Patent Assertion Entities and the Courts: Injunctive or Fee-Based Relief?

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Abstract:

A 2006 U.S. Supreme Court decision shifted the legal regime for patent litigation, encouraging district courts to rely more on license fees and less on injunctions as a remedy for patent infringement. This paper provides a simple model of a patent-owing "patent assertion entity" (PAE) and an infringing firm. These two parties bargain over a possible license fee after the PAE initiates an infringement lawsuit. Using the Nash bargaining solution, we compare a fee-based regime in which the court imposes a "fair value" license fee if there is no settlement with a regime in which failure to reach a settlement leads to an injunction that disrupts production. The injunctive regime always involves a settlement but, in a fee-based regime, settlements occur only for patents on drastic innovations (as defined by Arrow, 1962). For small incremental innovations, license fees are higher in an injunctive regime, as expected, and PAEs would prefer the injunctive regime. However, for higher value innovations, license fees are lower in the injunctive regime and PAEs would prefer the fee-based regime, contrary to the presumption that injunctive regimes necessarily favor PAEs.

Keywords: injunction, license fee, patent litigation, patent assertion entity, royalty

JEL classification codes: L40, O38, K20

Introduction:

Prior to mid-2006, an important aspect of U.S. patent infringement cases was the possibility of a court-imposed injunction forcing the infringing firm to stop using the patented technology. The threat of such "injunctive relief" would sometimes induce firms accused of infringement to agree to seemingly excessive settlements rather than risk a costly disruption of business arising from an injunction. For example, in early 2006, smartphone producer Research in Motion (RIM) settled a questionable patent infringement case filed by NTP for a license fee of $612.5 million rather than face a possible injunction that would suspend its U.S. sales.

The U.S. legal landscape changed in 2006, when the U.S. Supreme Court reached a rare unanimous decision in the patent infringement case *eBay v. MercExchange*. This important decision dramatically reduced the use of injunctive relief, as it affirmed that U.S. district courts should use injunctive relief only when standard remedies such as license fees or other monetary awards are not adequate. Concern about patent assertion entities[1] (PAEs), many of whom are pejoratively referred to as "patent trolls", was an important consideration for at least some of the justices in the eBay decision.[2]

But does replacing injunctive relief with fee-based relief necessarily restrain aggressive bargaining by PAEs? In this paper, we focus on the comparative economic properties of injunctive and fee-based legal regimes using a simple model that we believe captures important economic principles at work in patent infringement cases. Our primary research question is

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[1] PAEs are defined by the U.S. Federal Trade Commission (FTC, 2016) as "businesses that acquire patents from third parties and seek to generate revenue by asserting them against alleged infringers." The overlapping and similar term "non-practicing entity" (NPE) refers to patent holders who do not "practice" the patent.

[2] See *eBay Inc. v. MercExchange*, 547 U.S. 388 (2006), concurring opinion.

whether a PAE would, as intuition suggests, generally prefer a legal regime that relies on injunctions, or whether a PAE would sometimes prefer a fee-based legal regime.

We also consider the comparative effects of the two regimes more broadly, including the effects on infringing firms and on consumers. Our analysis incorporates Nash bargaining between a PAE and an infringing firm over a license fee. An important feature of the model is that incentives in Nash bargaining differ depending on whether the case is filed in an injunctive regime or a fee-based regime, implying different outcomes in the two regimes. We focus on the case in which the patented innovation is a cost-reducing or process innovation.

Using the distinction between “incremental” and “drastic” innovations first introduced by Arrow (1962), we show that the size of the patented innovation is important in determining which regime would be preferred by a PAE and in determining other economic effects of the infringement claim. If the cost-reducing value of the innovation is at the low end of the incremental category, PAEs would prefer a regime of injunctive relief, consistent with standard intuition. For innovations at the high end of the incremental category and for drastic innovations, we obtain the striking result that PAEs would prefer a fee-based regime.

The central insight is that a PAE has a lot to lose from an injunction as it earns no revenue in that case, and its potential loss from foregone license fee revenue is higher if the patented innovation is larger (more valuable). With sufficiently large innovations, the PAE has more to lose from an injunction than the infringing firm, and this weakens its bargaining power in the injunctive regime.

Section 2 of the paper provides a brief literature review along with relevant institutional background. Section 3 introduces our model, discusses our key assumptions, and shows how negotiated or court-imposed license fees affect output. Section 4 presents the analysis of Nash bargaining over the license fee for each legal regime and Section 5 undertakes an economic comparison of the two regimes. Section 6 examines an extension to our basic model in which the infringing firm has the option of working around the patented innovation by paying a fixed restructuring cost. Section 7 discusses extensions to consider uncertainty, litigation costs, fixed license fees, alternative fair value license fees, and asymmetric bargaining power. Section 8 provides concluding remarks and proofs of propositions are in the Appendix.

2. Institutional Background and Literature Review

We motivate our comparison of fee-based and injunctive legal regimes with reference to the U.S. Supreme Court decision of 2006 favoring the use of license fees. However, neither the pre-2006 nor the post-2006 period illustrates a pure form of either regime. Prior to 2006 U.S. courts could and often did impose license fees and after 2006 it was still possible for courts to use injunctions. Still, the 2006 decision makes this comparison of particular interest as it increased the relative importance of the fee-based approach. In addition, the comparison between injunctive and fee-based approaches is also relevant in other important contexts such as environmental policy.

There is a large literature on patent assertion entities. In principle, PAEs could play a valuable intermediation role in channeling resources to inventors by purchasing patents and licensing those patents to users, as in Hagiu and Yoffie (2013), or PAEs may develop specialized expertise in patent enforcement, as suggested by Haus and Juranek (2018). Also, Turner (2018) presents a model in which it may be efficient for some firms to specialize in discovery, earning revenue mainly from license fees. Such firms would therefore act as PAEs.

However, many PAEs are alleged to do little more than accumulate minor patents and initiate predatory lawsuits. The Federal Trade Commission (2016) provides a systematic assessment of PAEs that distinguishes between “portfolio PAEs” and “litigation PAEs”. We see this distinction as essentially between “legitimate PAEs” and “trolls”. The trolls account for about 96% of PAE infringement lawsuits.

Scott Morton and Shapiro (2016) and Cohen, Gurun, and Kominers (2016) provide (largely critical) overviews of patent troll activity. Lemley and Feldman (2016) argue that any intermediation benefits of PAEs are small compared to the extent of their lawsuit generation. Bessen, Ford, and Meurer (2011) provide a widely cited estimate of the cost of U.S. patent troll activity. An interesting formal model of patent trolls is given by Choi and Gerlach (2018).

Our analysis is related to the literature on patent policy and patent licensing. Gallini (2002) provides an overview of U.S. patent reforms and relevant theory underlying patent policy. Kamien (1992) reviews much of the early research on patent licensing. The use of the Nash bargaining solution to analyze patent licensing has been undertaken by several authors, including Shapiro (2010), who uses Nash bargaining over royalties to assess the effect of potential “hold-up” by patent owners. See also Kishimoto and Muto (2012) and Kishimoto (2020).

There is also an extensive literature on litigation and settlement of legal disputes more broadly, starting with the classic work of Landes (1971). Valuable overviews of this literature are provided by Spier (2007) and Daughety and Reinganum (2012). See Jeitschko and Kim (2012) for an analysis of preliminary injunctions in legal disputes. Crampes and Langinier (2002) provide a classic analysis of patent litigation using the Nash bargaining solution.

3. Model Preliminaries

The timeline of the model is as follows. At some point in the past, a producing firm incorporated a patented cost-reducing technology in its production process without obtaining a license. The patent is owned by a patent assertion entity (PAE) that produces no output itself and files a patent infringement claim against the infringing firm. A two-stage game follows filing of this claim. In the first stage, the PAE and the infringing firm engage in Nash bargaining over a license fee. If the parties reach agreement, they commit to the negotiated license fee. If they do not agree, the outcome is determined in court. In the second stage, the firm chooses its output to maximize profit conditional on the outcome of the first stage and pays any license fees owed to the PAE.

The legal regime may be either injunctive or fee-based. In the injunctive regime, the court imposes an injunction that prevents production by the infringing firm and neither the infringing firm nor the PAE earns any revenue. In the fee-based regime, we assume the court imposes a "fair value" per-unit license fee or royalty equal to the cost-reducing value of the patented innovation, after which the firm produces its implied profit-maximizing output. Such a license fee is consistent with the structure used by, among others, Anton and Yao (2006), who argue (p. 200) that it reflects both U.S. law and the empirical record. In Section 7, we consider alternative interpretations of fair value royalties.

One possible criticism of our use of per-unit royalties in the fee-based regime is that, in our simple model, (non-distortionary) fixed fees are more efficient than (distortionary) royalties. A welfare-maximizing court with full information could use a fixed fee, as in Hylton and Zhang (2017). However, the information requirements for the choice of an efficient fixed-license fee are higher than for a fair-value license fee as, in addition knowing the cost-reducing value of the innovation, knowledge of the level of demand is also required.

Nash bargaining takes place over a per-unit royalty in our base model, but we show in Section 7 that our primary results hold even if firms bargain over a fixed license fee. Our use of a royalty in bargaining reflects the assessment of Kamien (1992, p. 345) and many others that some form of royalty is the norm, although pure fixed fees are sometimes used. Kamien (1992) attributes the dominance of the royalty form to uncertainty and risk aversion. As discussed in Section 7, it is possible to incorporate uncertainty and risk aversion in our model.

Our assumption that an injunction forces the infringing firm to stop production reflects the idea that it is often prohibitively costly and time-consuming for a firm to change its production processes to work around an infringed patent. In an extension of the base model (Section 6), we allow the infringing firm to opt out of using the patented innovation by incurring some non-prohibitive restructuring cost so that it can continue production.

Our model is forward-looking in that we do not explicitly consider compensation for past sales. This would be strictly correct if the producing firm has set up its production facility and processes incorporating the patented innovation, but has not yet produced and sold any output. An alternative and more realistic assumption is that any compensation for past sales is negotiated or litigated as a distinct and additive part of the case and does not affect future license fee payments. Either way, as is common in the literature, we abstract from past production.

As in much of the literature, we assume that the firm faces a downward-sloping (inverse) linear demand function, $p = a - q$, where p is price and q is quantity. Without loss of generality, units are normalized so the slope is -1. The cost-reducing innovation reduces marginal cost, originally equal to c, by the amount v. Therefore, if the firm uses the innovation and pays the royalty, denoted r, its profit is

$$\pi = [p - (c - v)]q - rq \qquad (1)$$

We make the following additional five assumptions, most of which are familiar regularity conditions.

Assumptions:

A1. Marginal cost cannot be negative: $c - v \geq 0$.
A2. There is a market for the product: $a > c$.
A3. The patented innovation has value: $v > 0$.
A4. Value v is common knowledge.
A5. There are no litigation costs.

Assumptions A1 and A2 are standard feasibility requirements. A3 allows us to focus on cases in which the patented innovation has value. The alternative ($v = 0$) is easily analyzed but is of little interest and is therefore omitted. Assumption A4 abstracts from uncertainty over the value of innovation. It implies that the court is assumed to know the cost-reducing value of the innovation. A5 abstracts from litigation costs, which are empirically important but are not central to our focus here. Including uncertainty and litigation costs would not offset or undo the insights we identify, although it would add complications. We discuss the effects of relaxing A4 and A5 in Section 7.

The significance or size of the patented innovation is important. Following Arrow (1962) we define incremental, intermediate, and drastic innovations as follows.

Definitions:

D1. If $v < a - c$, the innovation is *incremental*.
D2. If $v = a - c$, the innovation is *intermediate.*
D3. If $v > a - c$, the innovation is *drastic*.

We follow the usual backward induction process to analyze the model, starting with the stage 2 output decision. In the absence of an injunction, the infringing firm maximizes profit as given by (1), conditional on the royalty r determined in stage 1. Letting r^P denote the prohibitive royalty at which output is zero, the profit-maximizing output for $r \leq r^P$ is

$$q(r) = (a - c + v - r)/2 \qquad (2)$$

As a royalty cannot be negative and increases in r above $r = r^{P}$ have no effect, without loss of generality, we restrict attention to $0 \leq r \leq r^{P}$. Setting $q(r) = 0$ in (2), the prohibitive royalty is

$$r^{P} = a - c + v \tag{3}$$

Since $a - c > 0$ (assumption A2), it follows from (3) that $v < r^{P}$. A royalty of amount v is not prohibitive so output $q(v)$ is strictly positive.

For linear demand and constant marginal cost, the firm's profit is the square of its output.

$$\pi(r) = q(r)^2 \tag{4}$$

From (2) and (4), the firm's output and profit are decreasing in the royalty for $r < r^{P}$. The profit of the PAE is simply its royalty revenue, denoted

$$L(r) = rq(r) \tag{5}$$

4. Nash Bargaining and Royalty Determination

The Nash bargaining solution can be found by maximizing the product of the net payoffs of the two parties relative to the "disagreement payoffs" they obtain if they fail to reach agreement and the payoffs are determined in court. We denote court-determined disagreement payoffs for the firm and the PAE as π^{o} and L^{o} respectively, yielding the following Nash product, denoted B (for "bargain").

$$B(r) \equiv (\pi(r) - \pi^{o})(L(r) - L^{o}) \tag{6}$$

The Nash product is maximized over the set $r \in [0, r^{P}]$. A "trivial solution" in which $B(r) = 0$ arises if no value of r is better for both parties than the outcome of adjudication. We treat this trivial solution as failure to reach a settlement. A settlement requires $B(r) > 0$ and a therefore a positive surplus for both players. The disagreement payoffs, π^{o} and L^{o}, depend on the legal regime, either injunctive or fee-based. As we will show, these payoffs are crucial for the bargaining outcome.

4.1 Preferences of the Firm and PAE

Before considering outcomes in court, it is useful to consider profit-maximizing royalty rates for both the infringing firm and the PAE. It follows immediately from (2) and (4) that the infringing firm always prefers a lower royalty and would, ideally, prefer no royalty at all. The properties of the PAE's preferred or optimal royalty, denoted r^*, are set out in Proposition 1, including a simple but important relationship between r^* and the cost-reducing value of the innovation. The proof of Proposition 1 and subsequent proofs that are not provided in the text are available in the Appendix.

Proposition 1: *The PAE-optimal royalty*

(i) The PAE's optimal royalty is always less than the prohibitive level and is, specifically, half the prohibitive royalty.

$$r^* = (a - c + v)/2 < r^P = a - c + v \tag{7}$$

(ii) The PAE's optimal royalty

a. exceeds the innovation's value, v, if the innovation is incremental ($v < a - c$);
b. equals v if the innovation is intermediate ($v = a - c$); and
c. is less than v if the innovation is drastic ($v > a - c$).

The PAE's optimal royalty is below the prohibitive level because it receives no revenue if the firm does not operate. For large (drastic) innovations, the PAE's revenue is maximized by a royalty that is less than the cost-reducing value of the innovation. For intermediate innovations ($v = a - c$), the PAE-optimal royalty equals v and for incremental innovations, the PAE prefers a royalty exceeding v.

4.2 Nash Bargaining in the Injunctive Regime

In the injunctive regime, legal adjudication leads to a shutdown in production and disagreement payoffs of zero for both parties. Letting superscript I identify variables for the injunctive regime, and setting π^0 and L^0 equal to zero in (6), the Nash product becomes

$$B^I(r) = \pi(r)L(r) \quad (8)$$

Maximizing (8) with respect to $r \in [0, r^P]$ yields the following first order condition.[3]

$$dB^I(r)/dr = \pi(r)L'(r) + L(r)\pi'(r) = 0 \quad (9)$$

The maximum is unique and the royalty, r^I, that satisfies (9) is the Nash bargaining solution.

Proposition 2 sets out the properties of this solution.

Proposition 2: *The Nash Bargaining Solution for the Injunctive Regime*

(i) In the injunctive regime, the firm and the PAE always settle on a negotiated royalty. The solution is unique and lies between the firm's preferred royalty of zero and the PAE-optimal royalty of r^*.

Try: In the injunctive regime, the firm and the PAE always settle. The Nash product is maximized at the unique royalty negotiated at the settlement.

and the maximum of the Nash product gives rise to a unique maximum of the Nash product for $r \in [0, r^P]$. reaches its unique maximum for r negotiated royalty maximizes

(ii) The negotiated royalty r^I is

$$r^I = (a - c + v)/4 = r^*/2 \quad (10)$$

(iii) The negotiated royalty

a. exceeds v if $v < (a - c)/3$ (for small incremental innovations);
b. equals v if $v = (a - c)/3$; and
c. is less than v if $v > (a - c)/3$ (for larger incremental innovations and all intermediate and drastic innovations).

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[3] Maximization of $B^I(r)$ for $r \in [0, r^P]$ implies $r \geq 0$ and $r \leq r^P$, but the constraints are not binding and do not affect the first order condition (9). The PAE earns no revenue at $r = 0$ and the firm does not produce at $r = r^P$, so $B^I(r)$ is at its minimum of zero at both $r = 0$ and $r = r^P$.

Figure 1: *The Nash Product for Royalty Negotiation in the Injunctive Regime*

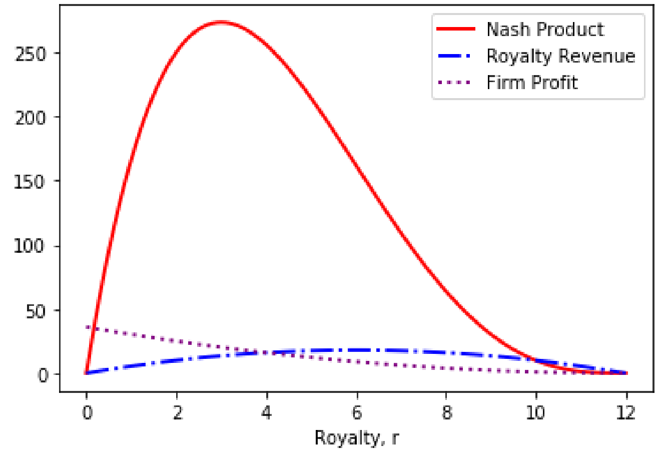

Figure 1 illustrates firm profit, royalty revenue, and the Nash product as functions of r for the case of $a = 24$, $c = 16$, and $v = 4$.[4] The Nash product is at its minimum of zero both at the infringing firm's preferred royalty of zero and at the prohibitive royalty of 12. The Nash bargaining solution, r^I, is 3 and the PAE-optimal royalty, r^*, is 6. As shown in the proof of Proposition 2 (see the Appendix), all admissible parameter values yield the same general shapes for these functions shown in Figure 1. In particular, the Nash product is always strictly concave for $r < r^*$. There is an inflection point at $r = r^*$ and the Nash product is strictly convex for $r > r^*$ up to the prohibitive level r^P. It follows that there is a unique maximum of the Nash product and, from Proposition 2, this maximum occurs at $r^I = r^*/2$.

One important feature of the injunctive regime is that for small incremental innovations ($v < (a - c)/3$), the infringing firm ends up paying a royalty that exceeds the cost-reducing value of the patented innovation. We will discuss this result more fully when we compare the injunctive and fee-based regimes in section 5, but fundamentally it is due to the PAE's bargaining advantage when its loss in revenue from an injunction is small relative to the loss in

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[4] It is algebraically convenient for these values to be divisible by 4, giving rise to whole numbers as solutions. Calculations were done using Python.

profit of the infringing firm. If it were costless for the firm to stop using the patented innovation when r exceeds v, it would, but that possibility is ruled out by assumption. (We relax this assumption in Section 6 of the paper.)

But why would the firm decide to use the innovation in its production structure if it rationally anticipated a possible royalty exceeding the innovation's value. We do not formally model this earlier decision. However, we could rationalize it by introducing uncertainty at that earlier stage over whether the innovation is covered by an existing patent. A firm's decision to use the innovation could have a positive expected value when that decision is made, even if the eventual outcome is sometimes a royalty that exceeds the value of the innovation. Our model starts after that uncertainty is resolved and we focus on the cases in which the innovation does infringe a patent.

4.3 Nash Bargaining in the Fee-Based Regime

If the parties fail to reach a negotiated settlement in the fee-based regime, the case goes to legal adjudication and a royalty of $r = v$ will be set. A royalty of v is never sufficient to shut down production (see (3)). Thus the disagreement outcomes, π^0 and L^0, in the Nash product (6) are the (strictly positive) profits of the firm and the PAE at $r = v$. Letting the superscript F identify variables associated with the fee-based regime, the Nash product for $r \in [0, r^P]$ is

$$B^F(r) = (\pi(r) - \pi(v))(L(r) - L(v)) \tag{11}$$

For incremental innovations, the firm and the PAE have diametrically opposed incentives. The firm prefers as low a royalty as possible and would never accept a royalty rate higher than the rate v available from legal adjudication. In contrast, the PAE prefers a royalty rate greater than v for incremental innovations (Proposition 1) and would never accept a rate less than the value v it could get in court. Therefore, no settlement is possible and, as set out in Proposition 3(i), the only possible outcome is an adjudicated royalty equal to v.

In contrast, for drastic innovations, there is always a settlement. The PAE-optimal royalty, r^*, is less than the royalty of v that the court would impose (Proposition 1), so both the firm and the PAE can be made better off by a settlement. Maximizing $B^F(r)$ from (11), we show in Proposition 3(ii) that, for a drastic innovation, a unique solution exists at a royalty, denoted r^F, that is between the infringing firm's favored royalty of zero and the PAE-optimal royalty. The royalty must therefore also be less than v (as $r^* < v$). Proposition 3(iii) solves for an explicit expression for the negotiated royalty, r^F.

Proposition 3: *The Nash Bargaining Solution for the Fee-Based Regime*

(i) In the fee-based regime with incremental and intermediate innovations ($v \leq a - c$), no settlement is reached. The court imposes a royalty equal to the cost-reducing value of the innovation, which is less than or equal to the PAE-optimal royalty: $r^F = v \leq r^*$.

(ii) With drastic innovations ($v > a - c$), the parties always settle and agree on a royalty that is strictly positive, but less than the PAE-optimal royalty, which is less than v: $0 < r^F < r^* < v$.

(iii) Letting $\omega \equiv 5(a - \mathrm{c}) + v$, the royalty in the fee-based regime can be expressed as:

$$\begin{aligned} r^F &= v \quad \text{if } v \leq a - c \\ &= r^* - \{[8(v^2 - (a-c)^2) + \omega^2]^{1/2} - \omega\}/8 \quad \text{if } v \geq a - c \end{aligned} \tag{12}$$

5. Comparing Injunctive and Fee-Based Legal Regimes

5.1 Comparing Injunctive and Fee-based Royalties

Based on the results in Section 4, it is possible to compare royalties in the two regimes.

Proposition 4: *Comparing Injunctive and fee-based Royalties*

(i) For small incremental innovations ($v < (a - c)/3$), the fee-based regime leads to a lower royalty than the injunctive regime. Royalty rates are equal if $v = (a - c)/3$.

(ii) For larger incremental innovations and for intermediate innovations ($(a - c)/3 < v \leq a - c$), the fee-based regime leads to a higher royalty than the injunctive regime.

(iii) For drastic innovations ($v > a - c$), the fee-based regime leads to a higher royalty than the injunctive regime.

Using the same parameter values as Figure 1 ($a = 24$ and $c = 16$), Figure 2 illustrates the royalties under each regime and the PAE's optimal royalty as functions of the innovation's cost-reducing value, v.

Figure 2: *Royalty Comparison (for a = 24; c = 16)*

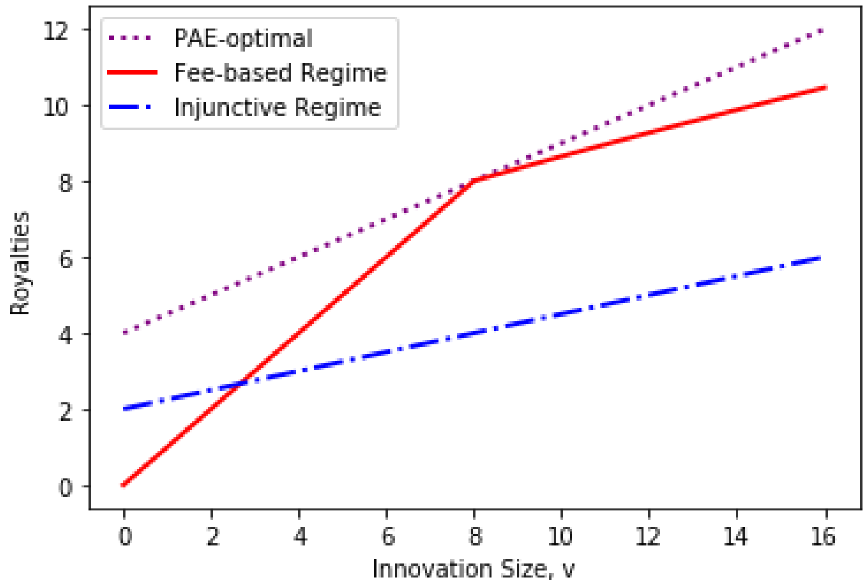


Figure 2 illustrates, in accordance with Proposition 4(i), that for $v < (a - c)/3$ (= 8/3) the royalty r^I negotiated under the injunctive regime exceeds the fair-value royalty, $r^F = v$, imposed by the court under the fee-based regime. But, the response of r^I to an increase in v is less than one and r^I is surpassed by r^F as v is increased above $(a - c)/3$. As Figure 2 shows, the royalty, r^F, in the fee-based regime has a kink at $v = a - c$ (= 8). For $v \leq a - c$, the court imposes the fair-value royalty of v, but for $v > a - c$, the parties settle on a royalty below v. The diagram also illustrates the fact that at $v = a - c$, the PAE's optimal royalty, r^*, is equal to v and hence equal to r^F, but for all other values of v, royalties in both regimes are strictly below r^*.

5.2 *Legal Regime Preferences*

Proposition 5 sets out the conditions under which the PAE or the firm would prefer a particular legal regime. In both regimes, the Nash-bargained royalty is never above the PAE-optimal royalty and always exceeds the infringing firm's preferred royalty of zero. Therefore, the PAE would always prefer the regime with the higher royalty and the firm would always prefer the other regime. As a result, the conditions under which the PAE or the firm prefers a particular legal regime depend solely on the conditions under which the royalty in one regime exceeds the royalty in the other regime. As these conditions are set out in Proposition 4, Proposition 5 follows immediately.

Proposition 5. Legal Regime Preferences

(i) For small incremental innovations ($v < (a - c)/3$), the PAE prefers the injunctive regime and the infringing firm prefers the fee-based regime.

(ii) For larger incremental innovations and for intermediate and drastic innovations, the PAE prefers the fee-based regime and the infringing firm prefers the injunctive regime.

Even though the injunctive regime imposes a greater threat of loss on the infringing firm than the fee-based regime, Proposition 5 shows that PAEs would prefer the fee-based regime in a wide range of cases. In practice, the majority of patents held by PAEs would be small relative to the difference, $a - c$, between the demand intercept and marginal cost, so the majority would fall into the "small incremental" category in which the PAE prefers the injunctive regime. Even so, it is striking that in a wide range of cases the natural intuition that injunctive regimes favor PAEs (and plaintiffs in patent cases more broadly) does not hold.

Understanding why the PAE or the infringing firm prefers one legal regime over the other requires consideration of the interaction of regime choice and innovation size. This explanation is outlined in the introduction but is specified in more detail here. Consider first the case in which the value of the patented innovation is small in relation to the size of $a - c$ so, from

Proposition 5(i), the PAE prefers the injunctive regime. In the injunctive regime, if the firm proposes a royalty equal to fair value, the PAE can say, in effect: "With only a low royalty of v on the table, the license revenue, $L(v) = vq(v)$, that I will lose from an injunction is small." The infringing firm has more to lose: At a royalty of v its profit is $\pi(v) = (q(v))^2 = (a - c)^2/4$ and, in this case, $a - c$ is large relative to v.

Given this imbalance in the loss from an injunction, it becomes understandable that for v sufficiently small (specifically for $v < (a - c)/3$), the infringing firm is willing to settle in the injunctive regime on a royalty that is above the true value of the innovation (see Proposition 2(iii)). This is better for the PAE (and worse for the firm) than the fee-based regime where all the PAE would get for $v < (a - c)/3$ is a royalty equal to the innovation's value (see Proposition 3(i)). This is the classic situation that concerned the U.S. Supreme Court in the eBay v. MercExchange case, in which the threat of an injunction gives the holder of a minor patent excessive bargaining power in negotiations with the infringing firm.

In contrast, for larger incremental innovations and for intermediate and drastic innovations, the outcomes are reversed: the PAE prefers the fee-based regime and the firm prefers the injunctive regime (see Proposition 5(ii)). For incremental and intermediate innovations, we have $r^F = v$ and if v is sufficiently large, negotiation reduces the royalty, r^I, in the injunctive regime below v. For drastic innovations, the PAE-optimal royalty, r^*, is less than v, so it is not hard to understand that the negotiated royalty must be below r^* and hence below v under both regimes. But now, for the PAE to be worse off under the injunctive regime, we require the much less obvious result that the negotiated royalty, r^I, in the injunctive regime is actually less than the negotiated royalty, r^F, in the fee-based regime.

To explain the lower royalty under the injunctive regime for drastic innovations, we examine the *difference* in relative loss to the PAE and the infringing firm from an injunction rather than an adjudicated of a royalty of v. In the injunctive regime, the PAE earns zero in the event of an injunction, but the loss is $L(v) = vq(v)$ relative to an adjudicated royalty of v in the fee-based regime. Comparing with the corresponding relative loss, $\pi(v)$, of the infringing firm and using (2), the difference in relative loss for the PAE versus the infringing firm is

$$L(v) - \pi(\text{v}) = q(v)(v - (a - c)/2) \qquad (13)$$

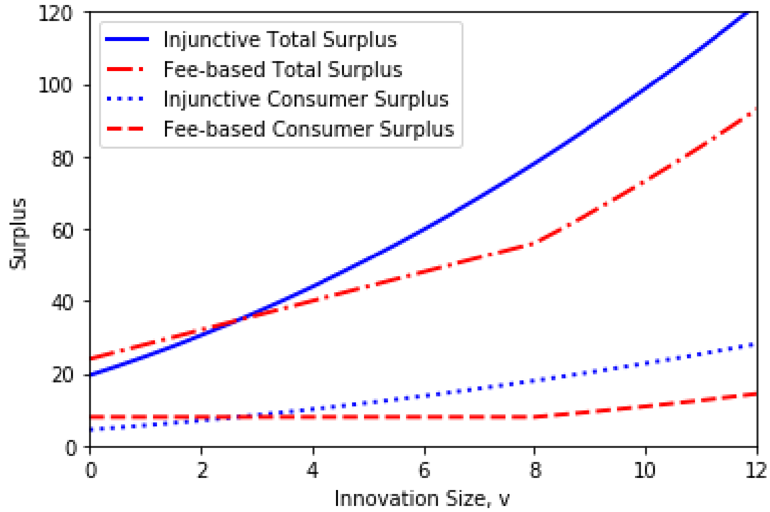
As (13) shows, for v sufficiently large, including for drastic innovations ($v > a - \text{c}$), the PAE suffers a higher relative loss than the infringing firm from an injunction relative to adjudication of $r = v$. As a result, the incentive to reach a settlement is higher for the PAE (and lower for the infringing firm) in the injunctive regime than the fee-based regime. For drastic innovations, the outcome is a settlement at a lower royalty in the injunctive regime than in the fee-based regime.

5.3 Surplus Comparisons for the Two Regimes

We can readily determine comparative levels of consumer and total surplus in the two regimes. As demand is linear, consumer surplus, given by $CS = q(r)^2/2$, is equal to half the infringing firm's profit and, from (2), is strictly decreasing in the royalty rate. Total surplus, the sum of CS and profit, is also decreasing in the royalty rate because a higher royalty rate increases price which moves the market further away from the efficient price and quantity.

As a result, consumer and total surplus are higher the lower is the license fee as determined on the basis of values of v in Proposition 4. Specifically, for small incremental innovations ($v < (a - c)/3$), the fee-based regime provides higher consumer surplus and higher total surplus, whereas the result is reversed for larger innovations. Figure 3 illustrates the effect of v on consumer and total surplus in each regime.

Figure 3: *Consumer and Total Surplus in Each Regime (for a = 24; c = 16)*



6. The Walk-Away Option for the Infringing Firm

In our base model, we assume that the infringing firm cannot "walk away" from the patented innovation, reflecting the typically high cost of restructuring production to circumvent an infringed patent. However, infringing firms can sometimes work around infringed patents. It is therefore interesting to consider the case in which the infringing firm has the option to stop using or "walk away" from the patented innovation at some fixed restructuring cost.

Using a superscript W (for "walk away") to denote variables under a walk-away option, we denote the fixed restructuring cost as k^W. If the firm stops using the innovation, its marginal cost reverts to c, but it does not pay a royalty so it produces output, $q^W = (a - c)/2$, and earns variable profit $(a - c)^2/4$. To make the walk-away option potentially feasible, we assume that after paying $k^W > 0$, the firm's profit, denoted, π^W, is strictly positive:

$$\pi^W \equiv (a - c)^2 /4 - k^W > 0 \tag{14}$$

The walkaway option constrains the royalty r acceptable to the infringing firm by the requirement that $\pi(r) \geq \pi^W$. If $\pi(r) < \pi^W$, it is profitable for the firm to walk away. (If the firm is indifferent (i.e. $\pi(r) = \pi^W$), we assume it continues to use the innovation). We let r^c denote the critical value of r such that $\pi(r^c) = \pi^W$. It can be shown that $r^c \in (v, r^P)$ for $k^W \in (0, (a-c)^2/4)$. As k^W becomes small, the walk-away profit π^W approaches $\pi(v) = (a-c)^2/4$ (see (14) and (2)), and r^c approaches v. As k^W becomes close to it upper limit of $(a-c)^2/4$, π^W approaches zero and r^c approaches the prohibitive royalty, r^P. Our base model, which does not have a walk-away option, can be interpreted as a situation where $k^W \geq (a-c)^2/4$.

The fee-based regime is unaffected by the walk-away option. The reasoning is as follows. In the fee-based regime, the royalty is less than or equal to v, so the profit of the infringing firm equals or exceeds $\pi(v) = (a-c)^2/4$. With the inclusion of the cost, k^W, of exercising the walk-away option, we have $\pi(v) > \pi^W$ from (14), so the firm will never walk away. The disagreement payoffs of the PAE and the firm are unchanged, and the Nash bargaining solution remains the same. This result is reported in Proposition 6(i).

In response to an injunction, the walk-away option allows the firm to restructure and earn a positive profit rather than shut down. Consequently, the firm's threat to walk away in the event of an injunction is credible and the profit π^W is the firm's new disagreement payoff. The PAE's disagreement payoff is unchanged since it receives no revenue whether the firm walks away or shuts down. The Nash product in the injunctive regime with a walk-away option is

$$B^{IW}(r) = (\pi(r) - \pi^W)L(r) \tag{15}$$

In the injunctive regime, the improvement in the infringing firm's disagreement payoff increases its payoff in Nash bargaining. As shown in Proposition 6(ii), the royalty that the firm pays is always reduced. The magnitude of this effect depends on the restructuring cost.

The PAE prefers the regime with the higher royalty (Proposition 5) and the walk-away option reduces the negotiated royalty in the injunctive regime, but has no effect in the fee-based regime. Consequently, as set out in Proposition 6(iii), the walk-away option can never increase the range of v for which the PAE prefers the injunctive regime.

Proposition 6: The Walk-away Option

(i) In the fee-based regime, the walk-away option has no effect.

(ii) In the injunctive regime, the royalty, r^{IW}, with the walk-away option is strictly less that the royalty, r^{I}, without the walk-away option.

(iii) Introducing a walk-away option never increases the range of v for which the PAE prefers the injunctive regime. If k^{W} is sufficiently small and $v < (a - c)/3$, then $r^{IW} < v < r^{I}$ and the walk-away option causes the PAE to shift its preference from the injunctive to the fee-based regime.

An implication of Proposition 6(iii) is that introducing the walk-away possibility causes the PAE to shift from preferring the injunctive regime to preferring the fee-based regime for some cases and never causes the reverse shift. Therefore, the walk-away option reinforces the main theme of the paper that the PAE may prefer the fee-based regime.

7. Extensions

7.1 Litigation Costs

Our model abstracts from litigation costs. If added to our analysis, litigation costs would include the cost of filing a claim (filing costs) and the cost of fighting the case in court (court costs). Filing costs would rule out some subset of cases in which potential gains to the PAE are less than the filing cost. Our model therefore would apply only to cases for which the subsequent gain to the PAE exceeds the filing cost. Such costs would be sunk by the time our model "begins" and would have no effect on the analysis.

Court costs that can be avoided if a settlement occurs (but not otherwise) would make settlement more attractive to the parties. This would have no effect on the injunctive regime, where settlement always occurs (Proposition 2). The fee-based regime would be affected in that some cases involving incremental and intermediate innovations would be settled rather than going through the full legal process. To determine the royalty, it would be necessary to specify how much of the court costs are borne by each party. Small court costs would have only small effects on the results.

7.2 Uncertainty

Another important simplification is that we abstract from uncertainty. Several important uncertainties may arise in the patent litigation process. First, a patent may be found invalid. Second, even if the court finds a patent to be valid, it may also find that it was not infringed. And even if a patent is both valid and infringed, any license fee or other compensation awarded is uncertain and may exceed or fall short of the patent's fair value. Such uncertainty is often emphasized, although Mazzeo, et al. (2013) find that monetary awards to patent holders are predictable with reasonable accuracy.

In our model, if the parties are risk neutral and information is symmetric, introducing any of these three uncertainties is straightforward as we can simply interpret the payoffs as expected values and the analysis is essentially unchanged. However, if parties are risk averse, as assumed in the literature on patent litigation insurance (such as Buzzacchi and Scellato, 2008), or if the uncertainty is asymmetric (as in the models of litigation in Gelbach, 2018) then uncertainty would have more significant effects on the analysis, although neither risk aversion nor informational asymmetries would invalidate the principles identified in our analysis.

7.3 Fixed License Fees

We assume license fees of the per-unit royalty form, which is common both in practice and in the literature. However, the basic insights generalize readily to the case in which the firms bargain over a fixed fee, F, instead of a royalty, r. In this case, the profit of the infringing firm is $\pi(0) - F$, where $\pi(0)$ is the profit from the innovation without a royalty. The return to the PAE is simply F. In the injunctive regime, the Nash product is $B^I(F) = (\pi(0) - F)F$ as the disagreement payoffs remain at zero. Maximizing this Nash product with respect to F implies that $F = \pi(0)/2$. The two firms share the profit from the innovation.

In the fee-based regime, legal adjudication gives rise to a per-unit license fee of v, so the Nash product is $B^F(F) = (\pi(0) - F - \pi(v))(F - L(v))$, where $\pi(v)$ and $L(v)$ are the disagreement payoffs as in (11). Maximizing B^F with respect to F yields $F = \pi(0)/2 + (L(v) - \pi(v))/2$. As with royalties, the negotiated fixed fee may be higher or lower than in the injunctive regime. It will be higher if v is large enough that $L(v) > \pi(v)$, which, from (13), applies if $v > (a - c)/2$. Therefore, the PAE prefers the fee-based regime for sufficiently large innovations, as with royalties.

The principle in this case is very similar to bargaining over a royalty. With large innovations, the injunctive regime gives the licensing firm a lot of bargaining power relative to the value of the innovation and the PAE actually does better by going to Court, where it can negotiate a higher license fee than under the injunctive regime. The same principle applies to related fee structures, such as two-part license fees with both a fixed component and a per-unit component. We therefore take the qualitative insights of our analysis as relatively robust.

7.4 Alternative Court-Determined Royalty Rates

Our analysis of the fee-based regime assumes a court-adjudicated default royalty of v. As the infringing firm always prefers a lower royalty, if the PAE-optimal royalty r^* is less than v, then both parties would prefer a royalty of r^* rather than v. Possibly the court would use r^* as the default royalty in such cases. As suggested by an anonymous reviewer, we therefore consider an extension in which the default royalty in the fee-based regime is the *minimum* of v and r^*.

This change has no effect if the innovation is incremental or intermediate as $r^* > v$ in those cases. However, for drastic innovations, we have $r^* < v$, so the court would use r^* as the default royalty instead of v. This reduction in the default royalty would make the fee-based regime even more attractive to the PAE, as it would now get its optimal license fee. This reinforces our main result that the PAE prefers a fee-based regime if the innovation is drastic.

Furthermore, *any* departure from a default royalty of v in the direction of the PAE-optimal royalty of r^* increases the appeal of the fee-based regime to the PAE. Departures in the other direction have the opposite effect but would still allow for cases in which the PAE prefers the fee-based regime. More generally, if the default royalty exceeds or falls short of v in equation (11), the analysis follows as before with a corresponding change in the threshold level of v at which the PAE would shift its regime preference.[5]

We emphasize that courts may use v as the default royalty even if both parties would prefer a lower royalty. The reason is that courts do not focus exclusively on the interests of the two parties. Courts may reject joint proposals from plaintiffs and defendants if those proposals are inconsistent with precedent and/or provide insufficient deterrence for future violations. Our reading of the case record is that both these factors are important in patent infringement cases.

As discussed in Shapiro (2010), Jarozs and Chapman (2012), and elsewhere, the most important U.S. legal precedent in this area is *Georgia-Pacific Corp. v. United States Plywood Corp* (1970). This case sets out fifteen different factors for courts to consider in setting royalties in patent infringement cases. Most are obvious practical considerations, such as the duration of the patent. However, the factors also include at least three distinct general approaches of relevance to our analysis.

[5] Large departures could have large effects on the analysis. If, for example, the default royalty was near zero, the PAE would never prefer fee-based regime. But small departures would leave that qualitative nature of the analysis essentially unchanged.

One approach is based on "the advantages of the patent property over the old modes" [of production] (factor 9). This approach is consistent with imposing v as the royalty, as v is precisely "the advantage of the patented innovation…". The second major approach is based on "realizable profit that should be credited to the invention" (factor 13). This approach is sometimes taken to imply that all of the profit attributable to the innovation should go to the licensor in infringement cases, reducing the infringer's benefit to zero. This is consistent with a default royalty of v, as the infringing firm would earn exactly the same amount it would have earned without the innovation.

The third major approach is based on a hypothetical negotiation between the parties "if both had been reasonably and voluntarily trying to reach an agreement" (factor 15). A Nash bargaining approach can be attempted but the disagreement payoffs are unclear as factor 15 implies that disagreement should not occur. Different specifications can lead to a royalty that can exceed, equal, or fall short of v.

Overall, we suggest that assuming a default royalty of v is useful starting point that is likely to be a good approximation in a wide range of cases. Furthermore, the major results are robust to a variety of plausible extensions. In addition to the extensions already discussed, Section 7.2. implies that allowing the default royalty to vary randomly around an expected value of v leaves the results unchanged if the parties are risk neutral. And our results are robust to small departures from v in expected value for the same reason as in the certainty case.

7.5 Asymmetric Bargaining Power

In our model, the position of either party improves when its disagreement payoff increases. However, bargaining power is symmetric in the sense that the infringing firm and the PAE each receive equal weight in the Nash product. This contrasts with most of the early work on license fees (as reviewed in Kamien, 1992) and in much subsequent work in which the licensor has most or all of the bargaining power. Such a situation arises if the licensor sets a posted price or, in a bilateral setting, can make a take-it-or-leave it (TILO) offer. Stamatopoulos (2020) provides a comparison of TILO offers and Nash bargaining in licensing situations.

In the injunctive regime of our model, the infringing firm would get nothing if it turns down a TILO offer as an injunction would follow. Therefore, the PAE could demand the PAE-optimal royalty and it would always be accepted as that is better than nothing for the infringing firm. In the fee-based regime, refusal of the offer by the firm results in a court-determined royalty of *v*, which limits the ability of the PAE to extract surplus. The PAE always prefers the injunctive regime in this (extreme) case in which it can make TILO offers.

Less extreme asymmetries in bargaining power can be modelled by using the generalized Nash bargaining solution as in, for example, Sempere Monerris and Vannetelbosch (2001). The generalized Nash product is $\mathrm{B}(r) \equiv (\pi(r) - \pi^{o})^{s}(L(r) - L^{o})^{1-s}$, where $0 \leq s \leq 1$. The symmetric case arises if $s = ½$, and is equivalent to our base model. If s is slightly less than ½, the outcome becomes slightly better for the PAE and the injunctive regime is favored by the PAE for a slightly larger range of *v*. In the limit as s approaches 0, all the bargaining power goes to the PAE and the outcome converges on the TILO outcome in which the PAE strictly prefers the injunctive regime for all values of *v*.

Concluding Remarks

This paper compares legal regimes based on injunctive and fee-based relief for the purposes of patent litigation. The comparison is motivated by the change that occurred in the U.S. legal system in 2006 when the U.S, Supreme Court shifted the default legal regime to one of fee-based relief.

One factor underlying this decision was a desire to reduce the bargaining power of patent assertion entities (PAEs) in negotiations with firms alleged to be infringing their patents. We show that for small innovations a PAE does have a bargaining advantage in an injunctive regime, would be able to bargain for a higher royalty, and would prefer that regime. Our more striking result is that for larger innovations, the PAE would prefer a fee-based regime and would be at a bargaining disadvantage in an injunctive regime, leading to a lower royalty in the injunctive regime. This finding provides a counterpoint to the general presumption that injunctive regimes necessarily favor PAEs.

Much of our analysis concerns the case in which an injunction to cease using an innovation prevents production. We relax this assumption to consider a walk-away option in which an infringing firm can restructure so as to comply with the injunction, yet continue to produce. With the walk-away option, there is a reduction in the negotiated royalty in the injunctive regime, but not in the fee-based regime. More broadly, the walk-away option makes the injunctive regime less attractive to the PAE and increases the range of cases in which the PAE would prefer a fee-based regime, reinforcing our main result.

How can the fee-based regime give rise to a higher bargained royalty than the injunctive regime? When the value of the patented innovation is high, injunctive relief is a two-edged sword for the PAE. While an injunction is still a costly threat to the infringing firm, it is also harmful to the PAE in that a reasonable royalty close to its cost-reducing value would be worth a lot to PAE and that value is lost if there is an injunction. The bargaining advantage shifts to the infringing firm in the sense that for a very valuable innovation, the PAE loses more from an injunction than does the infringing firm.

Furthermore, the general principle that an injunctive regime favors PAEs if patented innovations are small but favors the infringing firm for larger innovations seems robust to most directions of generalization. Most patents used by "patent trolls" as a basis for infringement claims are of minor relevance, as implied by the FTC (2016). Therefore, reducing the threat of injunctive relief has probably reduced the bargaining power of PAEs overall. But it is important to understand that for larger innovations, PAEs could actually prefer fee-based relief to injunctive relief.

We also consider welfare (surplus) effects. Taking the level of innovation as given, consumers are always better off with a lower price, which means they are always better off with a lower royalty. For small innovations, they are therefore better off with fee-based relief. However, the level of innovation is not necessarily given. Possibly a regime that favors PAEs would induce PAEs to pay more when acquiring patents from underlying inventors, increasing the incentives to innovate and leading to induced innovation. However, early stage analysis of the empirical record, as in Mezzanotti and Simcoe (2019), finds no evidence of any such induced innovation effect. Therefore, if most patents held by PAEs are for small innovations, as seems likely, then the shift to fee-based relief has probably benefited consumers.

Appendix

Proposition 1: *The PAE-optimal royalty*

(i) The PAE's optimal royalty is always less than the prohibitive level and is, specifically, half the prohibitive royalty.

$$r^* = (a - c + v)/2 < r^P = a - c + v \quad (7)$$

(ii) The PAE's optimal royalty

a. exceeds the innovation's value, v, if the innovation is incremental ($v < a - c$);
b. equals v if the innovation is intermediate ($v = a - c$); and
c. is less than v if the innovation is drastic ($v > a - c$).

Proof: (i) Maximizing $L(r) = rq(r)$ from (5) for $r \in [0, r^P]$, it follows from $L(0) = L(r^P) = 0$ that the constraints are not binding. From (5), (2) and $q'(r) = -½$, r^* satisfies the first order condition:

$$L'(r) = q(r) - r/2 = (a - c + v - 2r)/2 = 0 \quad (A1)$$

where $L''(r) = -1$. From the strict concavity of $L(r)$ for $r \in [0, r^P]$, it follows that $r^* = (a - c + v)/2$ satisfying (A1) is unique and is less than r^P as in (7).

(ii) If $v = a - c$, then $r^* = v$ from (7). The two inequality results follow immediately. ***

Proposition 2: *The Nash Bargaining Solution for the Injunctive Regime*

(i) In the injunctive regime, the firm and the PAE always settle on a negotiated royalty. The solution is unique and lies between the firm's preferred royalty of zero and the PAE-optimal royalty of r^*.

Try: In the injunctive regime, the firm and the PAE always settle. The negotiated royalty r^I gives rise to a unique maximum of the Nash Product, $B^I(r)$, for all $r \in [0, r^P]$.

(ii) The negotiated royalty r^I is

$$r^I = (a - c + v)/4 = r^*/2 \quad (10)$$

(iii) The negotiated royalty

a. exceeds v if $v < (a - c)/3$ (for small incremental innovations);
b. equals v if $v = (a - c)/3$; and
c. is less than v if $v > (a - c)/3$ (for larger incremental innovations and all intermediate and drastic innovations).

Proof: (i) and (ii). We first determine the sign $d^2B^I(r)/(dr)^2$ for $r \in [0, r^P]$. From $dB^I(r)/dr = \pi(r)L'(r) + L(r)\pi'(r)$ (see (9)) using (4) and (5), we obtain

$$dB^I(r)/dr = q(r)^2(q(r) - 3r/2) \qquad \text{(A2)}$$

From (A2) using $L'(r) = q(r) - r/2$ from (A1), we further obtain

$$d^2B^I(r)/(dr)^2 = -3q(r)(q(r) - r/2) = -3q(r)L'(r) \qquad \text{(A3)}$$

where $q(r) > 0$ for $r < r^P$. Since $L(r)$ is maximized at $r = r^* \in (0, r^P)$ (see (7)) and $L''(r) = -1 < 0$, we have $L'(r) > 0$ for $r \in [0, r^*)$ and, from (A3), $B^I(r)$ is strictly concave for $r \in [0, r^*)$:

$$d^2B^I(r)/(dr)^2 < 0 \text{ for } r \in [0, r^*) \qquad \text{(A4)}$$

For $r \in (r^*, r^P]$, we have $L'(r) < 0$ and from (A3), $B^I(r)$ is strictly convex for $r \in (r^*, r^P]$.

Maximizing $B^I(r) = \pi(r)L(r)$ as in (8), subject to $r \in [0, r^P]$, the constraints $r \geq 0$ and $r \leq r^P$ are not binding. If $r = 0$ or $r = r^P$, then, using $L(0) = 0$ and $L(r^P) = \pi(r^P) = 0$ (see (4) and (5)), $B^I(r)$ is at its minimum value of zero. If $r \in (0, r^P)$, then $B^I(r) > 0$ from $q(r) > 0$ and $\pi(r) > 0$ (see (4)) and from $L(r) > 0$ for $r > 0$ (see (5)).

Setting $dB^I(r)/dr = 0$ in (A2) and using $q(r)$ from (2), the Nash bargaining solution is $r^I = (a - c + v)/4$ as in (10). From (7), we obtain $r^I = r^*/2$ and hence $r^I \in (0, r^*)$. From the strict concavity of $B^I(r)$ for $r \in [0, r^*)$ (see (A4)) and $dB^I(r)/dr < 0$ for all $r \in [r^*, r^P)$ (from (A2) and (7)), $B^I(r^I)$ is the unique maximum of $B^I(r)$ for all $r \in [0, r^P]$.

(iii) Since $r^I - v = (a - c - 3v)/4$ from (12), we have $r^I - v > 0$ if $v < (a - c)/3$, $r^I = v$ if $v = (a - c)/3$ and $r^I - v < 0$ if $v > (a - c)/3$.***

Lemma 1: If $r \in [0, r^*]$ where $r^* < v$ or if $r \in [0, v]$ where $v < r^*$, then the Nash product in the fee-based regime, $B^F(r)$, is strictly concave in r:

$$d^2B^F(r)/(dr)^2 = -3q(r)L'(r) + q(v)L'(v) < 0 \qquad \text{(A5)}$$

Proof: From (8) and (11) we obtain $B^F(r) = B^I(r) - [\pi(r)L(v) + \pi(v)(L(r) - L(v))]$ and hence

$$dB^F(r)/dr = dB^I(r)/dr - [\pi'(r)L(v) + \pi(v)L'(r)] \quad (A6)$$

From (A6) using $\pi''(r) = -q'(r) = ½$ and $L''(r) = -1$, we further obtain $d^2B^F(r)/(dr)^2 = dB^I(r)/(dr)^2 + \pi(v) - L(v)/2$ where from (4) and (5), $\pi(v) - L(v)/2 = q(v)(q(v) - v/2) = q(v)L'(v)$. Using $d^2B^I(r)/(dr)^2 = -3q(r)L'(r)$ from (A3), we obtain $d^2B^F(r)/(dr)^2$ as in (A5).

Now examining the sign of (A5), since $v < r^P$ (see (3)), we have $q(r) > 0$ for all $r \in [0, v]$. If $r \in [0, r^*)$ and $r^* \leq v$, using $L'(r) > 0$ for $r \in [0, r^*)$ and $L'(v) \leq 0$ for $r^* \leq v$ in (A5), we obtain $d^2B^F(r)/(dr)^2 < 0$. If $r \in [0, v]$ and $v < r^*$, then $L'(v) > 0$. Rearranging (A5), we obtain $d^2B^F(r)/(dr)^2 = -q(r)(3L'(r) - L'(v)) - L'(v)(q(r) - q(v)) < 0$ since $r \leq v$ and $L''(r) < 0$ imply $L'(r) \geq L'(v)$ and $q'(r) < 0$ implies $q(r) \geq q(v)$. ***

Proposition 3: *The Nash Bargaining Solution for the Fee-Based Regime*

(i) In the fee-based regime with incremental and intermediate innovations ($v \leq a - c$), no settlement is reached. The court imposes a royalty equal to the cost-reducing value of the innovation, which is less than or equal to the PAE-optimal royalty: $r^F = v \leq r^*$.

(ii) With drastic innovations ($v > a - c$), the parties always settle and agree on a royalty that is strictly positive, but less than the PAE-optimal royalty, which is less than v: $0 < r^F < r^* < v$.

(iii) Letting $\omega \equiv 5(a - c) + v$, the royalty in the fee-based regime can be expressed as:

$$r^F = v \quad \text{if } v \leq a - c$$
$$= r^* - \{[8(v^2 - (a - c)^2) + \omega^2]^{1/2} - \omega\}/8 \quad \text{if } v \geq a - c \quad (12)$$

Proof: Maximizing $B^F(r) = (\pi(r) - \pi(v))(L(r) - L(v))$ as in (11), subject to $r \in [0, r^P]$, the constraints $r \geq 0$ and $r \leq r^P$ are not binding. This follows since if $r = 0$, then $L(0) - L(v) < 0$ and $B^F(r) < 0$ and if $r = r^P$, $\pi(r^P) = 0$ and $\pi(r^P) - \pi(v) < 0$. Hence the Nash bargaining solution, $r = r^F$ satisfies $dB^F(r)/dr = 0$.

From (11), the effect of an increase in r on $B^F(r)$ under the fee-based regime is

$$dB^F(r)/dr = (L(r) - L(v))\pi'(r) + (\pi(r) - \pi(v))L'(r) \quad (A7)$$

(i) From Proposition 1(ii), we have $v \leq r^*$ if and only $v \leq a - c$. For $r \in [0, v)$, it follows from $L''(\mathrm{r}) = -1 < 0$ from (A1) and $r < r^*$ that $L'(r) > 0$ and $L(r) - L(v) < 0$. Using $\pi'(r) = 2q(r)\mathrm{q}'(r) = -q(\mathrm{r}) < 0$, we also have $\pi(r) - \pi(v) < 0$ for $r \in [0, v)$. Hence, from (A7), we have $\mathrm{dB}^{\mathrm{F}}(r)/\mathrm{d}r > 0$ for $r \in [0, v)$ and $v \leq r^*$, which rules out a settlement for $v \leq a - c$. Since $\mathrm{dB}^{\mathrm{F}}(v)/\mathrm{d}r = 0$ from (A7), the unique maximum of $\mathrm{B}^{\mathrm{C}}(r)$ for $r \in [0, v]$ is at $r = v$ (the trivial solution). Consequently, if $v \leq a - c$, the Court imposes a royalty, $r^{\mathrm{F}} = v$ where $v \leq r^*$.

(ii) We have $r^* < v$ if and only $v > a - c$ (Proposition 1(ii)). To show that the parties will settle at some $r \in (0, r^*)$, we rule out a settlement for $r \in [r^*, v)$. From $L''(r) < 0$ and $r \geq r^*$ we obtain $L'(r) \leq 0$ and from $r < v$ that $L(r) - L(v) > 0$. Using $\pi'(r) = -q(r) < 0$ from (4), we also have $\pi(r) - \pi(v) < 0$ for $r \in [r^*, v)$. Using (A7), it then follows that $\mathrm{dB}^{\mathrm{F}}(r)/\mathrm{d}r < 0$ for $r \in [r^*, v)$. We have shown above that $\mathrm{dB}^{\mathrm{F}}(0)/\mathrm{d}r > 0$. From the strict concavity of $\mathrm{B}^{\mathrm{F}}(r)$ for $r \in [0, r^*]$ and $r^*(s) < v$ (see Lemma 1), there exists a unique $r^{\mathrm{F}} \in (0, r^*)$ that maximizes $\mathrm{B}^{\mathrm{F}}(r)$.

(iii) The royalty, r^{F}, satisfies the first order condition, $\mathrm{dB}^{\mathrm{F}}(r)/\mathrm{d}r = 0$ (see (A7)). To obtain an explicit expression for r^{F}, we manipulate (A7) into a form with terms in $v - r$. Using $L(r) = rq(r)$ and $L(v) = vq(v) = v(a - \mathrm{c})/2$ from (5) and (2), we obtain $L(r) - L(v) = (v - r)(r - (a - c))/2$. Letting $\rho \equiv v/(a - c)$, we can write $v - (a - c) = (\rho - 1)(a - c)$ and $L(r) - L(v) = -(v - r)[v - r - (\rho - 1)(a - c)]/2$. Since $\pi'(r) = -q(r) = -[v - r + a - c]/2$ from (4) and (2), we have

$$(L(r) - L(v))\pi'(r) = (v - r)[(v - r)^2 + (2 - \rho)(a - c)(v - r) - (\rho - 1)(a - c)^2]/4 \qquad \text{(A8)}$$

From $\pi(r) - \pi(v) = (q(r))^2 - (q(v))^2 = (v - r)(v - r + 2(a - c))/4$ from (4) and (2) and $L'(r) = (2(v - r) - (\rho - 1)(a - c))/2$ from (A1), we obtain

$$(\pi(r) - \pi(v))\mathrm{L}'(r) = (v - r)[2(v - r)^2 + (5 - \rho)(a - c)(v - r) - 2(\rho - 1)(a - c)^2]/8 \qquad \text{(A9)}$$

Substituting (A8) and (A9) into (A7), we can express the first order condition as

$$\mathrm{dB}^{\mathrm{F}}(r)/\mathrm{d}r = (v - r)[4(v - r)^2 + 3(3 - \rho)(a - c)(v - r) - 4(\rho - 1)(a - c)^2]/8 = 0 \qquad \text{(A10)}$$

We now solve for r^F from (A10)). Since (A10)) is a cubic function, it has 3 potential solutions. One solution is $r = v$, which, as shown in part (i), applies for $v \leq a - c$. For $v > a - c$, setting the quadratic expression in square brackets in (A10)) equal to zero, we obtain

$$r^F = v + (a - c)\{3(3 - \rho) - [9(3 - \rho)^2 + 64(\rho - 1)]^{1/2}\}/8 \qquad \text{(A11)}$$

A second root of the quadratic implies $r^F > v$, so it is not the Nash bargaining solution. From part (ii), if $v > a - c$, then r^F is unique and satisfies $r^F < r^* < v$. Since $9(3 - \rho)^2 + 64(\rho - 1) = 8(\rho^2 - 1) + (5 + \rho)^2$ and $v - r^* = (v - (a - c))/2 = (\rho - 1)(a - c)/2$, we can express (A11) as

$$r^F = r^* - (a - c)\{[8(\rho^2 - 1) + (5 + \rho)^2]^{1/2} - (5 + \rho)\}/8 \qquad \text{(A12)}$$

If $v > a - c$, then $\rho - 1 > 0$ and (A12) shows that $r^F < r^*$. Letting $\omega \equiv (5 + \rho)(a - c) = 5(a - c) + v$ and using $(a - c)^2(\rho^2 - 1) = v^2 - (a - c)^2$ in (A12), we obtain (12) as was to be proven. ***

Proposition 4: *Comparing Injunctive and fee-based Royalties*

(i) For small incremental innovations ($v < (a - c)/3$), the fee-based regime leads to a lower royalty than the injunctive regime. Royalty rates are equal if $v = (a - c)/3$.

(ii) For larger incremental innovations and for intermediate innovations ($(a - c)/3 < v \leq a - c$), the fee-based regime leads to a higher royalty.

(iii) For drastic innovations ($v > a - c$), the fee-based regime leads to a higher royalty than the injunctive regime.

Proof: (i) and (ii). Comparing royalties for $v \leq a - c$, the royalty is $r^F = v$ in the fee-based regime and, rearranging (10), the royalty is $r^I = v + (a - c - 3v)/4$ in the injunctive regime. Consequently $r^F - r^I = (3v - (a - c))/4$ and the results follow immediately.

(iii) If $v > a - c$ then both r^F and r^I are less than r^* where $r^* < v$ (Propositions 1 and 2). From the strict concavity of $B^F(r)$ for $r \in [0, r^*]$ where $r^* < v$ (Lemma 1), we have $r^F > r^I$ if and only if $dB^F(r^I)/dr > 0$. Setting $dB^I(r)/dr = 0$ in (A6), we obtain $dB^F(r^I)/dr = -[\pi'(r^I)L(v) + \pi(v)L'(r^I)]$.

It follows from $L'(r) = q(r) - r/2$ from (A1) and (A2) that r^{I} satisfies $dB^{I}(r^{I})/dr = q(r)^2(L'(r^{I}) - r^{I}) = 0$, which implies $L'(r^{I}) = r^{I}$. Using $\pi'(r^{I}) = - q(r^{I})$, $L(v) = vq(v)$ and $\pi(v) = (q(v))^2$, we further obtain $dB^{F}(r^{I})/dr = q(v)[vq(r^{I}) - r^{I}q(v)]$. Finally, using $q(r^{I}) = q(v) + (v - r^{I})/2$ and $q(v) + v/2 = r^*$ from (2) and (7), $dB^{F}(r^{I})/dr$ reduces to

$$dB^{F}(r^{I})/dr = q(v)(v - r^{I})r^* \quad \text{(A13)}$$

From (A13), we have $dB^{F}(r^{I})/dr > 0$, and hence $r^{I} < r^{F}$, if and only if $v - r^{I} > 0$. Since $v - r^{I} > 0$ if and only if $v > (a - c)/3$ (Proposition 2(iii)), it follows that if $v > a - c$ then $r^{I} < r^{F}$. ***

Proposition 6: The Walk-away Option

(i) In the fee-based regime, the walk-away option has no effect.

(ii) In the injunctive regime, the royalty, r^{IW}, with the walk-away option is strictly less that the royalty, r^{I}, without the walk-away option.

(iii) Introducing the walk-away option never increases the range of v for which the PAE prefers the injunctive regime. If k^{W} is sufficiently small and $v < (a - c)/3$, then $r^{IW} < v < r^{I}$ and the walk-away option causes the PAE to shift its preference from the injunctive to the fee-based regime.

Proof: (i) proved in the text.

(ii) To show $r^{IW} < r^{I}$, from (15) and (8), we have $B^{IW}(r) = B^{I}(r) - \pi^{W}L(r)$, which implies

$$dB^{IW}(r)/dr = dB^{I}(r)/dr - \pi^{W}L'(r) \quad \text{(A14)}$$

From (A14), using $L'(0) = q(0)$, $dB^{I}(0)/dr = q(0)^3$ (see (A1) and (A2)), we obtain $dB^{IW}(0)/dr = q(0)(q(0)^2 - \pi^{W}) > 0$ from (2) and (14), which implies $r^{IW} > 0$.

To maximize $B^{IW}(r)$ subject to $r \leq r^{c}$, letting $\mathscr{L}(r) \equiv B^{IW}(r) + \mu(r^{c} - r)$ where μ denotes the Lagrange multiplier, it follows from (A14) that $r^{IW} > 0$ satisfies the Kuhn-Tucker conditions:

$$d\mathscr{L}(r)/dr = dB^{I}(r)/dr - \pi^{W}L'(r) - \mu = 0$$

$$d\mathscr{L}(r)/d\mu = r^{c} - r \geq 0; \;\; \mu \geq 0; \;\; \mu(d\mathscr{L}(r)/d\mu) = 0 \quad \text{(A15)}$$

From $dB^{I}(r)/dr = q(r)^2(q(r) - 3r/2)$ (see (A2)), $L'(r) = q(r) - r/2$ (see A1) and (A15), we obtain

$$d\mathscr{L}(r)/dr = q(r)^2(q(r) - 3r/2) - \pi^{W}(q(r) - r/2) - \mu = 0 \quad \text{(A16)}$$

Using (2) and $r^{\mathrm{I}} = (a - c + v)/4$ from (10), we have $q(r) - 3r/2 = 2(r^{\mathrm{I}} - r)$. From (A16),

$$\mathrm{d}\mathscr{L}(r)/\mathrm{d}r = 2(q(r)^2 - \pi^{\mathrm{W}})(r^{\mathrm{I}} - r) - \pi^{\mathrm{W}} r - \mu = 0 \tag{A17}$$

It can be seen from (A17) and $\pi(r^{\mathrm{IW}}) = q(r^{\mathrm{IW}})^2 \geq \pi^{\mathrm{W}}$ (from $\pi(r^{\mathrm{c}}) = \pi^{\mathrm{W}}$) that if $r^{\mathrm{I}} - r^{\mathrm{IW}} \leq 0$ then $\mathrm{d}\mathscr{L}(r^{\mathrm{IW}})/\mathrm{d}r < 0$, which contradicts $\mathrm{d}\mathscr{L}(r^{\mathrm{IW}})/\mathrm{d}r = 0$ and proves that $r^{\mathrm{I}} > r^{\mathrm{IW}}$.

(iii) The PAE prefers the regime with the higher royalty (Proposition 5). Since $r^{\mathrm{IW}} < r^{\mathrm{I}}$ from part (ii) and the fee-based regime is unchanged, the walk-away option cannot increase the range of v for which the PAE prefers the injunctive regime. To prove that the walk-away option can cause a shift from the injunctive to the fee-based regime, from (A16), using $q(r) - 3r/2 = 2(r^{\mathrm{I}} - r)$ from the proof of part (ii), $\pi^{\mathrm{W}} \equiv q(v)^2 - k^{\mathrm{W}}$ from (14) and $q(v) - v/2 = (a - c - v)/2$, we obtain:

$$\mathrm{d}\mathscr{L}(r)/\mathrm{d}r = 2(q(r)^2 - q(v)^2)(r^{\mathrm{I}} - r) - rq(v)^2 + k^{\mathrm{W}}(a - c - v)/2 - \mu = 0 \tag{A18}$$

Suppose $q(r^{\mathrm{IW}})^2 - q(v)^2 \leq 0$. If $k^{\mathrm{W}} < 2vq(v)/(a - c - v)$ and $v < a - c$, it follows from (A18), using $r^{\mathrm{I}} - r^{\mathrm{IW}} > 0$ from part (ii), that $\mathrm{d}\mathscr{L}(r^{\mathrm{IW}})/\mathrm{d}r < 0$, which contradicts $\mathrm{d}\mathscr{L}(r^{\mathrm{IW}})/\mathrm{d}r = 0$, proving that $q(r^{\mathrm{IW}})^2 - q(v)^2 > 0$ and hence $r^{\mathrm{IW}} < v$. If $v < (a - c)/3$, then $v < r^{\mathrm{I}}$ from Proposition 2(iii). If $k^{\mathrm{W}} < 2vq(v)/(a - c - v)$ and $v < (a - c)/3$, then $r^{\mathrm{IW}} < v < r^{\mathrm{I}}$ and the result follows. ***

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