetry in investment costs across countries is reflected by the assumption that firm $H$, in the developed country, requires an investment $F(q)$ to produce a product with quality $q$, whereas firm $L$, in the LDC, requires an investment of $\gamma F(q)$, where $\gamma \geq 1$. Otherwise, the two firms are identical for any given value of $q$. The investment cost, $F(q)$, and the marginal investment cost, $F'(q)$, are assumed to be strictly increasing in quality for all $q \in (0, \infty]$. Following Ronnen (1991), for the existence and uniqueness of equilibrium, we also assume that $F''(q) \geq 0$ and that $F'(q)$ becomes infinite in the limit as $q$ becomes very large. The total and marginal costs of the first unit of quality are assumed to be zero (i.e. $F(0) = F'(0) = 0$) so as to make it profitable for both firms to enter. In summary, we assume:\footnote{Two classes of functions satisfying (1) are $F(q) = a q^n$ for $n \geq 2$ and $F(q) = q(e^q - 1)$, where $a > 0$.}

$$ F(0) = F'(0) = 0; \quad F'(q) > 0 \quad \text{and} \quad F''(q) > 0 \quad \text{for} \quad q > 0; $$
$$ \lim_{q \to \infty} F'(q) = \infty; \quad F''(q) \geq 0. $$

(1)

Also, to focus on investment decisions, we assume that marginal and average production costs per unit of quality are constant and, for simplicity, we let these costs be zero.\footnote{Our results apply if the total production cost for output $x$ of quality $q$ is $C = c x q$, where $c \geq 0$ is constant.}

If firms are identical ($\gamma = 1$ in our setting), Ronnen (1991) has shown that for Bertrand competition at stage two, conditions (1) are sufficient to ensure the
existence of a unique global equilibrium with respect to the qualities produced of each product. However, the allocation of qualities across firms is indeterminate. Nevertheless, as we show in Zhou et al. (2000), for both Bertrand and Cournot competition, we can address the case in which policy decisions are made knowing that the LDC will produce the lower quality product by assuming that $\gamma$ is sufficiently large. Consequently, letting superscripts $L$ and $H$ indicate variables associated with firms $L$ and $H$ respectively, the ratio of high to low quality is given by $r = q^H / q^L \geq 1$.

Consumers vary based on a taste parameter for quality, denoted $\theta$, which is uniformly distributed on $[0, 1]$. Each consumer purchases at most one unit of the differentiated good and obtains a (linear) utility, $u q$, from consumption of quality $i$ for $i = L, H$. Since consumers pay a price, $P$, for quality $q$, the quality-adjusted price is $p^i = P^i / q^i$ for $q^i > 0$ and consumer surplus for taste $\theta$ can be represented by $C = C^i(q^i, p^i; \theta) = \theta q^i - P^i = q^i (\theta - p^i)$. Assuming a reservation surplus of zero, consumers purchase the good if and only if $C^i > 0$, which requires $q^i > 0$. For $C^i > 0$, we then define $\tilde{\theta}$ to represent the value of $\theta$ at which a consumer would be indifferent between high and low quality.\(^{15}\) It then follows that consumers with $\theta \in (\tilde{\theta}, 1]$ purchase $q^H$, consumers with $\theta \in (p^i, \tilde{\theta}]$ purchase $q^L$ and consumers with $\theta \in [0, p^i]$ for $p^i > 0$ do not purchase the differentiated good. Also, when qualities differ, i.e. for $r > 1$, the quantities demanded of the low and high quality goods, denoted by $x^L$ and $x^H$ respectively, are given by\(^{16}\),

$$x^L = \tilde{\theta} - p^L = r (p^H - p^L) / (r - 1),$$
$$x^H = 1 - \tilde{\theta} = 1 - (r p^H - p^L) / (r - 1).$$

3. Investment policy and quality choice under Bertrand competition

Assuming prices are determined by Bertrand competition after firms have committed to quality, we develop the model of quality choice in subsection 3.1. Policies towards investment in quality are then investigated in 3.2 and 3.3 for the LDC and developed country respectively.

3.1. The two-stage model of firm behavior: Bertrand competition

As is standard in these models, we first examine the second stage equilibrium in which each firm sets its price to maximize its profit, taking the price of the other

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\(^{14}\)There are no income effects since, implicitly, utility is assumed to be separable in a second homogeneous good. This homogeneous good also acts behind the scenes to achieve trade balance.

\(^{15}\)For $r > 1$, setting $C^i(q^i, p^i; \tilde{\theta}) = C^i(q^H, p^H; \tilde{\theta})$, we obtain $\tilde{\theta} = (p^H - p^L) / (r - 1)$.

\(^{16}\)If $r = 1$, then since $p^H = p^L$, the good is purchased for $\theta \in (p^i, 1]$ and $x^i = (1 - p^i) / 2$ for $i = L, H$. As we show, qualities always differ at the Nash equilibrium (see also Motta, 1993).
firm as given. Since the qualities \( q^L \) and \( q^H \) are committed at the first stage, this is equivalent to choosing quality-adjusted prices, \( p^i = P^i / q^i \) for \( i = L, H \). Recalling that production costs are zero, the firms earn profits from production equal to their respective revenues, denoted by \( R^i = p^i x^i \) for \( i = L, H \). Solving for the Bertrand equilibrium we obtain \( p^L = p^H / 4 \) and, as shown in (A.4) of Appendix A, revenues as a function of quality can be expressed in the convenient forms, 

\[
R^L(q^L, q^H) = \varphi(r)q^L \quad \text{and} \quad R^H(q^L, q^H) = 4\varphi(r)q^H
\]

where \( \varphi(r) = p^L x^L = r(r - 1)/(4r - 1)^2 \) and \( \varphi'(r) > 0 \). As these expressions show, firm \( H \) earns four times the revenue per unit of quality than does its low-quality rival. Also, firm \( L \)'s revenue is increasing in \( q^L \), whereas firm \( H \)'s revenue is decreasing in \( q^L \): i.e. using subscripts \( L \) and \( H \) to represent partial derivatives with respect to \( q^L \) and \( q^H \) respectively, we obtain

\[
R^L = \varphi'(r) > 0, \quad R^H = -4(r)^2 \varphi'(r) < 0.
\]

Since an increase in \( q^H \) and a reduction in \( q^L \) both increase the separation of products as measured by \( r = q^H / q^L \), both firms enjoy higher revenue as the products become more differentiated. These higher revenues reflect the fact that greater product differentiation reduces price competition, leading to an increase in quality-adjusted prices (and a reduction in both outputs) at the Bertrand equilibrium (see (A.3)).

Turning to the determination of quality, we first define \( s^i \) to represent the proportion of the cost of investment in quality covered by a subsidy to firm \( i \) for \( i = L, H \). Assuming that \( s^L < 1 \), with \( s^L < 0 \) corresponding to a tax, both firms face a strictly positive cost of investment at stage 1, with profits for firm \( L \) and firm \( H \) respectively given by:

\[
\begin{align*}
\pi^L(q^L, q^H; s^L) &= R^L(q^L, q^H) - \gamma(1 - s^L)F(q^L), \\
\pi^H(q^L, q^H; s^H) &= R^H(q^L, q^H) - (1 - s^H)F(q^H).
\end{align*}
\]

Setting \( q^L \) to maximize \( \pi^L \), taking \( q^H \) as given, and setting \( q^H \) to maximize \( \pi^H \), taking \( q^L \) as given, it follows from (4) that the Nash equilibrium qualities, \( q^L \) and \( q^H \), satisfy the first order conditions:

\[
\begin{align*}
\pi^L_L = R^L_L - \gamma(1 - s^L)F'(q^L) = 0, \\
\pi^H_H = R^H_H - (1 - s^H)F'(q^H) = 0,
\end{align*}
\]

where marginal revenue from an increase in quality is given by \( R^L_L = \varphi(r) - r\varphi'(r) \) and \( R^H_H = 4\varphi(r) + r\varphi'(r) \) for firms \( L \) and \( H \) respectively. The products are not identical since, as shown in (A.5), \( R^L_L > 0 \) only if \( r > 7/4 \). Also, from (A.6) and (A.7), the second order and stability conditions are satisfied globally.

In deciding on quality, the firms face two basic considerations. The first is the profitability of the location in quality space based on revenues and the cost of

\[\text{If } s^i \geq 1, \text{ then the cost of investment is zero or negative and quality would increase excessively.}\]
investment in quality for a given distance from the rival’s quality as measured by the quality ratio, \( r \). The second is the effect of a change in the quality ratio, which determines the degree of price competition. For firm \( L \), since an increase in \( q^L \) reduces the gap between the products (holding \( q^H \) fixed), the associated increase in price competition tends to limit the gain from an increase in quality. Nevertheless, firm \( L \) has an incentive to set \( q^L > 0 \) (for any \( q^H \)) because the assumptions \( F(0) = 0 \) and \( F'(0) \) from (1) ensure that its marginal profit from a very low quality is always strictly positive. By contrast, the prospect of reduced price competition favors an increase in quality by firm \( H \), but the extent of the increase is limited by the rising marginal cost of investment in quality.

As illustrated in Fig. 1, since each firm’s marginal revenue from own quality is increasing in the other firm’s quality (i.e. \( R^{HH}_L > 0 \) and \( R^{LL}_H > 0 \)), the reaction functions, denoted \( q^H = \rho^H(q^L) \) and \( q^L = \rho^L(q^H) \) for firms \( H \) and \( L \) respectively, have positive slopes (see (A.8)), making the products strategic complements in quality space. Since an increase in \( q^L \) raises price competition by reducing the gap between qualities, firm \( H \) has an incentive to also raise \( q^H \) so as to help ease this competition. Conversely, the reduced competition from an increase in \( q^H \) gives firm \( L \) the room to raise \( q^L \) so as to better position its product. The second order and stability conditions ensure that firm \( L \) has a steeper reaction function than does firm \( H \) and hence that the curves cross at the unique Nash-equilibrium point (shown as \( N \)). Since \( r = q^H / q^L > 1 \), the reaction functions both lie above the (dotted) 45° line.

3.2. LDC investment policy towards the low-quality product

We now turn to the effects of an LDC subsidy (or tax) set at stage 0, prior to investment in quality. Since the subsidies, \( s^L \) and \( s^H \), by the LDC and developed
country respectively are applied directly to investment and since investment costs are sunk at stage 1, there is no change in the second-stage price equilibrium for given levels of quality. As for the first stage quality game, the first order conditions (5) define \( q^L = q^L(s^L, s^H) \) and \( q^H = q^H(s^L, s^H) \) and, as set out in Proposition 1(i) below, an investment subsidy by the LDC increases both \( q^L \) and \( q^H \), enhancing the quality levels chosen by both firms. This result follows since an increase in \( q^L \) due to the subsidy causes firm \( H \) to also increase quality (\( q^L \) and \( q^H \) are strategic complements). However, since the increase in \( q^H \) is not sufficient to prevent a fall in the quality ratio, \( r = q^H/q^L \), the outcome is a greater similarity of products. Firm \( L \) enjoys higher profits, but the profits of firm \( H \) are reduced due to the fall in quality-adjusted prices and the increasingly high cost of investment in quality.

Since all of the good is exported, welfare in the LDC, denoted \( W^L \), is simply the profit from firm \( L \)'s exports less the cost of the subsidy, \( s^L \), to taxpayers.\(^{18}\) Letting \( s^L \) maximize \( W^L \) taking the policy, \( s^H \), of the developed country as given, we show in Proposition 1(ii) that \( s^L > 0 \) and hence that the LDC has a unilateral incentive to subsidize investment in quality. The proof of Proposition 1 and all subsequent proofs concerning the Bertrand case are set out in Appendix A.

**Proposition 1.** Assume Bertrand competition and \( s^H \) held fixed.

(i) An increase in \( s^L \), (a) raises both \( q^L \) and \( q^H \), but \( r = q^H/q^L \) falls, causing \( p^L \) and \( p^H \) to fall, but \( x^L \) and \( x^H \) to rise; (b) increases profit, \( \pi^L \), in the LDC and reduces \( \pi^H \).

(ii) LDC welfare is maximized by an investment subsidy, \( s^{L*} = \frac{R_p(q^H/dq^L)}{\gamma F'(q^L)} > 0 \).

Following Spencer and Brander (1983), the LDC policy can be understood from the insight that \( s^{L*} \) maximizes the LDC's rents from exports by shifting the equilibrium to what would have been the Stackelberg leader–follower point in quality space with firm \( L \) as the leader and no subsidy.\(^{19}\) Since \( q^H \) is increasing in \( q^L \) (firm \( H \)'s quality reaction function has a positive slope) and since, for any \( q^L \), firm \( L \) benefits from an increase in \( q^H \) (i.e. \( R_p^H > 0 \)), firm \( L \), as the leader, would raise \( q^L \) above the Nash-equilibrium level\(^{20}\). It follows that for Nash behavior by firms in quality space, LDC welfare is maximized by a subsidy to investment so as to achieve the same higher level of \( q^L \). Fundamentally, the subsidy corrects for the

\(^{18}\)\( W^L(s^L, s^H) = \pi^L(q^L; q^H; s^L) - s^L \gamma F(q^L) = R^L(q^L, q^H) - \gamma F(q^L) \) for \( q^L = q^L(s^L, s^H) \) and \( q^H = q^H(s^L, s^H) \).

\(^{19}\)As a leader, firm \( L \) would set \( q^L \) to maximize \( \pi^L(q^L, p^H(q^L; s^H), 0) \), which equals \( W^L(s^L, s^H) = R^L(q^L, q^H) - \gamma F(q^L) \) for \( p^H = p^H(q^L; s^H) \). The difficulty is to show that \( \pi^L(q^L, p^H(q^L; s^H), 0) \) is strictly concave in \( q^L \) and hence that \( W^L \) is locally concave at \( s^{L*} \). This concavity, together with a large \( \gamma \) ensures that the LDC does not attempt to leapfrog \( q^L \) above \( q^H \) (see Zhou et al., 2000).

\(^{20}\)More formally, \( d\pi^L/dq^L = \pi^L_1 + R^L_2(dq^H/dq^L) > 0 \) at \( s^{L*} = 0 \).
fact that, by taking $q^H$ as given, firm $L$ sets $q^L$ too low due to its overestimate of the increase in price competition from an increase in $q^L$. Since $r = q^H / q^L$ falls, the subsidy actually makes the products more similar, causing quality-adjusted prices, $p^L$ and $p^H$, to fall, but firm $L$’s revenue, $R^L = p^L \times q^L$, nevertheless increases due to a higher volume of higher-quality exports (see Proposition 1(i)). Consequently, the LDC gains from a better positioning of its product in relation to consumer preferences as both firms move up the quality ladder.

These results are illustrated in Fig. 2. Starting from the Nash equilibrium at point $N$, the subsidy, $s^L$, shifts firm $L$’s quality reaction function to the right (shown as the dashed line), resulting in a new Nash equilibrium at point $S$. There is a net increase in LDC profit at the expense of firm $H$ and, as a result, the LDC moves to a higher iso-welfare contour (from $L_1$ to $L_2$) while the developed country, country $H$, moves to a lower iso-welfare contour (from $H_1$ to $H_2$). Since the contour $L_2$, based on firm $L$’s profits less the cost of the subsidy, is tangent to firm $H$’s reaction function at $S$, point $S$ also represents the outcome if firm $L$ were a Stackelberg leader in quality space in the absence of a subsidy.

3.3. Developed country policy towards the high-quality product

As set out in Proposition 2(i) below, an investment subsidy, $s^H$, set by country $H$ also causes the quality of both products to rise. However, in contrast to the effect of $s^L$, the quality ratio $r = q^H / q^L$ increases, making the products more differentiated. This difference arises because $s^H$ directly raises $q^H$ which raises $r$. 

![Fig. 2. The LDC’s optimal subsidy: Bertrand competition.](image-url)
and $s^L$ directly raises $q^L$, which lowers $r$. Although the rival firm also raises quality in each case (since $q^L$ and $q^H$ are strategic complements), the initial effect dominates. The differing effects of the two policies, $s^L$ and $s^H$, on the degree of product differentiation also translates into differing effects on profits. Whereas an increase in $s^L$ decreases the profit of firm $H$, the lessening of competition due to an increase in $s^H$ serves to boost the profits of the rival low-quality firm. Interestingly, it is not obvious that an increase in $s^H$ would raise the profit of firm $H$. The problem arises because the increase in $q^L$ from an increase in $q^H$ due to $s^H > 0$ reduces the increase in firm $H$’s revenue, potentially offsetting the direct effect of the subsidy in reducing firm $H$’s costs for any given $q^H$. However, since the convexity of the investment cost function limits the extent to which both $q^H$ and $q^L$ rise, we are able to use conditions (1) to demonstrate the result.

As for the choice of policy, letting $W^H$ denote welfare in country $H$, we show in Proposition 2(ii), that for any given value of $s^L$, country $H$ has an incentive to tax the investment of its firm, leading to $s^{H*} < 0$, where $s^{H*}$ denotes the optimal unilateral policy. The tax reduces $q^H$ and the profits of firm $H$, but since the fall in profits is less than the increase in tax revenue, domestic welfare is nevertheless increased.

**Proposition 2.** Assume Bertrand competition and $s^L$ held fixed.

(i) An increase in $s^H$, (a) raises both $q^H$ and $q^L$, but $r = q^H/q^L$ increases, causing $p^H$ and $p^L$ to rise, but $x^H$ and $x^L$ to fall; (b) raises revenue, $R^H$, and the profits, $\pi^H$ and $\pi^L$, of both firms.

(ii) Welfare in country $H$ is maximized by an investment tax, $s^{H*} = R^H_L \left( dq^L/dq^H \right)/F^{q^H} < 0$.

These results are illustrated in Fig. 3. Starting from the Nash equilibrium at point $N$, the tax, $s^{H*}$, shifts down the reaction function of firm $H$ (shown as the dashed line), resulting in a new Nash equilibrium at point $S$. As a result, the developed country moves to a higher iso-welfare contour (from $H1$ to $H3$) while the LDC moves to a lower iso-welfare contour (from $L1$ to $L3$). Since $H3$ is tangent to the reaction function of the LDC firm, point $S$ also represents the outcome if firm $H$ were a Stackelberg leader and $s^H = 0$.

Since firm $L$ lowers its quality as $q^H$ is reduced (firm $L$’s reaction function has a positive slope) and since firm $H$ gains revenue from a reduction in $q^L$ (i.e. $R^H_L < 0$), firm $H$, as a Stackelberg leader, would reduce $q^H$ below the level implied.

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3. $W^H(s^L, s^H) = \pi^H(q^L, q^H, s^H) - s^H F(q^H) = R^H(q^L, q^H) - F(q^H)$ for $q^L = q^L(s^L, s^H)$ and $q^H = q^H(s^L, s^H)$. To ensure that $s^{H*}$ exists, we assume local concavity of $W^H$ at $s^H = s^{H*}$. As shown in Zhou et al. (2000), letting $E(q) = qF^{q}(q)/F^{q}(q)$ represent the responsiveness (or elasticity) of $F^{q}(q)$ with respect to $q$, this holds if $E(q) > 0$ is constant. If $F(q) = aq^\alpha$, then $E(q) = n - 1$ and the result applies for $n \geq 2$. 


Fig. 3. Country $H$’s optimal tax: Bertrand competition.

by the Nash equilibrium. Consequently, given Nash behavior in quality space, country $H$ gains from an investment tax so as to reduce $q^H$. Fundamentally, the tax corrects for the fact that, by taking $q^L$ as given, firm $H$ sets $q^H$ too high due to its overestimate of the effect of an increase in $q^H$ in reducing price competition. Although the products become more similar, causing the revenues of both firms to fall (see Proposition 2(i)), the total rent (profit plus tax revenue) earned by country $H$ is increased due to the saving in investment costs as $q^H$ falls.

To explain why the policy is a tax in country $H$ and a subsidy in the LDC, recall that firm $H$ gains from a reduction in the quality, $q^L$, of its rival, whereas firm $L$ gains from an increase in $q^H$. Since in both cases, quality reaction functions are positively sloped, by reducing $q^H$, the tax in country $H$ serves to reduce $q^H$ and conversely, by raising $q^L$, the subsidy in the LDC serves to raise $q^H$. However, since the LDC would like to see an increase in $q^H$, but country $H$ taxes $q^H$, and since country $H$ would like to see a reduction in $q^L$, but the LDC subsidizes $q^L$, when both countries intervene, these unilateral incentives for policy tend to undermine the goal of raising profits from exports.

As the above argument suggests, aggregate or joint welfare of the two producing countries can be increased if LDC policy is switched to a tax on investment in quality and developed country policy is switched to a subsidy. These joint policies, denoted $s^{HJ}$, $s^{HL}$ ($J$ for joint) for the LDC and country $H$ respectively, differ from unilateral policy by taking into account the effects of each firm’s choice of quality

$^{22}$More formally, $d\pi''/dq'' = \pi''_H + K''_L(dq^L/dq'') < 0$ at $\pi''_H = 0$. 


on its rival’s profit. Thus the gain to firm $H$ from reduction in $q^L$ favors an LDC tax and the gain to firm $L$ from an increase in $q^H$ favors a subsidy in country $H$. In fact, as set out in Proposition 3, the policy in each country depends only on the cross effect of own quality on the other firm’s revenue and on the own cost of increasing quality. Since the joint policies increase the quality gap between the products so as to reduce price competition in the third-country market, the joint gain to the producing countries is achieved at the expense of consumers, who pay higher quality-adjusted prices and purchase lower quantities of both products.\(^3\)

**Proposition 3.** For Bertrand competition, the jointly optimal policies $(s^L, s^H)$ involve an investment tax in the LDC and an investment subsidy in country $H$: i.e. $s^L = R^H_\gamma F' (q^L) < 0$ and $s^H = R^L_\gamma F' (q^H) > 0$.

4. Investment policy and quality choice under Cournot competition

We now turn to the case of Cournot competition in which firms choose output levels at stage 2 after committing to quality levels at stage 1. The game played by firms is set out in 4.1 and the respective effects of LDC and developed country policies towards quality are explored in 4.2 and 4.3.

4.1. The two-stage model of firm behavior: Cournot competition

Since production costs are zero, at the second stage Cournot equilibrium, each firm $i$ sets output, $x^i$, to maximize its revenue $R^i = p^i x^i q^i$ for $i = L, H$, taking the output of the other firm and the qualities, $q^L$ and $q^H$, as given. Using a superscript $c$ to distinguish functions at the Cournot equilibrium, it is shown in (B.3) of Appendix B that equilibrium revenues, $R^c_L(q^L, q^H)$ and $R^c_H(q^L, q^H)$ for firm $L$ and $H$ respectively are both decreasing in the quality of the other firm: i.e.

$$R^c_L = -2r/(4r - 1)^3 < 0, \quad R^c_H = -4(r^2/(2r - 1)/(4r - 1)^3 < 0. \quad (6)$$

Since an increase in $q^H$ and a reduction in $q^L$ both increase $r$, (6) shows that the revenues of the two firms respond in opposite directions with respect to a greater separation of products (holding own quality fixed), with firm $L$’s revenue falling and firm $H$’s revenue rising. Thus, in contrast with the Bertrand case, firm $L$ now gains as the products become more similar, whereas firm $H$ gains from a greater separation of products as before.

These results can be understood by first examining the inverse demand functions, $p^L = 1 - (x^L + x^H)$ and $p^H = 1 - x^L/r - x^H$ (from (2)), which represent

\[^3\]The reduction in $q^L$ due to the LDC tax may seem to suggest a broadening in market sales, but since $p^L$ and $p^H$ rise and $x^L$ and $x^H$ fall (see (A.3)), there is clearly a move towards monopoly pricing.
consumer willingness to pay per unit of quality for the low and high quality goods respectively. As these functions show, holding \( x^L \) and \( x^H \) fixed, a greater separation of qualities (an increase in \( r \)) shifts up the demand curve for good \( H \), raising the willingness of consumers to pay for the high-quality good (i.e., \( \frac{\partial p^H}{\partial r} = x^L/(r^2) > 0 \)), but the willingness to pay for the low-quality good is unchanged (i.e., \( \frac{\partial p^L}{\partial r} = 0 \)). Due to this asymmetry in demand, the Cournot response to a widening of the quality gap is for firm \( H \) to expand output (taking firm \( L \)’s output as given) and for firm \( L \) to respond by contracting output (since \( x^L \) and \( x^H \) are strategic substitutes). As is typical in Cournot models, the net effect is an increase in aggregate output, as measured by \( x^L + x^H \), which tends to reduce prices, and in fact the quality-adjusted price of the low-quality good does fall. However, despite this pressure for lower prices, the quality-adjusted price of the high-quality product is increased: i.e. from (B.2), we obtain, \( dx^L/dr = dp^L/dr < 0 \) and \( dx^H/dr = dp^H/dr > 0 \). This increase in \( p^H \) can be explained due to the more than offsetting effect of the initial increase in consumer willingness to pay for high quality as the gap between qualities increases. Since quality-adjusted price and output both fall for firm \( L \) and both rise for firm \( H \), it follows immediately that firm \( L \)’s revenue must fall and firm \( H \)’s revenue must rise as the products become more differentiated. \(^2\)

Now considering the stage 1 choice of quality, firms \( L \) and \( H \) earn respective profits given by:

\[
\Pi_L(q^L, q^H) = R^L(q^L, q^H) - (1 - s^L)F(q^L),
\]

\[
\Pi_H(q^L, q^H) = R^H(q^L, q^H) - (1 - s^H)F(q^H).
\]  

(7)

Setting own quality to maximize own profit, taking the quality of the other firm as given, the Nash equilibrium qualities, \( q^L \) and \( q^H \), satisfy the first order conditions:

\[
\Pi_L = R^L_q - (1 - s^L) F'(q^L) = 0, \quad \Pi_H = R^H_q - (1 - s^H) F'(q^H) = 0.
\]  

(8)

As reflected in \( R^L_q > 0 \) (see (B.4)), the gain to firm \( L \) from a narrowing of the quality gap gives it an incentive to increase \( q^L \), which reduces \( r \), holding \( q^H \) fixed. For firm \( H \), analogously to Bertrand competition, a greater separation in qualities raises revenue leading it to also want to raise \( q^H \). However, for both firms, the profitability of an increase in quality is limited by the rising marginal cost of investment in quality.

The second order and stability conditions (see (B.5)), are assumed to hold locally at the Nash equilibrium in qualities. However, satisfaction of these

\[^2\] Similar reasoning shows the contrast with Bertrand competition. By raising consumer willingness to pay, an increase in \( r \) causes firm \( H \) to raise price (taking the price of firm \( L \) as given) and firm \( L \) responds by also raising price (\( p^L \) and \( p^H \) are strategic complements). Outputs fall, but the revenues of both firms rise.
conditions is made difficult by the fact that firm $L$’s marginal revenue with respect to own quality increases as the products become more similar, making $R_{LL}^L > 0$. This places restrictions on the form of $F(q)$, since the marginal cost of investment needs to increase sufficiently fast to make the choice of $q^L$ determinate. Also, despite the gain to firm $L$ from a greater similarity of products, it is not hard to show (see Appendix B) that in equilibrium, $q^H > q^L$ and hence $r > 1$.

As shown by the positive slope of firm $H$’s reaction function, denoted $q^H = \rho^{cH}(q^L)$, in Fig. 4, firm $H$ continues to view $q^L$ as a strategic complement to $q^H$ and, as under Bertrand competition, responds to an increase in $q^L$ by also increasing quality (i.e. $dq^H/dq^L = -R_{HL}^H/P_{HL}^H > 0$). But for firm $L$, since $R_{LL}^L$ is increased by a greater similarity of products, it follows (see (B.6)) that an increase in $q^H$ reduces firm $L$’s marginal revenue (i.e. $R_{LL}^L < 0$) leading firm $L$ to view $q^H$ as a strategic substitute to $q^L$. Consequently, as shown by the negative slope of $q^L = \rho^{cL}(q^H)$ in Fig. 4, firm $L$ responds to an increase in $q^H$ by reducing quality (i.e. $dq^L/dq^H = -R_{HL}^L/P_{HL}^L < 0$).

4.2. LDC investment policy towards the low-quality product

As set out in Proposition 4(i) for Cournot competition, an investment subsidy, $s^L$, applied to firm $L$ by the LDC raises the qualities of both products, but the

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25 Zhou et al. (2000) use analytical methods to explore the restrictions on $F(q)$ implied by the second order and stability conditions. These conditions hold locally for the commonly used cost function, $F(q) = q^2/2$ and also for $F(q) = q^n F''(q^*)/F'(q^*) \geq 2$, which includes $F(q) = aq^n$ for $a > 0$ and $n \geq 3$.

---

Fig. 4. Quality reaction functions: Cournot competition.
direct effect of \( s^L \) in raising \( q^L \) dominates the increase in \( q^H \), making the products more similar. Firm \( L \) enjoys higher profits, but the profits of firm \( H \) are reduced.\(^{26}\)

While these effects have the same signs as in the Bertrand case, there are, however, some critical differences behind the scenes, since, as shown in Proposition 4(ii), the LDC has a unilateral incentive to tax the investment of its firm under Cournot competition, whereas a subsidy raises LDC welfare in the Bertrand case. This tax policy may initially seem hard to understand, since the LDC tax lowers the profit of firm \( L \) and at the same time, since firm \( H \) benefits from a reduction in \( q^L \), raises the profit of firm \( H \) and hence welfare in the developed country. The proof of Proposition 4 and subsequent propositions concerning the Cournot case are set out in Appendix B. Letting \( W_{LL} = \Pi^L(q^L, q^H, s^L) - s^L\gamma F(q^L) \) denote LDC welfare, the optimal policy, \( s^{L*} \), is derived assuming that \( W_{LL} \) is locally concave.\(^{27}\)

**Proposition 4.** Assume Cournot competition and \( s^H \) held fixed.

(i) An increase in \( s^L \), (a) raises both \( q^L \) and \( q^H \), but \( r = q^H/q^L \) falls, causing a fall in \( x^H \) and \( p^H \), a rise in \( x^L \) and \( p^L \) and a fall in aggregate output, \( x^L + x^H \); (b) increases profit, \( \Pi^L \), in the LDC and reduces \( \Pi^H \).

(ii) LDC welfare is maximized by an investment tax, \( s^{L*} = R^L_{HH} (dq^H/dq^L)/\gamma F'(q^L) < 0 \).

To understand the reason for a tax, we again appeal to the correspondence of the model with a Stackelberg leader–follower model in which firm \( L \) is the leader and \( s^L = 0 \). First recall that firm \( H \) reduces \( q^H \) in response to a reduction in \( q^L \) (firm \( H \)’s quality reaction function has a positive slope) and for a given \( q^L \), a reduction in \( q^H \) reduces consumer willingness to pay for the high-quality good, causing firm \( H \) to cut output. The Cournot response is for firm \( L \) to expand output and, because of the overall fall in output, firm \( L \) also gains from a higher quality-adjusted price and hence a higher revenue as \( q^H \) falls (i.e. \( R^L_{HH} < 0 \)). As this suggests, firm \( L \), as a Stackelberg leader in quality, would reduce \( q^L \) below the Nash equilibrium level\(^{28}\).

Correspondingly, in a situation of Nash behavior in quality space, the LDC achieves the same choice of quality from its investment tax. Fundamentally the tax corrects for the fact that, taking \( q^H \) as given, firm \( L \) sets \( q^L \) too high due to its overestimate of the extent to which the quality gap narrows as \( q^L \) is increased.

\(^{26}\)It is hard to prove that \( d\Pi^L/ds^L > 0 \) (see (B.9)) because the increase in \( q^H \) due to \( s^L > 0 \) tends to reduce firm \( L \)’s revenue, which partly offsets the effect of the subsidy in reducing firm \( L \)’s costs for a given \( q^L \).

\(^{27}\)Zhou et al. (2000) show that \( dW_{LL}/(ds^L) < 0 \) at \( s^{L*} \) if \( E(q^L) = q^L F'(q^L)/F(q^L) \geq 2 \) and \( \sigma(q) = (F'(q^L))^2 - F'(q^L)F^2(q^L) \approx 0 \). We have \( \sigma(q) > 0 \) if \( F(q) = aq^n \) for \( n \geq 2 \) or if \( F(q) = q(e^{q}-1) \).

\(^{28}\)More formally, for Cournot competition, \( d\Pi^L/dq^L = \Pi^L_{L} + R^L_{HH} (dq^H/dq^L) < 0 \) at \( \Pi^L_{L} = 0 \).
As illustrated in Fig. 5, the LDC tax shifts the quality reaction function of firm \( L \) in towards the origin (shown by the dashed line) and both countries move to higher iso-welfare contours. The tax reduces firm \( L \)'s profit (see Proposition 4(i)(b)), but, taking into account the tax revenue, LDC welfare is nevertheless increased. Since \( r = q^H / q^L \) rises (see Proposition 4(i)(a)), the actual effect of the tax is to increase the difference between qualities, leading to a shift in consumer spending towards the higher-quality product (\( p^H \) and \( x^H \) rise and \( p^L \) and \( x^L \) fall). This benefits firm \( H \), but the revenue and profit of firm \( L \) fall. Consequently (similar to the gain to country \( H \) from its tax under Bertrand competition), the benefit of the tax to the LDC arises from the savings in the cost of investment in quality.

With respect to the question as to why a switch from Bertrand to Cournot competition causes LDC policy to switch from an investment subsidy to an investment tax, it is useful to first point out that, since firm \( H \) raises \( q^H \) in response to an increase in \( q^L \) under both forms of competition and since firm \( L \) takes \( q^H \) as given at the Nash quality equilibrium, both cases involve an overestimate by firm \( L \) as to the effect of an increase in \( q^H \) in making the products more similar. The difference arises because the greater separation of products due an increase in \( q^H \) raises firm \( L \)'s revenue in the Bertrand case (i.e. \( R^L_H > 0 \)) and reduces firm \( L \)'s revenue in the Cournot case (i.e. \( R^L_H < 0 \)). Consequently, firm \( L \) sets \( q^L \) too low in the Bertrand case and too high in the Cournot case for maximum profit. To correct for this, the LDC policy of a subsidy under Bertrand competition and a tax under Cournot competition moves firm \( L \) (and hence firm \( H \)) up the quality ladder under Bertrand competition and down the quality ladder under Cournot competition.
4.3. Developed country policy towards the high-quality product

As shown in Proposition 5(i), an investment subsidy, \( s^H \), applied to investment in quality by firm \( H \) raises \( q^H \), but in this Cournot setting (in contrast to the Bertrand case), firm \( L \)'s quality reaction function has a negative slope and \( q^L \) falls. As might be expected, firm \( H \)'s profits rise, but firm \( L \)'s profits are reduced due to the negative effects of greater product separation on the output and quality-adjusted price of the low quality good. Also, as shown in Proposition 5(ii), a shift from Bertrand to Cournot competition gives country \( H \) an incentive to subsidize rather than tax the investment of its firm. Similar to Proposition 4, the optimal subsidy, \( s^{cH*} \), is derived assuming that \( W^H \) is locally concave.\(^{29}\)

**Proposition 5.** Assume Cournot competition and \( s^L \) held fixed.

(i) An increase in \( s^H \), \( (a) \) raises \( q^H \) but reduces \( q^L \), causing \( r = q^H / q^L \), \( x^H \) and \( p^H \) to rise and \( x^L \) and \( p^L \) to fall; \( (b) \) increases profit \( \Pi^H \), but reduces \( \Pi^L \).

(ii) Welfare in country \( H \) is maximized by an investment subsidy, \( s^{cH*} = R^H_L (dq^L/dq^H)^F(q^H) / F'(q^H) > 0 \).

As illustrated in Fig. 6, the subsidy, \( s^{cH*} \), shifts up the quality reaction function

\(^{29}\)As Zhou et al. (2000) show, \( W^H \) is locally concave at \( s^{cH*} \) under the same conditions developed for Proposition 4, namely, \( E(q^H) = q^H F'(q^H) / F'(q^L) \geq 2 \) and \( \sigma(q) = (F'(q))^2 - F'(q) F''(q) \geq 0 \).
of firm $H$ (shown as the dashed line), moving the equilibrium from point $N^*$ to point $S^*$, which, as before, corresponds to the Stackelberg leader–follower point with firm $H$ as the leader and $s^{\text{SH}} = 0$. Since firm $H$ gains from a greater separation of products and $q^L$ falls as $q^H$ is increased, the Stackelberg outcome involves an increase in $q^H$ above the Nash equilibrium level, leading to the gain from an investment subsidy. The investment subsidy corrects for the fact that, at the Nash equilibrium in quality space, firm $H$ underestimates the extent to which products become more differentiated as $q^H$ is increased. The effect of the subsidy is to widen the quality gap, leading to an increase in the revenue of firm $H$, due to a shift in consumer spending towards the high-quality good ($p^H$, $x^H$ and $q^H$ all rise, but $p^L$ and $x^L$ fall from Proposition 5(i)).

Finally, as in the Bertrand case, the jointly optimal investment policy differs from unilateral policy by taking into account the cross effects of the quality chosen by each firm on its rival’s profit. Since firm $H$ gains from the widening of the quality gap due to a reduction in $q^L$ and firm $L$ gains from the narrowing of the quality gap due to a reduction in $q^H$, joint profit maximization involves a move by both firms down the quality ladder. Consequently, as shown in Proposition 6, the policy requires that each country tax investment. Relative to the Nash-policy equilibrium, the joint choice of policies increases the investment tax in the LDC and results in a switch from a subsidy to a tax in the developed country. These joint policies have an ambiguous effect on the size of the quality gap\textsuperscript{30}. Also, a change in the quality gap has mixed effects on prices ($p^H$ rises and $p^L$ falls as $r$ is increased). Thus, in contrast to the Bertrand case, there is no clear relationship between the size of the quality gap and the ability to raise prices at the expense of third country consumers. This suggests that the source of the joint producer gain from coordinated policy under Cournot competition is primarily due to the saving in investment costs as both firms move down the quality ladder.

**Proposition 6.** For Cournot competition, the jointly optimal policies $(s^{\text{SL}, j}, s^{\text{SH}, j})$ involve an investment tax in both countries with $s^{\text{SL}, j} = \frac{D}{q^L} \gamma F'(q^L) < 0$ and $s^{\text{SH}, j} = \frac{D}{q^H} F'(q^H) < 0$. The LDC sets a higher tax than at the Nash-policy equilibrium: i.e. $s^{\text{SL}, j} < s^{\text{SL}*} < 0$.

5. Conclusion

This paper develops the implications of strategic trade theory for policies targeted at the quality of exports. The analysis involves a three-stage game in which an LDC and a developed country attempt to reposition their firms in product

\textsuperscript{30}The effect of the joint policy on $r = q^H/q^L$ depends on $F''(q)$ and the efficiency gap, $\gamma$. 
quality space through taxes and subsidies on investment. The two firms (one in each country) first make an investment determining the quality of their product and then compete on the basis of either Bertrand or Cournot competition in a third country export market.

There are two basic considerations in determining the profitability of a particular location in quality space. First, for a given difference between own quality and the quality of the rival firm, there is the profitability of the location based on revenue and the investment costs required to reach that quality. Higher quality products tend to command higher revenues, but this tends to be offset by the fact that the cost of investment in quality is increasing at an increasing rate. The second consideration is the extent of the difference or gap between the quality of the two products, but the role played by this gap differs depending on the nature of product market competition. For Bertrand competition, a greater difference in qualities relaxes price competition, raising the profits of both firms, whereas, for Cournot competition, the profits of firm H increase as before, but firm H also raises output, leading to a reduction in the output, price and profits of firm L. Consequently, firm H gains from a greater difference in qualities under both forms of competition, whereas firm L gains in the Bertrand case but not the Cournot case. Related reasoning shows that firm H responds to an increase in $q^L$ by also raising $q^H$ under both forms of competition and that firm L raises $q^H$ in response to an increase in $q^L$ under Bertrand competition, but does the opposite under Cournot competition. These differences in incentives towards a greater separation of products and with respect to the direction of response to an increase in the rival’s quality are at the heart of the explanation for the opposing policy prescriptions arising under the two market structures.

For the LDC, unilateral policy involves a subsidy to investment in quality under Bertrand competition and a tax under Cournot competition. At the Nash equilibrium in qualities, each firm takes its rival’s quality as fixed, but since under both Bertrand and Cournot competition, firm H responds to an increase in firm L’s quality by also increasing quality, firm L overestimates the extent to which the quality gap will narrow as it raises its quality. In the Bertrand case, since firm L’s profits are increasing in the quality gap, this causes firm L to position its product too low on the quality ladder. By contrast, in the Cournot case, firm L is better off as the quality gap narrows and it sets its quality too high. Consequently, LDC policy involves an investment subsidy so as to move firm L (and hence firm H) up the quality ladder in the Bertrand case and an investment tax so as to move firm L (and firm H) down the quality ladder in the Cournot case. Since under Bertrand competition, the LDC subsidy hurts firm H, this fits with the typical strategic trade story in which the gain to one country is at least partly at the expense of the rival foreign firm. However, it is interesting that this conflict does not apply for Cournot competition, since the tax by the LDC actually raises the profits of firm H and hence welfare in the developed country.

For the developed country, unilateral policy is reversed, with a tax on
investment in quality under Bertrand competition and a subsidy to investment in quality under Cournot competition. Since firm H gains from a widening in the quality gap and hence from a reduction in \( q^L \) under both forms of competition, in each case the policy is aimed at reducing \( q^L \). Since firm L raises \( q^L \) in response to an increase in \( q^H \) under Bertrand competition but lowers \( q^L \) under Cournot competition, taking \( q^L \) as given, firm H sets too high a quality in the Bertrand case (explaining the investment tax) and too low a quality in the Cournot case (explaining the investment subsidy).

Producing countries may also coordinate their policies so as to maximize joint profits. For Bertrand competition, a coordinated strategy involves a widening of the quality gap between the LDC and the developed country as a means of reducing price competition in the third country market. Thus the LDC would tax its firm while the developed country would subsidize its firm. Under Cournot competition, since each firm gains from a move of its rival down the quality ladder (narrowing the quality gap for firm L and widening it for firm H), both governments tax quality. Consequently, a main source of the joint gain is a saving in the costs of investment by both firms. For both Bertrand and Cournot competition, a joint welfare maximizing strategy shifts the focus of government policy from attempting to enhance domestic welfare by modifying the behavior of the firm in the other country, to also modifying the behavior of its own firm so as to enhance the profit of the firm in the other country.

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Appendix A. Bertrand competition

A.1. Second-stage price competition

Expressing \( R^L = q^L p^L (\tilde{\theta} - p^L) \) and \( R^H = q^H p^H (1 - \tilde{\theta}) \) for \( \tilde{\theta} = (rp^H - p^L)/(r - 1) \) from (2), it follows, using \( \partial \tilde{\theta}/\partial p^L = -1/(r-1) \) and \( \partial \tilde{\theta}/\partial p^H = r/(r-1) \) for \( r = q^H/q^L \), that \( p^L \) and \( p^H \) satisfy the first order conditions:

\[
\begin{align*}
\partial R^L/\partial p^L &= q^H (p^H - 2p^L)/(r - 1) = 0, \\
\partial R^H/\partial p^H &= q^H [1 - (2rp^H - p^L)/(r - 1)] = 0. \\
\end{align*}
\]

Letting \( \Omega = (\partial^2 R^L/(\partial p^L)^2)(\partial^2 R^H/(\partial p^H)^2) - (\partial^2 R^L/(\partial p^L)(\partial p^H))^2 \), the second order and stability conditions are also satisfied:
\[ \frac{\partial^2 R^L}{(\partial p^L)^2} = -2q^H/(r-1) < 0, \quad \frac{\partial^2 R^H}{(\partial p^H)^2} = -2r q^H/(r-1) < 0, \相等\frac{\partial^2 \Omega}{\partial (p^L)^2} = (q^H)^2/(4r-1)/(r-1)^2 > 0. \相等 (A.2) \]

Solving (A.1), we obtain \( p^L = (r-1)/(4r-1), \quad p^H = 2p^L \) and \( \bar{\theta} = (2r-1)/(4r-1) \). Since \( x^L = r/(4r-1) \) and \( x^H = 2x^L \) (from (2)), it then follows that

\[ dp^H/dr = 2(dp^L/dr) = 6/(4r-1)^2 > 0, \]
\[ dx^H/dr = 2(dx^L/dr) = -2/(4r-1)^2 < 0. \相等 (A.3) \]

Since \( p^L x^L = p^H x^H / 4 \), letting \( \varphi(r) = p^L x^L = p^H x^H / 4 = r(r-1)/(4r-1)^2 \), revenues can be expressed as:

\[ R^L(q^L, q^H) = \varphi(r) q^L, R^H(q^L, q^H) = 4\varphi(r) q^H, \相等 (A.4) \]

where \( \varphi(r) = (2r+1)/(4r-1)^2 > 0 \) and \( \varphi^2(r) = -2(8r+7)/(4r-1)^2 < 0. \]

From (A.4), we have

\[ R^L_\ell = \varphi(r) - r\varphi'(r) = (r^2)/(4r-1)^3 > 0 \text{ for } r > 7/4, \]
\[ R^H_\ell = 4(\varphi(r) + r\varphi'(r)) = 4r(4r^2 - 3r + 2)/(4r-1)^3 > 0. \相等 (A.5) \]

A.2. First-stage quality competition

Firm \( i \) for \( i = L, H \) sets \( q^i \) to maximize profit \( \pi^i \) as in (4), leading to first order conditions (5). The second order conditions are satisfied, since, from (5), using \( R^L_\ell = (r^2)\varphi^2(r)/q^L < 0, \quad R^H_\ell = 4(2\varphi'(r) + r\varphi''(r))/q^L = -8(5r + 1)/q^L (4r-1)^2 < 0 \) (from (A.5)) and \( F''(q) > 0 \) from (1), we obtain:

\[ \pi^L_\ell = R^L_\ell - \gamma(1-s^L)F'(q^L) < 0, \]
\[ \pi^H_\ell = R^H_\ell - (1-s^H)F'(q^H) < 0. \相等 (A.6) \]

Since \( R^L_{1H} = -r\varphi'(r)/q^L = -R^L_\ell/r > 0 \) and \( R^H_{1H} = -R^H_\ell = -4r(2\varphi'(r) + r\varphi''(r))/q^L = 8r(5r + 1)/q^L (4r-1)^3 > 0 \), we also obtain \( R^L_{1H} R^H_{1H} - R^L_{1H} R^H_{1H} = 0 \) and hence, letting \( D = \pi^L_\ell \pi^H_{1H} - \pi^L_{1H} \pi^H_\ell \), we obtain

\[ D = -(1-s^H)F'(q^H)R^L_\ell - \gamma(1-s^L)F'(q^L)\pi^H_\ell > 0. \相等 (A.7) \]

Since the allocation of qualities to firms is determinate (the LDC firm produces the low-quality good), it follows from (A.6) and (A.7) that the equilibrium is unique and globally stable. The slopes of the reaction functions in quality space are given by:

\[ dq^H/dq^L = -R^H_{1H}/\pi^H_{1H} = r R^H_{1H}/\pi^H_{1H} > 0, \]
\[ dq^L/dq^H = -R^L_{1H}/\pi^L_\ell = R^L_{1H}/r \pi^L_\ell > 0. \相等 (A.8) \]
A.3. Proof of Proposition 1

(i) (a) Totally differentiating (5) and applying Cramer’s rule, it follows, using (6.6), (6.7) and (3) that \( dq^L/dx^L = -\gamma F'(q^L)\pi_{dH}/D > 0 \) and \( dq^H/dx^H = \gamma F'(q^H)R_{dH}/D > 0 \). From \( r = q^H/q^L \), \( dq^H/dq^L = R_{dH}/\pi_{dH} \) (see (A.8) and (A.6)), we also obtain \( dr/dq^L = [(dq^H/dq^L) - r]/q^L = r(1 - s^L)F'(q^L)/\pi_{dH}q^L < 0 \) and hence \( dr/dx^L = (dr/dq^L)(dq^L/dx^L) < 0 \). It then follows, using (A.3), that \( dp^L/dx^L < 0 \) and \( dx^L/ds^L > 0 \) for \( i = L, H \). (b) From (4) using (5) and signing expressions from (3) and part (i)(a), we obtain

\[
d\pi^L/dx^L = R_{\pi^L}^L(dp^L/dx^L) + \gamma F(q^L) > 0,
\]

\[
d\pi^H/dx^L = R_{\pi^H}^L(dp^H/dx^H) < 0. \tag{A.9}
\]

(ii) Setting \( s^L \) to maximize \( W^L = \pi^L - s^L\gamma F(q^L) \), it follows, using (A.9), that \( dW^L/dx^L = R_{dH}^L(dp^H/dx^L) - s^L\gamma F'(q^L)(dq^L/dx^L) = 0 \) and hence \( s^L = R_{dH}^L(dp^H/dq^L)/\gamma F'(q^L) > 0 \) from (3) and (A.8). To show \( d^2W^L/(dx^L)^2 < 0 \) at \( s^L \), from \( W^L = R_{\pi^L}^L(q^L, q^H) - \gamma F(q^L) = \pi^L(q^L, q^H, 0) \) and \( q^H = \rho^H(q^L; s^L) \), we obtain \( dW^L/dx^L = (d\pi^L/dx^L)(dq^L/dx^L) \) for \( d\pi^L/dx^L = dq^L(q^L, \rho^H(q^L), 0)/dq^L \) and hence \( d^2W^L/(dx^L)^2 = (d^2\pi^L/(dx^L)^2)(dq^L/dx^L)^2 + (d\pi^L/dx^L)(d^2q^L/dx^L)^2 \). Since \( dW^L/dx^L = d\pi^L/dx^L = 0 \) at \( s^L \), it remains to show that \( d^2W^L/(dx^L)^2 < 0 \) for \( \pi^L = \pi^L(q^L, q^H, 0) \) where \( q^H = \rho^H(q^L; s^L) \). This is demonstrated in Zhou et al. (2000). \( \square \)

A.4. Proof of Proposition 2

(i) (a) Totally differentiating (5), it follows, using (6.6), (6.7) and (3), that \( dq^H/ds^H = -F'(q^H)\pi_{dH}/D > 0 \) and \( dq^H/ds^H = F'(q^H)R_{LH}/D > 0 \). From \( r = q^H/q^L \) and (A.6), we also obtain \( dr/dq^H = (1 - r(dq^L/dq^H))/q^L = -(1 - s^L)\gamma F'(q^L)/\pi_{dH}q^L > 0 \) and hence \( dr/ds^H = (dr/dq^H)(dq^H/ds^H) > 0 \). It then follows, using (A.3), that \( dp^L/ds^H > 0 \) and \( dx^L/ds^H < 0 \) for \( i = L, H \). (b) Since \( dR^H/ds^H = R_{\pi^L}^H(dp^L/ds^L) + R_{\pi^H}^H(dp^H/ds^H) \), using (3), (A.5), (A.6), part(i)(a) and \( R_{LH}^* = -R_{LH}^L/r \), we obtain

\[
dR^H/ds^H = F'(q^H)[R_{\pi^H}^H\gamma(1 - s^L)F'(q^L) - 4R_{LH}^L\varphi(r)]/D > 0.
\]

From (4) and (5), using (3) and part (i)(a), we also obtain:

\[
d\pi^L/ds^H = R_{\pi^L}^H(dp^L/ds^H) > 0, \ d\pi^H/ds^H = R_{\pi^H}^H(dp^H/ds^H) + F(q^H). \tag{A.10}
\]

To prove \( d\pi^H/ds^H > 0 \), from (A.10), using \( dq^L/ds^H = -F'(q^H)R_{LH}^L/rD \) (from part (i)(a) and \( R_{LH}^* = -R_{LH}^L/r \)), we first obtain \( d\pi^H/ds^H = -[R_{\pi^L}^H R_{LH}^L F'(q^H) - rD F'(q^H)]/rD \). Letting \( Z = R_{\pi^L}^H F'(q^H) + r(1 - s^L)F'(q^L)F(q^H) \) and using (A.7), this implies \( d\pi^H/ds^H = -[R_{\pi^L}^H Z/rD + \gamma(1 - s^L)F'(q^L)F(q^H)\pi_{dH}/D] \). Since \( R_{LH}^L < 0, R_{\pi^H}^H < 0 \) and \( F(q^H) > 0 \), it remains to show that \( Z > 0 \). Using \( R_{LH}^L = 4(\varphi(r) + r \varphi'(r)) = 4R_{LH}^L + 8r \varphi'(r) \) from (A.5) and \( R_{LH}^* = -4r \varphi'(r) \) from (3), we
first obtain $R^H = 2Rx^L - rR^H/2$. Imposing the first order conditions (5), this implies $R^H = 2\gamma(1 - s^R)F'(q^R) - r(1 - s^H)F'(q^H)/2$ and hence, letting $T(q) = 2F'(q)F(q) - (F'(q))^2$, we obtain $Z = r(1 - s^H)T(q^H)/2 + 2\gamma(1 - s^R)F'(q^R)F'(q^H)$. Since $F''(q) > 0$ from (1), we obtain $T'(q) = 2F(q)F''(q) > 0$ and since $T(0) = 0$, we have $T(q) \geq 0$ and $Z > 0$.

(ii) From $W^H = \pi^H - s^H F(q^H)$ and (A.10), we obtain $dW^H/ds^H = [R^H(dq^L/ds^L) - s^H F'(q^H)](dq^H/ds^H)$ and hence $s^H = R^H(dq^L/dq^H)/F'(q^H) < 0$ from (3) and (A.8). We assume $d^2W^H/(ds^H)^2 < 0$ at $s^H = s^{H*}$, which, as shown by Zhou et al. (2000) (see their Lemma 2), holds if $E(q)/qF''(q) = F'(q)$ is constant. The result applies if $F(q) = aq^n$ for $a > 0$ and $n \geq 2$. □

A.5. Proof of Proposition 3

Let $J = J(s^L, s^H) = W^L(s^L, s^H) + W^H(s^L, s^H)$ represent joint welfare, where $W^L(s^L, s^H) = \pi^L(q^L, q^H, s^L) - s^L \gamma F(q^L)$ for $\gamma = \gamma^L = 1$ and $i = L, H$. Using (4) and $\pi^L = 0$ from (5), it then follows that $dW^L/ds^L = R^L(dq^L/ds^L) - s^L \gamma F'(q^L)(dq^L/ds^L)$ and $dW^H/ds^H = R^H(dq^L/ds^H) - s^H \gamma F'(q^H)(dq^L/ds^H)$ for $i \neq j$. Hence $s^L$ and $s^H$ satisfy the first order conditions:

\[
\begin{align*}
dJ/ds^L &= (R^H - s^L \gamma F'(q^L))(dq^L/ds^L) + (R^L - s^H \gamma F'(q^L))(dq^L/ds^L) = 0, \\
dJ/ds^H &= (R^H - s^L \gamma F'(q^L))(dq^L/ds^H) + (R^L - s^H \gamma F'(q^L))(dq^L/ds^H) = 0.
\end{align*}
\]

Using $R^H < 0$ and $R^L > 0$ from (3), we then obtain $s^{LH} = R^L/dF'(q^L) < 0$ and $s^{HL} = R^H/dF'(q^H) > 0$. □

Appendix B. Cournot competition

B.1. Second-stage quantity competition

Since each firm $i$ sets $x^i$ to maximize its revenue $R^i = p^i x^i q^i$ for $i = L, H$ taking the output of the other firm as given, it follows, using $p^i = 1 - (x^L + x^H)$ and $p^H = 1 - x^L/r - x^H$ (from inverting (2)), that $x^L$ and $x^H$ satisfy the first order conditions

\[
\begin{align*}
\partial R^L/\partial x^L &= [1 - (2x^L + x^H)]q^L = 0, \\
\partial R^H/\partial x^H &= [1 - (x^L/r + 2x^H)]q^H = 0.
\end{align*}
\]

(B.1)

The second order and stability conditions (analogous to (A.2)) are also satisfied since $\partial^2 R^L/\partial x^L = -2q^L < 0$, $\partial^2 R^H/\partial x^H = -2q^H < 0$ and $\Omega = q^L(4q^H - q^H) > 0$. From (B.1), we obtain $x^L = p^L = r/(4r - 1)$, $x^H = p^H = (2r - 1)/(4r - 1)$ and $\theta = (r p^H - p^L)/(r - 1) = 2r/(4r - 1)$ from which it follows that
\[ \frac{dx^L}{dr} = \frac{dp^L}{dr} = -\frac{1}{(4r-1)^2} < 0, \]
\[ \frac{dx^H}{dr} = \frac{dp^H}{dr} = \frac{2}{(4r-1)^2} > 0. \] (B.2)

Letting \( \omega(r) = (r)^2/(4r-1)^2 \) and \( \psi(r) = (2r-1)^2/(4r-1)^2 \), revenues can be expressed as \( R^L(q^L, \cdot, q^H) = \omega(r)q^L \) and \( R^H(q^L, \cdot, q^H) = \psi(r)q^H \). Using \( \omega'(r) = -2r/(4r-1)^3 < 0 \) and \( \psi'(r) = 4(2r-1)/(4r-1)^3 > 0 \), it follows that an increase in the rival’s quality always reduces own revenue: i.e.

\[ R^L_{q^L} = \omega'(r) = -2r/(4r-1)^3 < 0, \]
\[ R^H_{q^H} = - (r)^2\psi'(r) = -4(2r-1)/(4r-1)^3 < 0. \] (B.3)

Also, an increase in own quality always raises own revenue: i.e.

\[ R^L_{q^L} = \omega(r) - r\omega'(r) = (4r+1)(r)^2/(4r-1)^3 > 0, \]
\[ R^H_{q^H} = \psi(r) + r\psi'(r) = 16(r)^3 - 12(r)^2 + 4r-1)/(4r-1)^3 > 0. \] (B.4)

**B.2. First-stage quality competition**

Firm \( i \) for \( i = L, H \) sets \( q^i \) to maximize profit, \( \Pi^i \), as in (7), leading to first order conditions (8). To show \( q^L < q^H \), it follows from (B.3) that \( R^L_{q^H} = 4R^L_{q^L} + 1/(4r-1)^2 > R^L_{q^L} \) for all \( q^H \equiv q^L > 0 \). Setting \( q^H = q^L \) in \( \Pi^H \) and recalling that \( \gamma \) is large, we obtain \( \Pi^H = R^H - (1-s^H)F'(q^H) > R^L - \gamma(1-s^L)F'(q^L) \) and hence \( \Pi^H > \Pi^L \). This contradicts \( \Pi^L = \Pi^H = 0 \) and also shows that if \( q^H = q^L \) satisfies \( \Pi^L = 0 \), then firm \( H \) has an incentive to increase \( q^H \) above \( q^L \).

We require that the following second order and uniqueness conditions hold locally at the Nash equilibrium values of \( q^L \) and \( q^H \) satisfying (8):

\[ \Pi^L_{q^L q^L} < 0, \Pi^H_{q^H q^H} < 0, D^L = \Pi^L_{q^L q^L} + \Pi^H_{q^H q^H} > 0, \] (B.5)

where

\[ \Pi^L_{q^L q^L} = R^L_{q^L q^L} - \gamma(1-s^L)F''(q^L) < 0 \quad \text{and} \quad \Pi^H_{q^H q^H} = R^H_{q^H q^H} - (1-s^H)F''(q^H). \]

From differentiation of (B.4), the cross derivatives of profit and revenue satisfy

\[ \Pi^L_{q^L q^H} = R^L_{q^L q^H} = -R^L_{q^L} > 0, \Pi^H_{q^H q^L} = R^H_{q^H q^L} = -R^H_{q^H} > 0, \] (B.6)

which implies \( R^L_{q^L} R^H_{q^H} - R^H_{q^H} R^L_{q^L} = 0 \) and hence, using \( \Pi^H_{q^H} = R^H_{q^H} - (1-s^H)F''(q^H) \) that

\[ D^L = - (1-s^H)F''(q^H) \Pi^L_{q^L q^L} - \gamma(1-s^L)F''(q^L) R^H_{q^H q^H}. \] (B.7)

For firm \( H \), we have \( R^H_{q^H} = -8(r-1)/(4r-1)^3 q^L < 0 \) for \( r > 1 \) and since \( F''(q) > 0 \), it follows from (B.7) that \( D^L > 0 \) and hence (B.5) is satisfied if \( \Pi^L_{q^L q^L} < 0 \). However, since \( \omega''(r) = 2(8r+1)/(4r-1)^4 > 0 \) and hence (from (B.3)), \( R^L_{q^L q^L} = (r)^2\omega''(r)/q^L > 0 \), we require that \( F''(q) > 0 \) be sufficiently large to ensure \( \Pi^L_{q^L q^L} < 0 \). As shown by Zhou et al. (2000) (see their Lemma 4), \( \Pi^L_{q^L q^L} < 0 \) holds locally if \( F(q) = q^2/2 \) or if \( E(q^2) = q^2F''(q^2)/F'(q^2) > 2 \), which is satisfied for \( F(q) = aq^n \) for \( n \geq 3 \).
From (8), (B.5) and (B.6), firm \( L \)'s reaction function, \( q^L = \rho^L(q^H) \), is negatively sloped in the neighborhood of equilibrium, whereas \( q^H = \rho^H(q^L) \) has a positive slope; i.e.

\[
\frac{dq^L}{dq^H} = - R_{LH}^* / II_{LH}^* < 0, \quad \frac{dq^H}{dq^L} = - R_{HL}^* / II_{HL}^* > 0. \tag{B.8}
\]

### B.3. Proof of Proposition 4

(i) (a) Totally differentiating (8), we obtain \( dq^L / ds^L = - \gamma F'(q^L) II_{HH}^* / D^L > 0 \) and \( dq^H / ds^L = \gamma F'(q^L) R_{HL}^* D^L > 0 \) from (B.3) and (B.5). Analogous to the proof of Proposition 1 (i)(a), we then obtain \( dr / dq^L = r(1 - s^L) F''(q^L) II_{HH}^* q^L < 0 \) and hence \( dr / ds^L = (dr / dq^L)(dq^L / ds^L) < 0 \). Using (B.2), it also follows that \( dx^L / ds^L = (dx^L / dr)(dr / ds^L) = dp^L / ds^L > 0 \), \( dx^H / ds^L = dp^H / ds^L < 0 \) and \( d(x^L + x^H) / ds^L < 0 \). (b) From (7), first order conditions (8), (B.3) and part (i)(a), we obtain:

\[
\frac{dII^H}{ds^L} = R_{HL}^* (dq^L / ds^L)^2 < 0, \quad \frac{dII^L}{ds^L} = R_{HHL}(dq^L / ds^L) + \gamma F(q^L). \tag{B.9}
\]

To prove \( dII^L / ds^L > 0 \), from (B.9), using \( dq^H / ds^L = - \gamma r R_{HHL} F'(q^L) / D^L \) (from part (i)(a) and \( R_{HHL}^* = - r R_{HHL}^* \)), we first obtain \( dII^L / ds^L = \gamma [\frac{- R_{HHL}^* R_{HHL}^* F'(q^L)}{D^L} + 2 \gamma F(q^L)] / D^L \). Letting \( Z^* = r R_{HL}^* F'(q^L) + \gamma (1 - s^L) F'(q^L) F(q^L) \), it then follows, using \( D^L \) as in (B.7) that \( dII^L / ds^L = - \gamma [R_{HHL}^* Z^* + (1 - s^L) F'(q^L) F(q^L) II_{HL}^* / D^L \). From (B.5), \( R_{HHL}^* < 0 \) and \( F'(q^L) > 0 \), it remains to show that \( Z^* \geq 0 \). Since \( R_{HHL}^* = 2 F'(q^L) F^2(q^L) / 2 + (r - 1)(F'(q^L))^2 / 2r \) + \( r F'(q^L) / 2(4r - 1)^3 \) for \( T(q) = 2 F'(q) F^2(q) / 2 + (r - 1)(F'(q)^2) / 2r \) + \( r F'(q^L) / 2(4r - 1)^3 \) for \( T(q) = 2 F'(q) F^2(q) / 2 + (r - 1)(F'(q)^2) / 2r \) + \( r F'(q^L) / 2(4r - 1)^3 \), this implies \( T(q) \geq 0 \) and hence \( Z^* > 0 \), proving the result.

(ii) From \( W^{L} = II^L + s^L \gamma F(q^L) \) and (B.9), it follows that \( dW^{L} / ds^L = [R_{HHL}(dq^L / ds^L) + s^L \gamma F'(q^L)][(dq^L / ds^L)^2 \) and hence \( s^L R_{HHL}^* (dq^L / ds^L) / F'(q^L)^2 < 0 \) from (B.3) and (B.8). We assume \( dW^{L} / (ds^L)^2 < 0 \) at \( s^L = * \), which, from Zhou et al. (2000), holds for \( E(q^L) = q^L F'(q^L) / F(q^L)^2 \geq 2 \) and \( \sigma(q) = (F'(q)^2 - F'(q))^2 / F(q)^2 F(q)^2 \geq 0 \). These both apply for \( F(q) = aq^L \) for \( n \geq 2 \).

### B.4. Proof of Proposition 5

(i) (a) From (8), (B.3) and (B.5), we obtain \( dq^H / ds^L = - F'(q^L) II_{HL}^* / D^L > 0 \) and \( dq^L / ds^H = F'(q^L) R_{HL}^* / D^L < 0 \). Since \( dr / dq^L = - (1 - s^L) \gamma F''(q^L) q^L II_{HL}^* < 0 \), we also have \( dr / ds^L = (dr / dq^L)(dq^L / ds^L) > 0 \). Using (B.2), we obtain \( dx^L / ds^H = (dx^L / dr)(dr / ds^H) = dp^L / ds^H < 0 \) and \( dx^H / ds^L = dp^H / ds^L > 0 \). (b) From (7) using (8), (B.3) and part (a), we obtain \( dII^L / ds^H = R_{LI}^* (dq^L / ds^H) + F(q^L)^2 > 0 \) and \( dII^L / ds^L = R_{HL}^* (dq^L / ds^L) + F(q^L)^2 > 0 \). (ii) Similar to the proof of Proposition 2, we obtain \( s^L R_{HHL}(dq^L / ds^H) / F'(q^L)^2 > 0 \). The same conditions (\( E(q^L) \geq 2 \) and \( \sigma(q) \geq 0 \)) are sufficient for local concavity of \( W^{L} \) as for \( W^{L} \) (see proof of Proposition 4).
B.5. Proof of Proposition 6

Joint welfare is given by $J = W^L + W^H$ where $W^x = W^x(s^L, s^H) = \Pi^x(q^L, q^H, s^L, s^H) - s^L \gamma F(q^L) - s^H \gamma F(q^H)$ for $\gamma^L = \gamma = \gamma^H = 1$ and $i = L, H$. Using $\Pi^i = 0$ from (8) and $R_i^i < 0$ for $i \neq j$ from (B.3), we obtain $dW^x/ds^j = R^x_i dq^j / ds^i < 0$ for $i \neq j$. At the policies $(s^L*, s^H*)$, since $dW^L/ds^j = dW^H/ds^j = 0$, it follows that $dJ/ds^j = dW^L/ds^j < 0$ and $dJ/ds^H = dW^L/ds^H < 0$ and hence $s^L < s^L*$ and $s^H < s^H*$. Similar to Proposition 3 we also obtain $s^{L/L} = R^L_{s^L} / \gamma F(q^L) < 0$ and $s^{H/H} = R^H_{s^H} / F(q^H) < 0$. □

References

and asymmetric costs, NBER working paper 7536.