Transformers

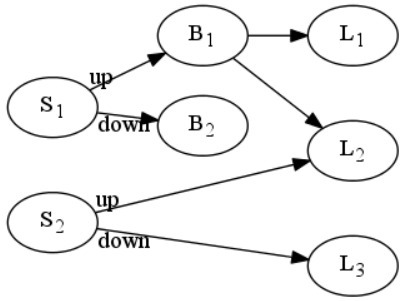
November 11, 2016

In the ELEC problem, you're given a network of electrical wires which can be represented as a directed, acyclic graph (DAG) with three types of nodes:

- "Switch" nodes supply power. They have **no** wires coming in and two wires going out labeled "up" and "down". They also have a switch. If the switch is in the up position, then power (electricity) flows into the up wire. If the switch is in the down position, then power flows into the down wire.
- "Branch" nodes can have one wire coming in (which may or may not carry power) and any number of wires going out. If the wire coming in carries power, then all wires going out also carry power. Otherwise, none of the wires carries power.
- "Load" nodes represent electrical devices that must be powered. They have one or more wires coming in and none going out. If any wire coming in carries power, the load is powered. Otherwise, it is not.

The solution to an ELEC instance is YES if some configuration of the switches powers all the loads; otherwise, it's NO.

1. Indicate a configuration of the switches in the following network that powers all the loads by writing "up" or "down" on each switch node. (Switch nodes are labeled S, branch nodes B, and load nodes L.)



2. Give a reduction from SAT to ELEC. *Hint:* Consider that a variable can be positive or negated, the positive (or negated) literal can appear in many clauses, and each clause needs at least one true literal in it.

1 NP-Completeness

You have already shown that ELEC is NP-hard with your reduction from SAT to ELEC above. Now, you will show that ELEC is in NP. (Together, the fact that it is NP-hard and in NP makes it NP-complete.)

1. Give a (polynomial-length) certificate for ELEC instances where all loads can be powered. *Hint:* We already asked you for your "solution" to an ELEC problem above. What form does such a solution take?

2. Give an algorithm that takes polynomial time in the size of an ELEC instance to determine whether all loads can be powered in that instance, given a certificate like the the one you describe above.