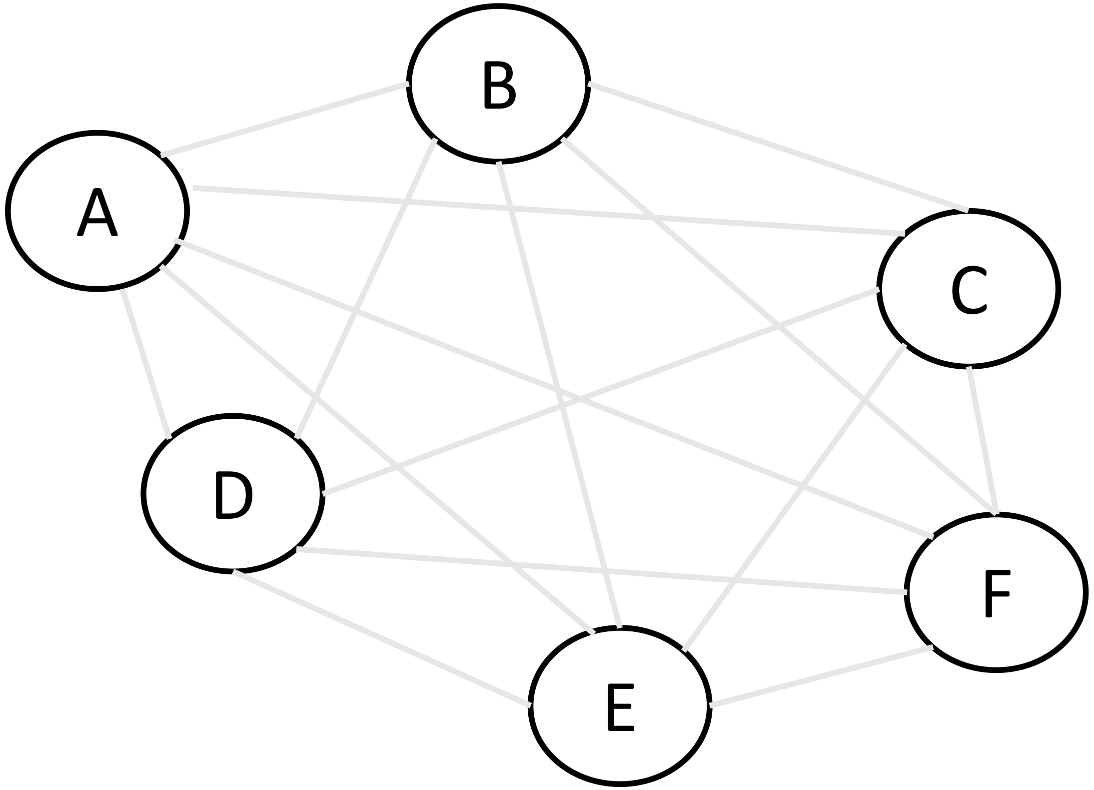
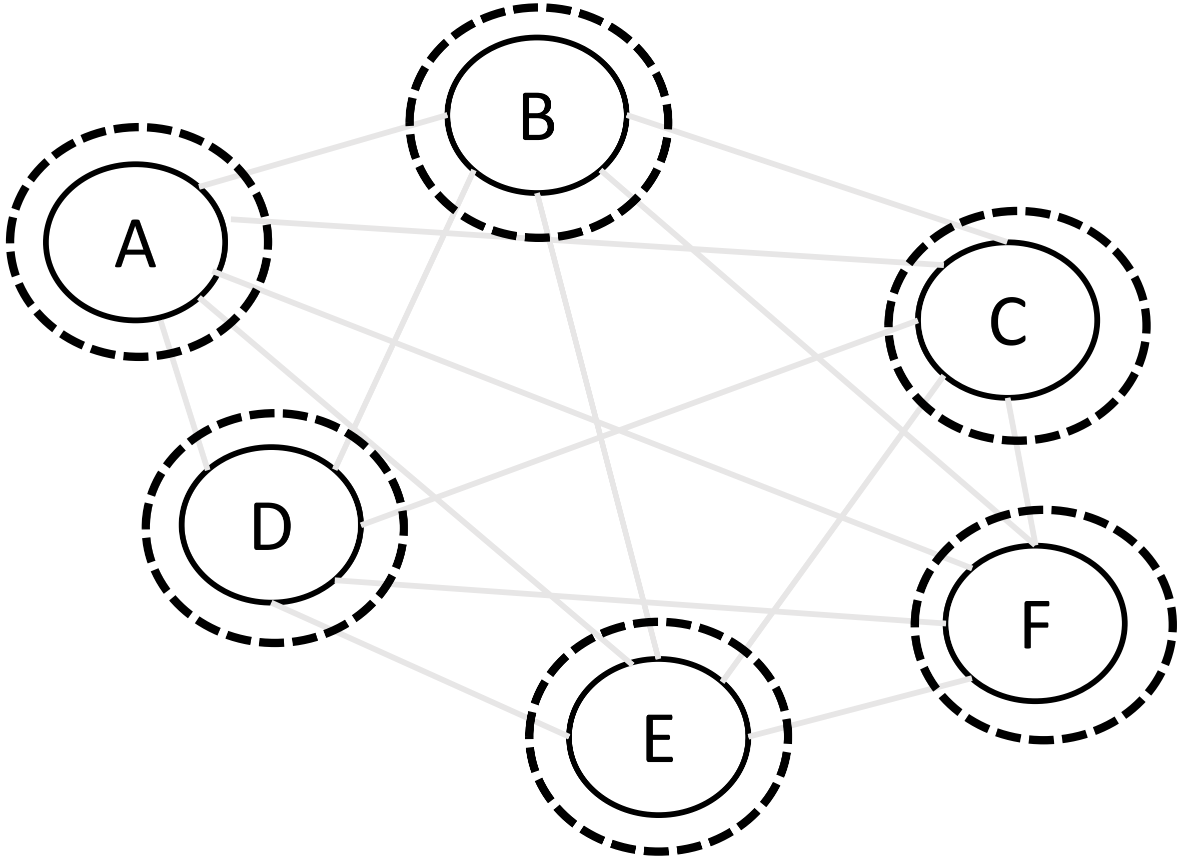
We imagined an instance of problem…



$k = 3$

And reasoned about a solution $\mathcal{G}$ made by our algorithm:



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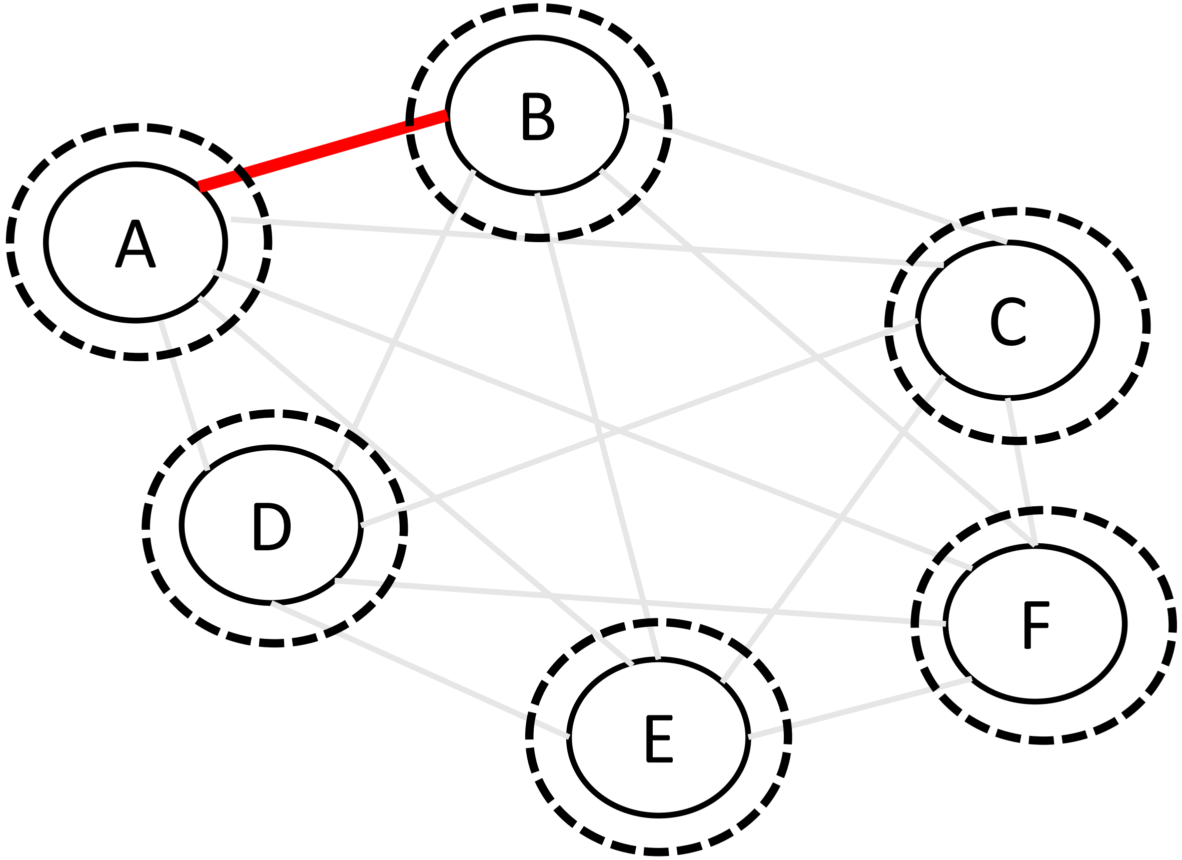
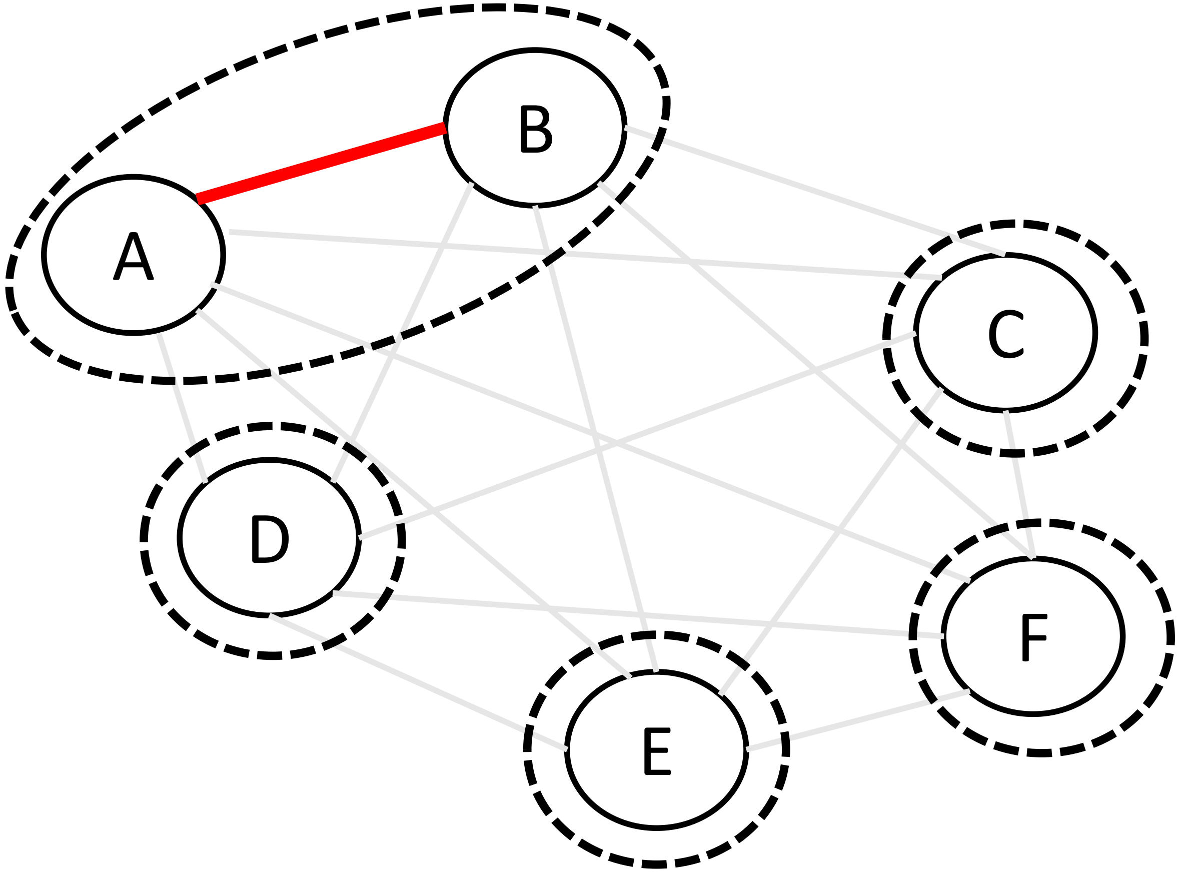


$k = 3$

Set of red edges = E' (picked in step 4(a))

And reasoned about a solution $\mathcal{G}$ made by our algorithm:



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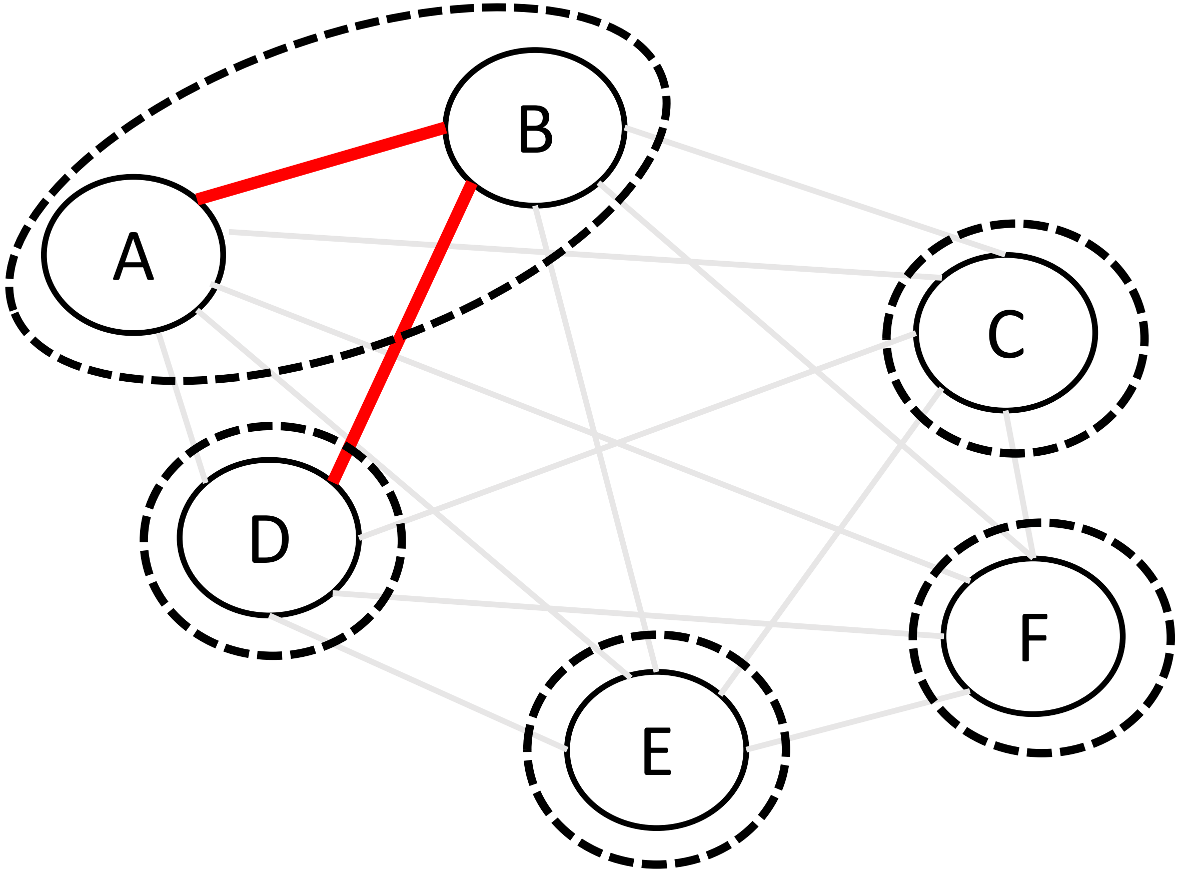
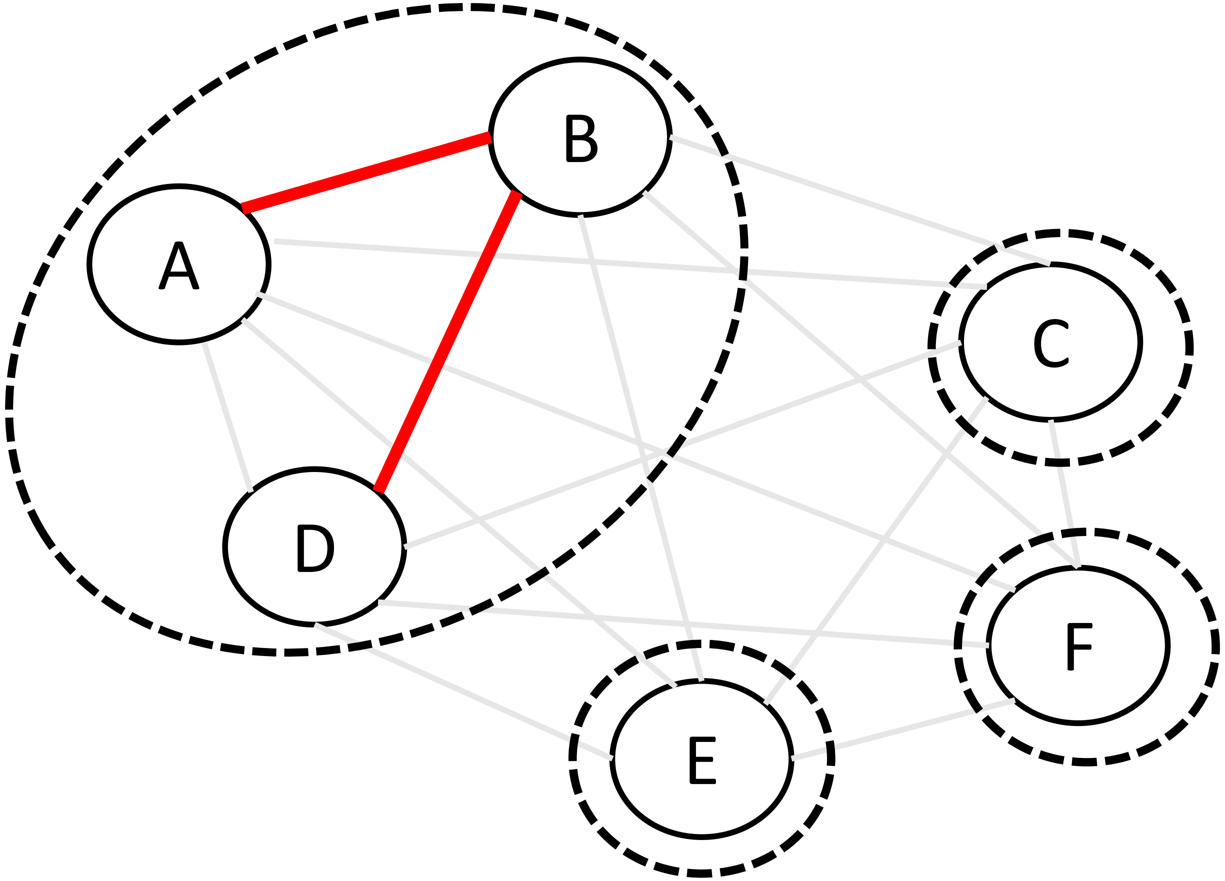


$k = 3$

Set of red edges = E' (picked in step 4(a))

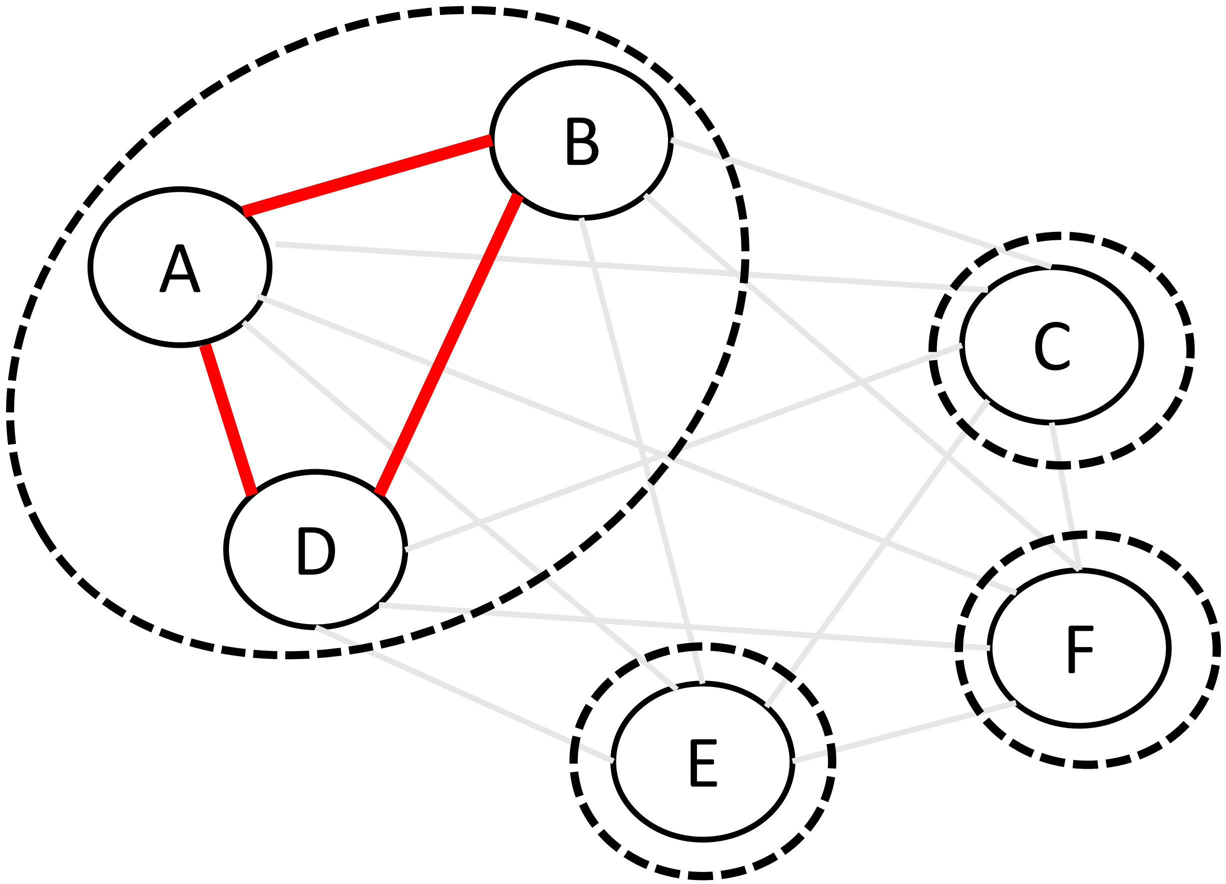
And reasoned about a solution $\mathcal{G}$ made by our algorithm:



$k = 3$

Set of red edges = E' (picked in step 4(a))

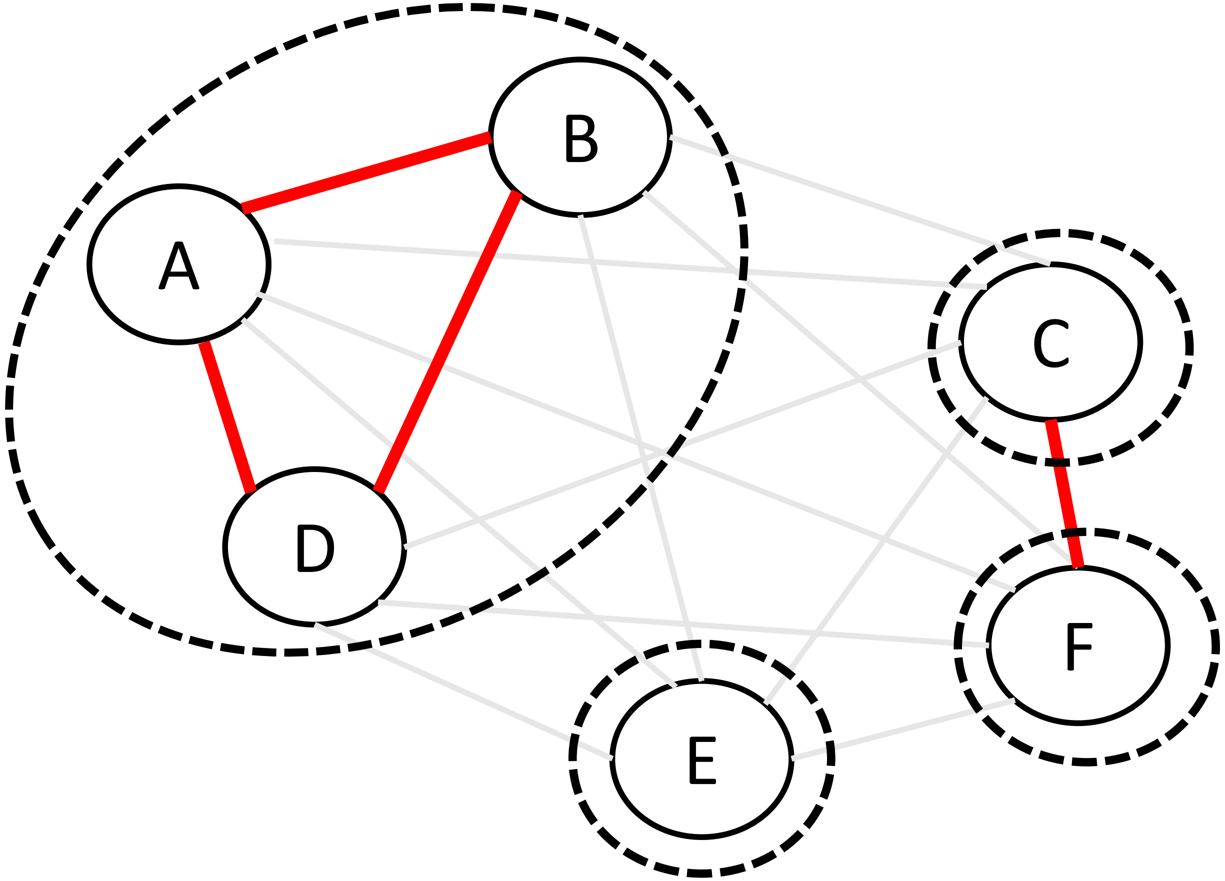
And reasoned about a solution $\mathcal{G}$ made by our algorithm:



$k = 3$

Set of red edges = E' (picked in step 4(a))

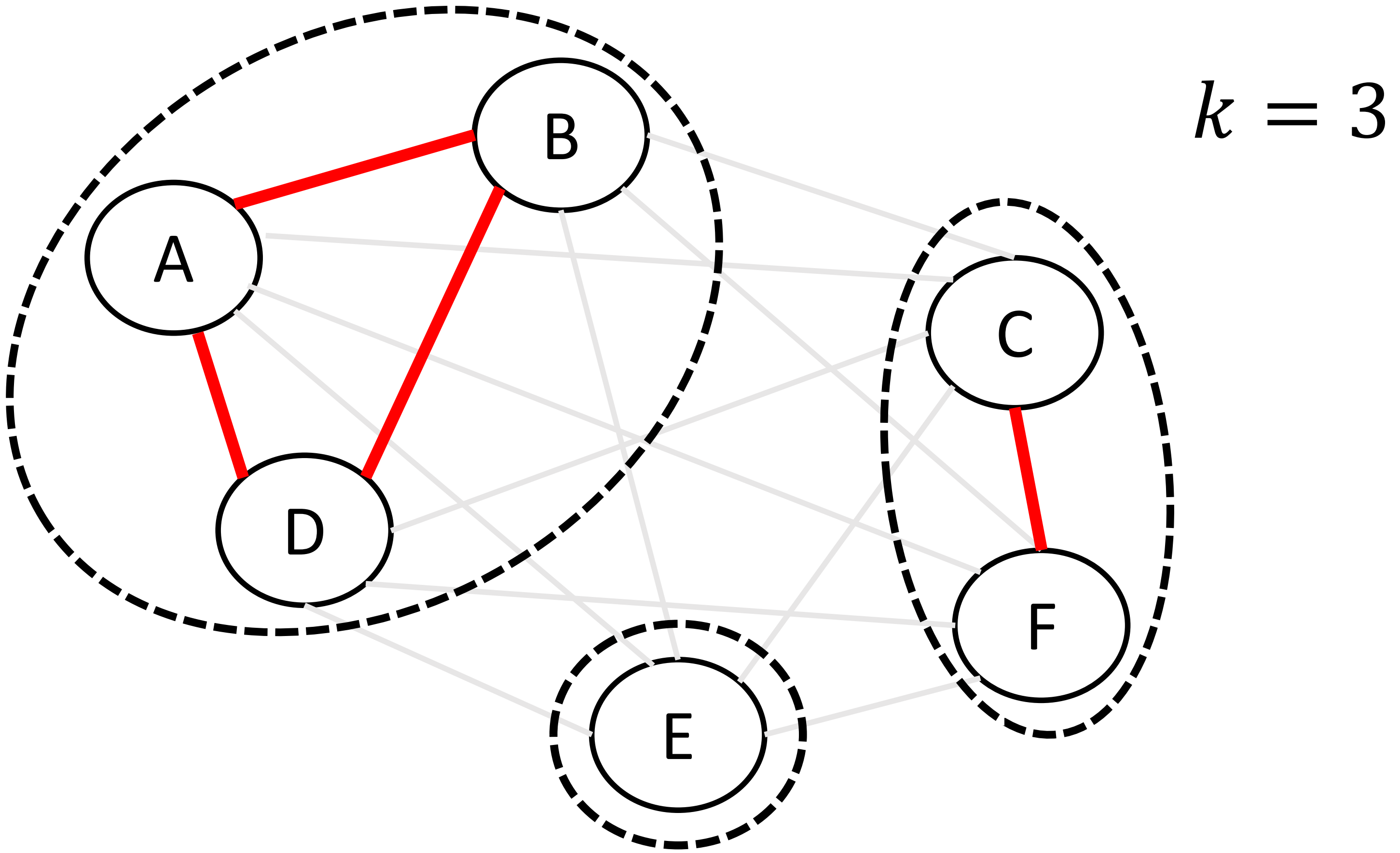
And reasoned about a solution $\mathcal{G}$ made by our algorithm:



$k = 3$

Set of red edges = E' (picked in step 4(a))

And reasoned about a solution $\mathcal{G}$ made by our algorithm:



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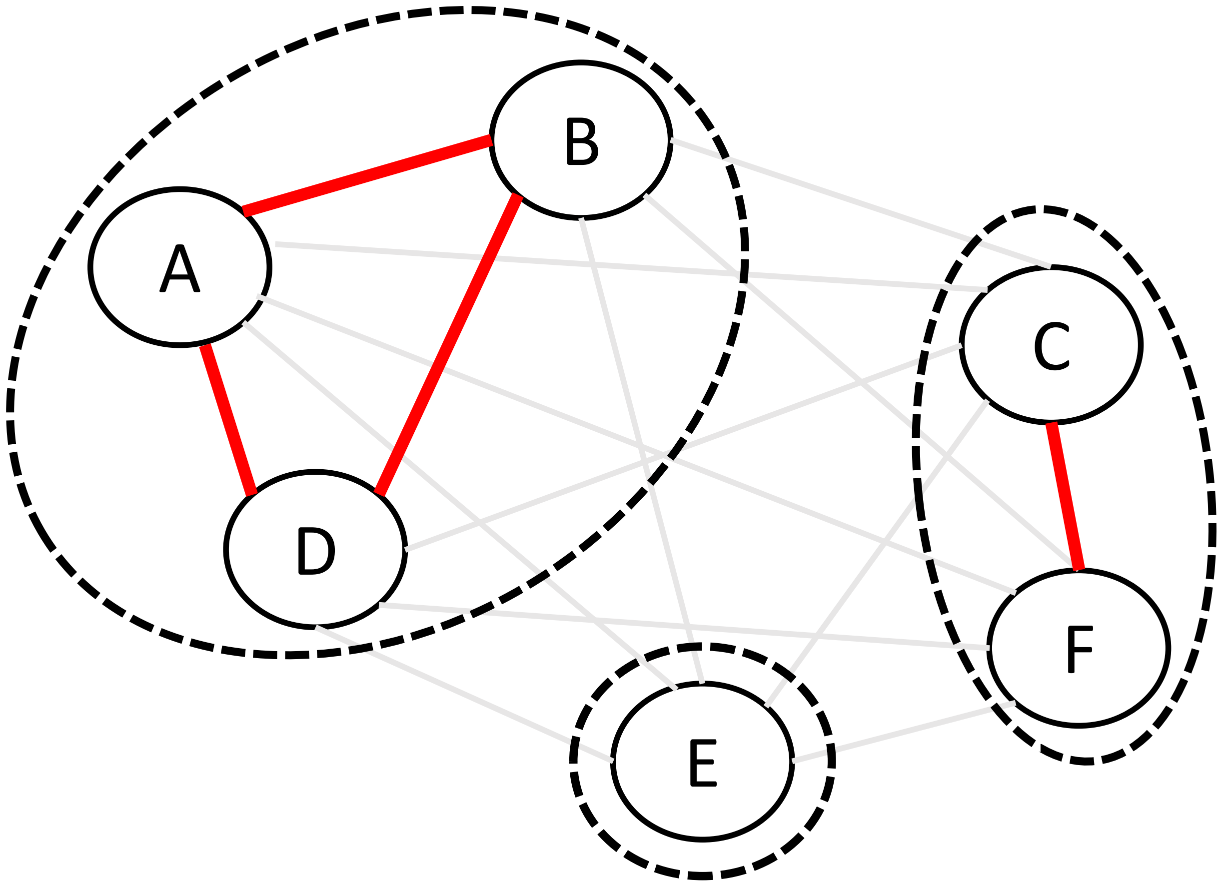
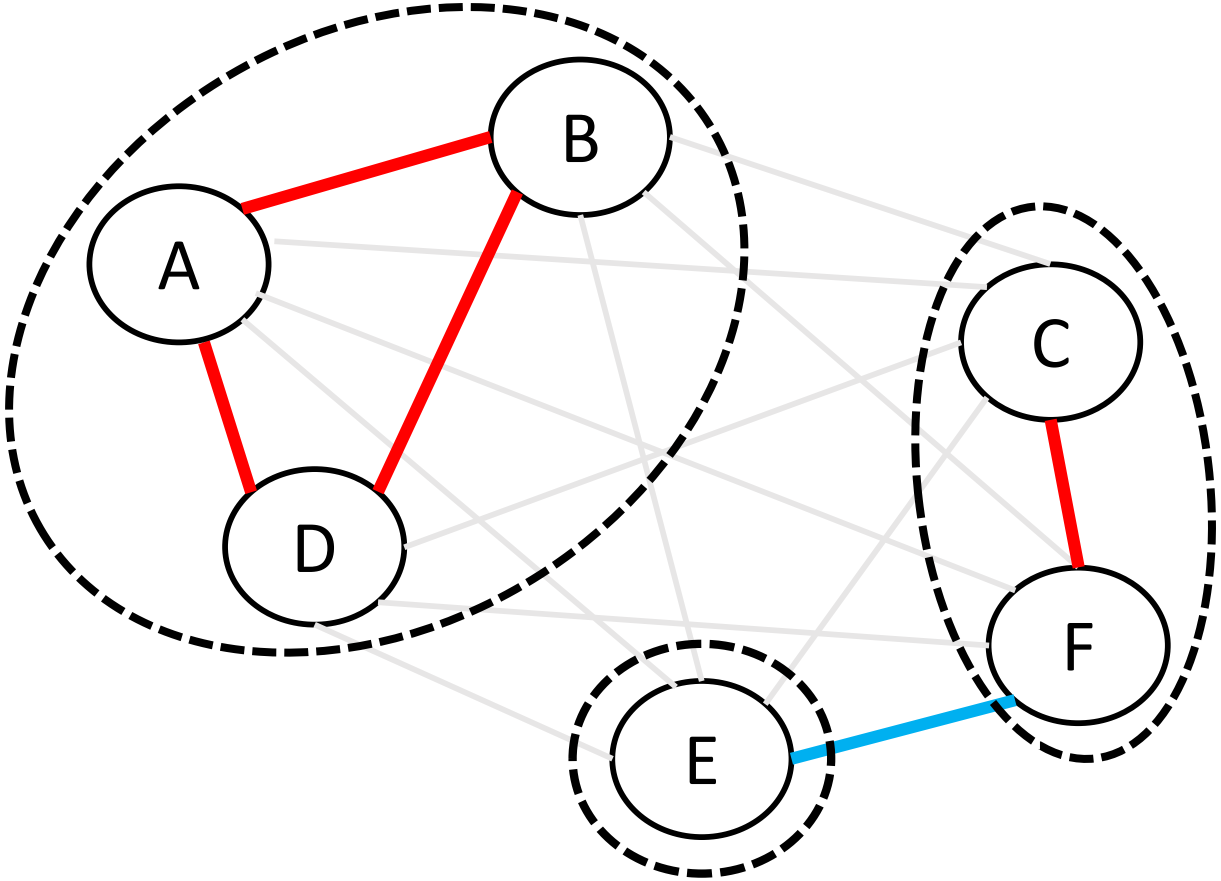


$k = 3$

Done! A complete greedy solution

Set of red edges = E' (picked in step 4(a))

And reasoned about a solution $\mathcal{G}$ made by our algorithm:

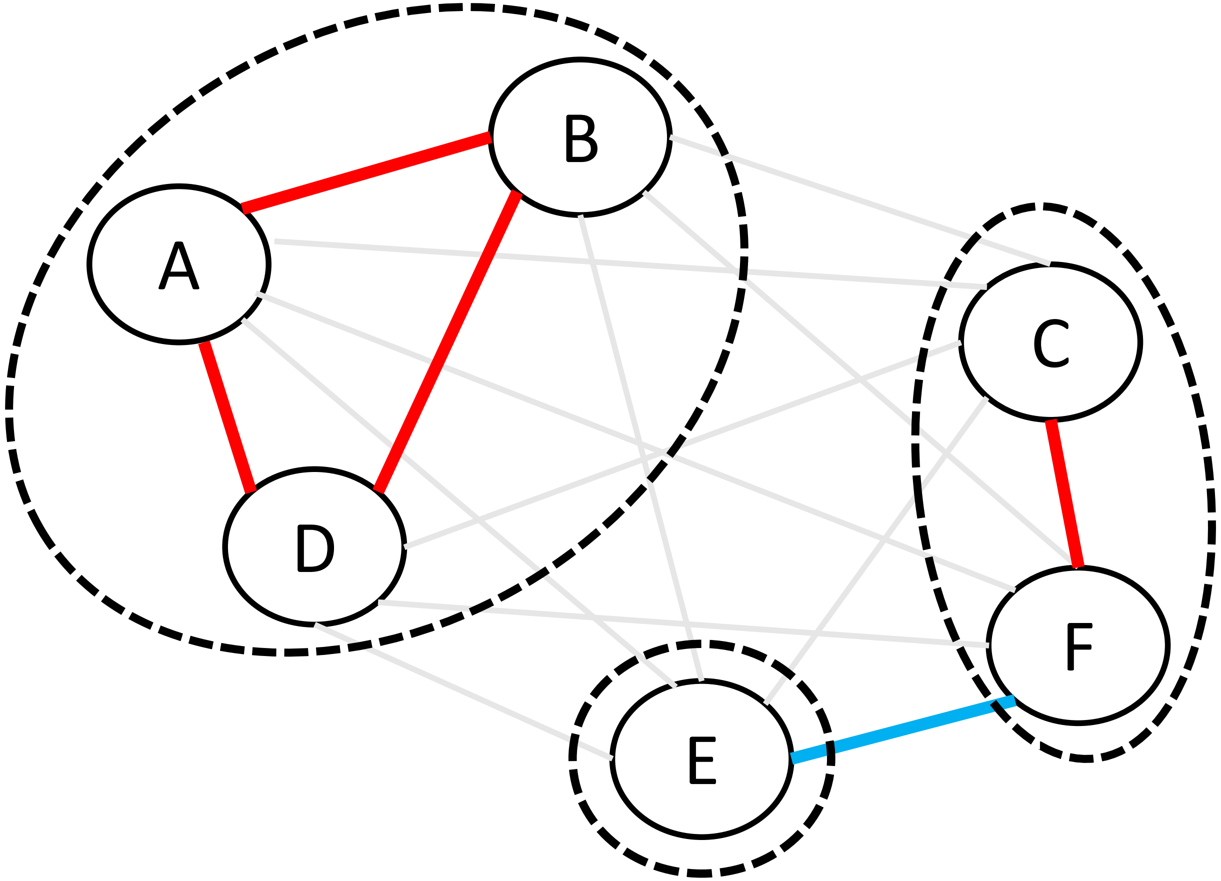


$k = 3$

Set of red edges = E' (picked in step 4(a))

The blue edge defines the cost of our greedy solution

And reasoned about a solution $\mathcal{G}$ made by our algorithm:

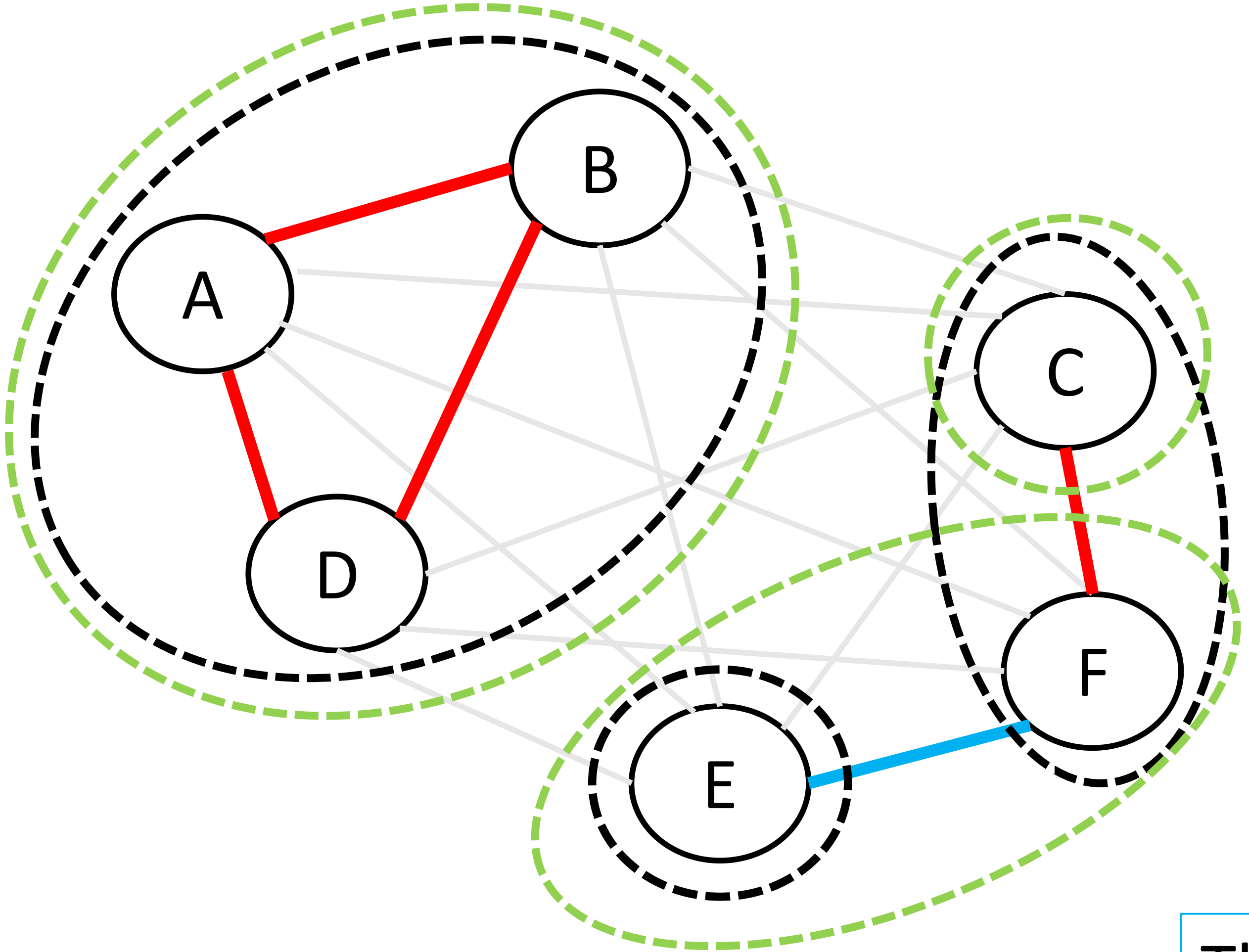


Proceeding with the proof: compare $\mathcal{G}$ to a **different** optimal solution $\mathcal{O}$

Set of red edges = E' (picked in step 4(a))

The blue edge defines the cost of our greedy solution

Case 1: some edges of E' are inter-category in $\mathcal{O}$

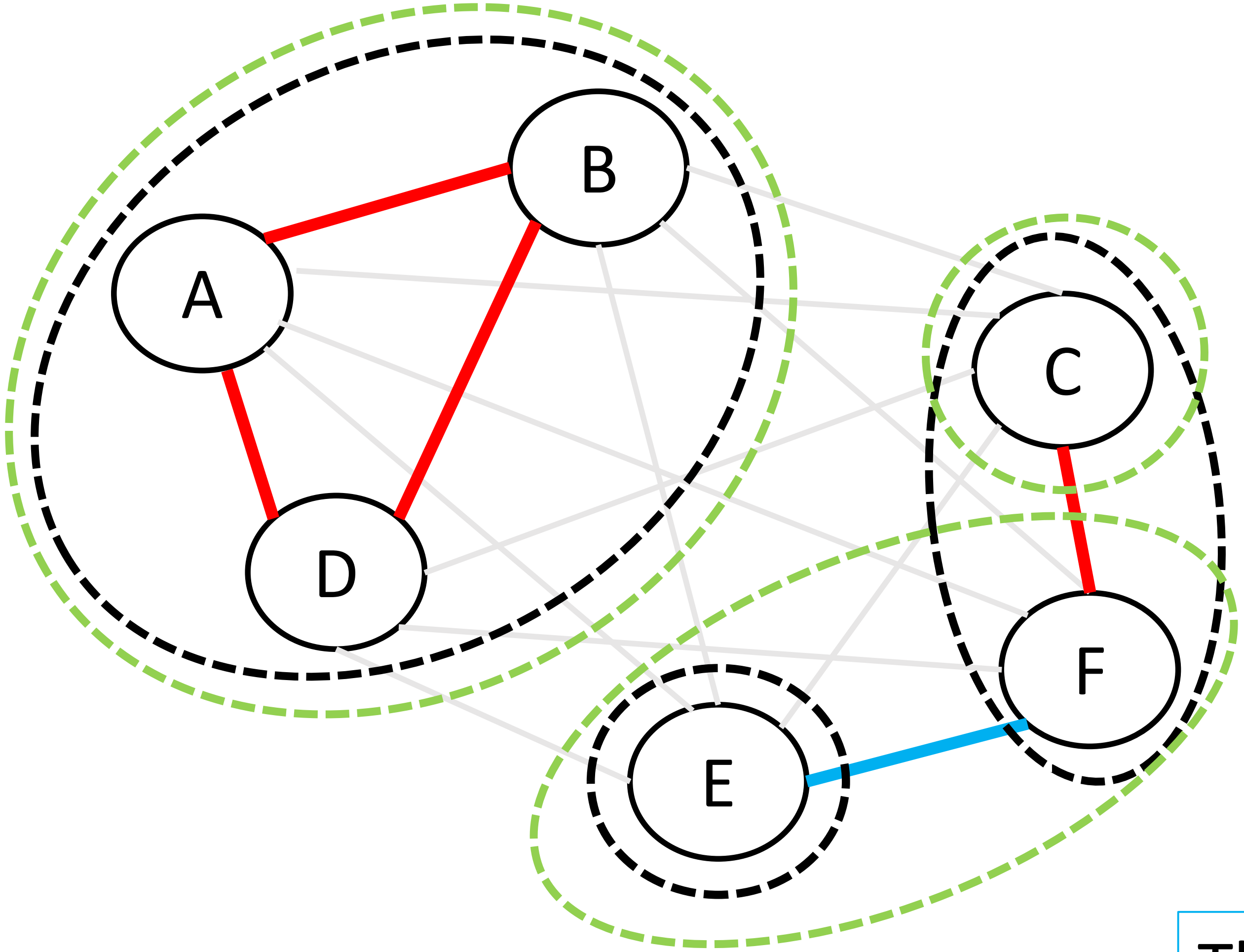


Example: $\mathcal{O}$ is in green.

The blue edge defines the cost of our greedy solution

Set of red edges = E' (picked in step 4(a))

Case 1: some edges of E' are inter-category in $\mathcal{O}$



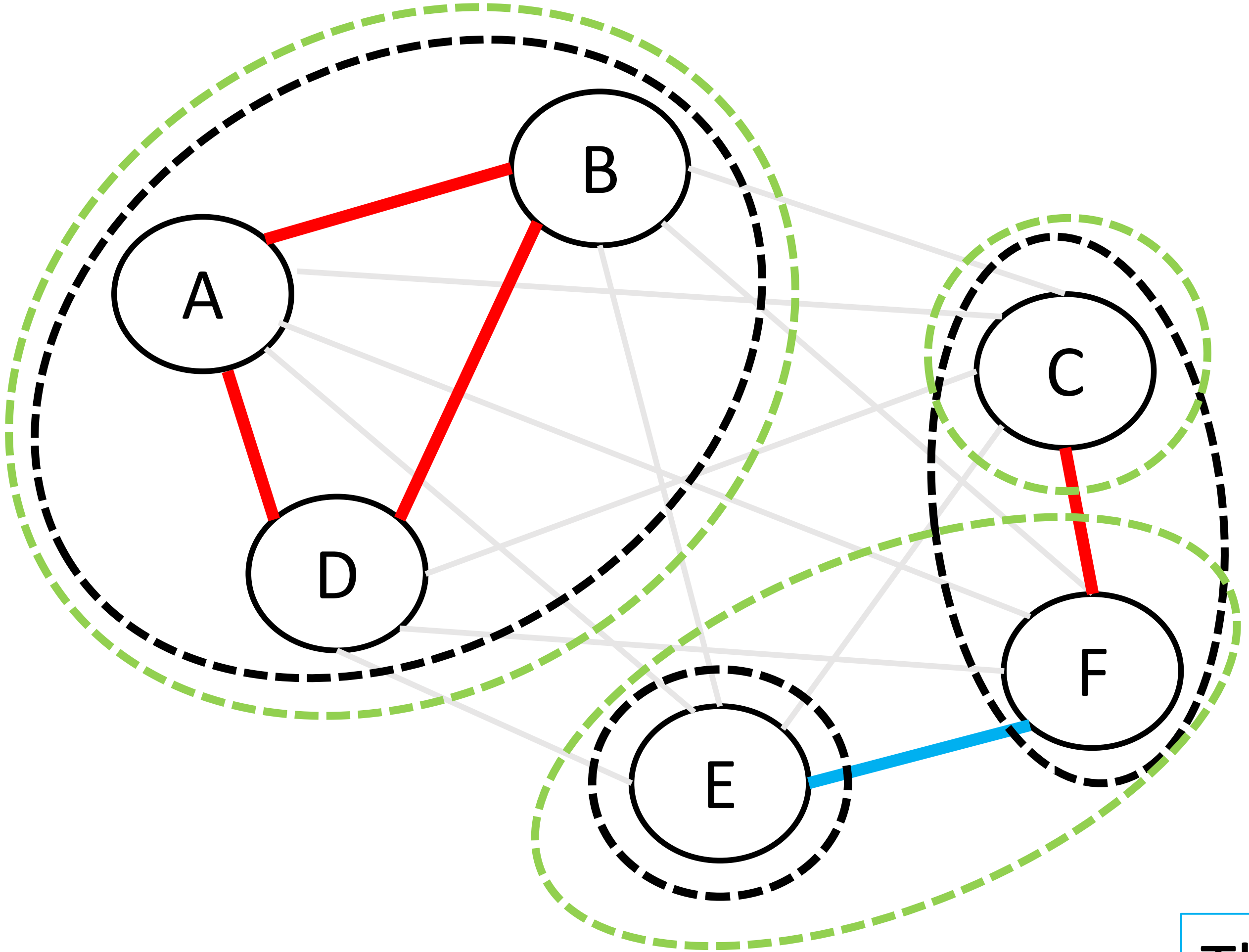
Example: $\mathcal{O}$ is in green.
Which edge defines Cost($\mathcal{O}$)?

A. (C,F)
B. (E,F)
C. Another red edge
D. Impossible to determine

Set of red edges = E' (picked in step 4(a))

The blue edge defines the cost of our greedy solution

Case 1: some edges of E' are inter-category in $\mathcal{O}$



Example: $\mathcal{O}$ is in green. (C,F) defines Cost($\mathcal{O}$). How does the weight of (C,F) compare to the weight (E,F)?

A. weight(C,F) $\leq$ weight(E,F)

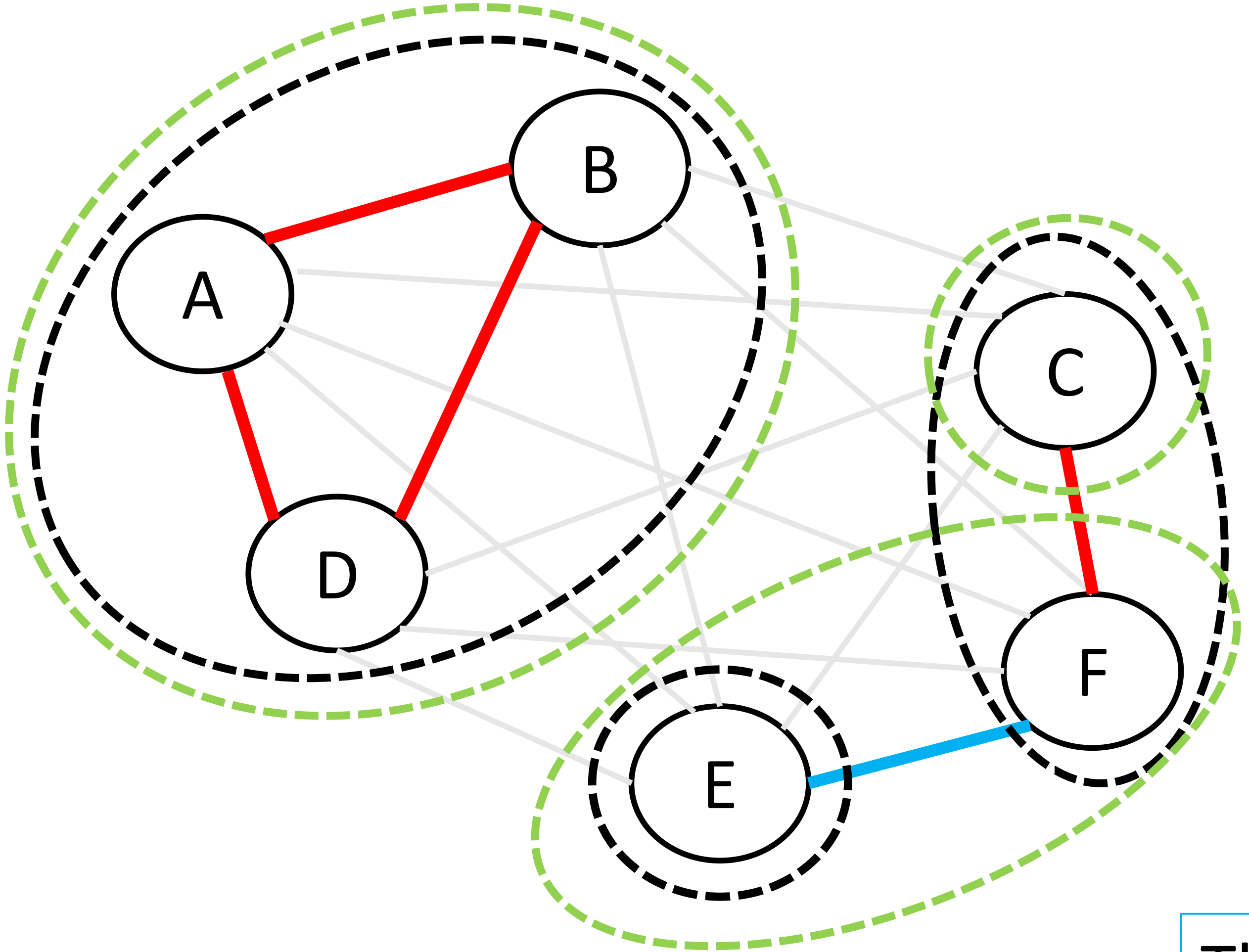
B. weight(C,F) $\geq$ weight(E,F)

C. Impossible to determine

Set of red edges = E' (picked in step 4(a))

The blue edge defines the cost of our greedy solution

Case 1: some edges of E' are inter-category in $\mathcal{O}$

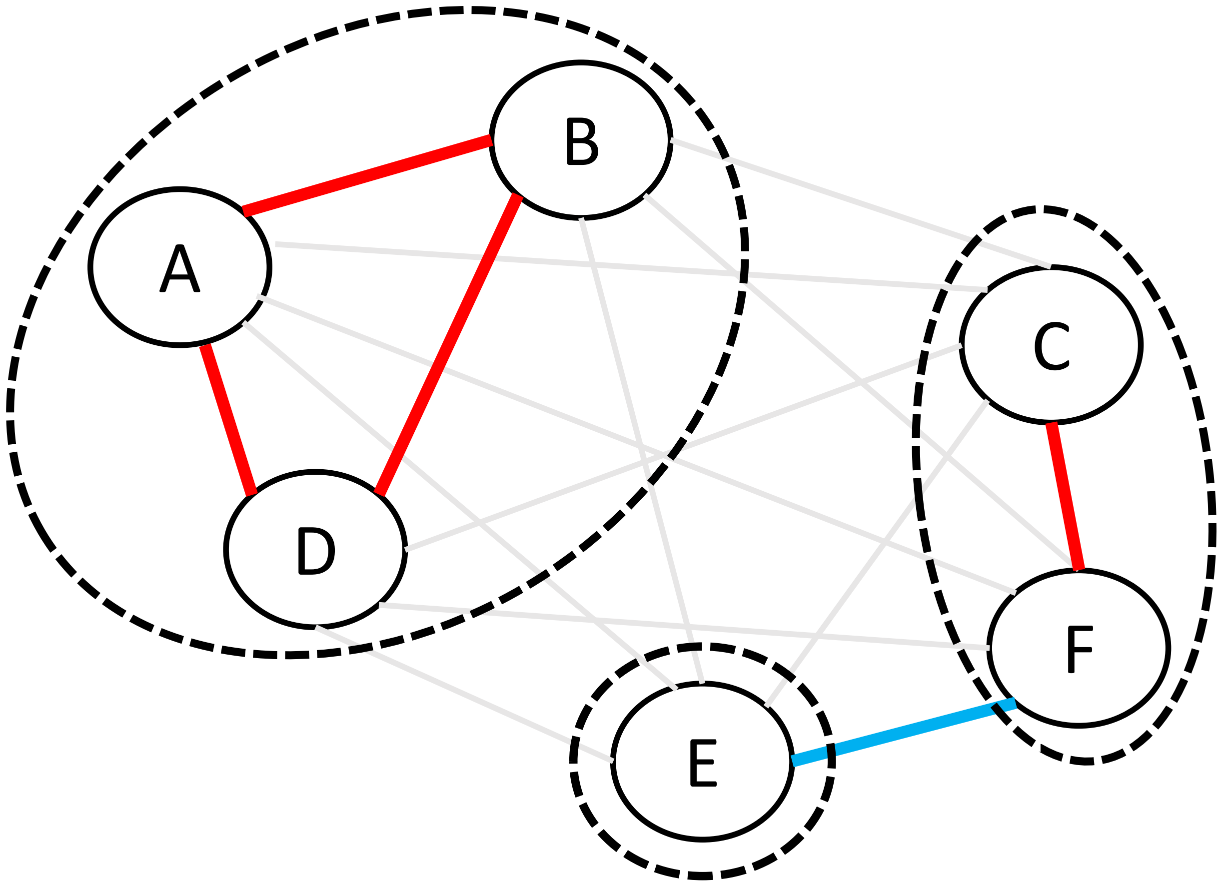


Example: $\mathcal{O}$ is in green.
(C,F) defines Cost($\mathcal{O}$), and has **equal or greater weight** than the blue edge:
$\rightarrow$ Cost($\mathcal{G}$) $\leq$ Cost($\mathcal{O}$)

Set of red edges = E' (picked in step 4(a))

The blue edge defines the cost of our greedy solution

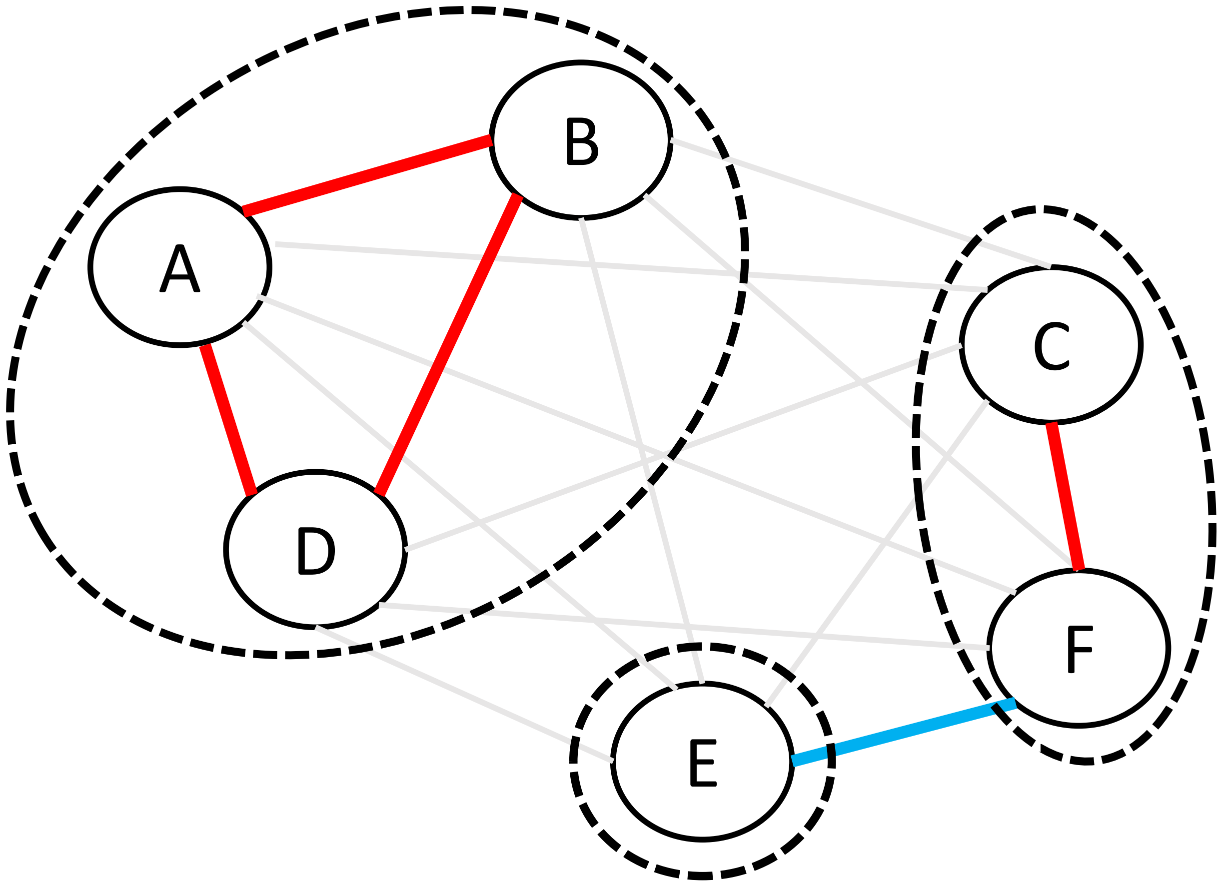
Case 2: all edges of E' are intra-category in $\mathcal{O}$



Set of red edges = E' (picked in step 4(a))

The blue edge defines the cost of our greedy solution

Case 2: all edges of E' are intra-category in $\mathcal{O}$

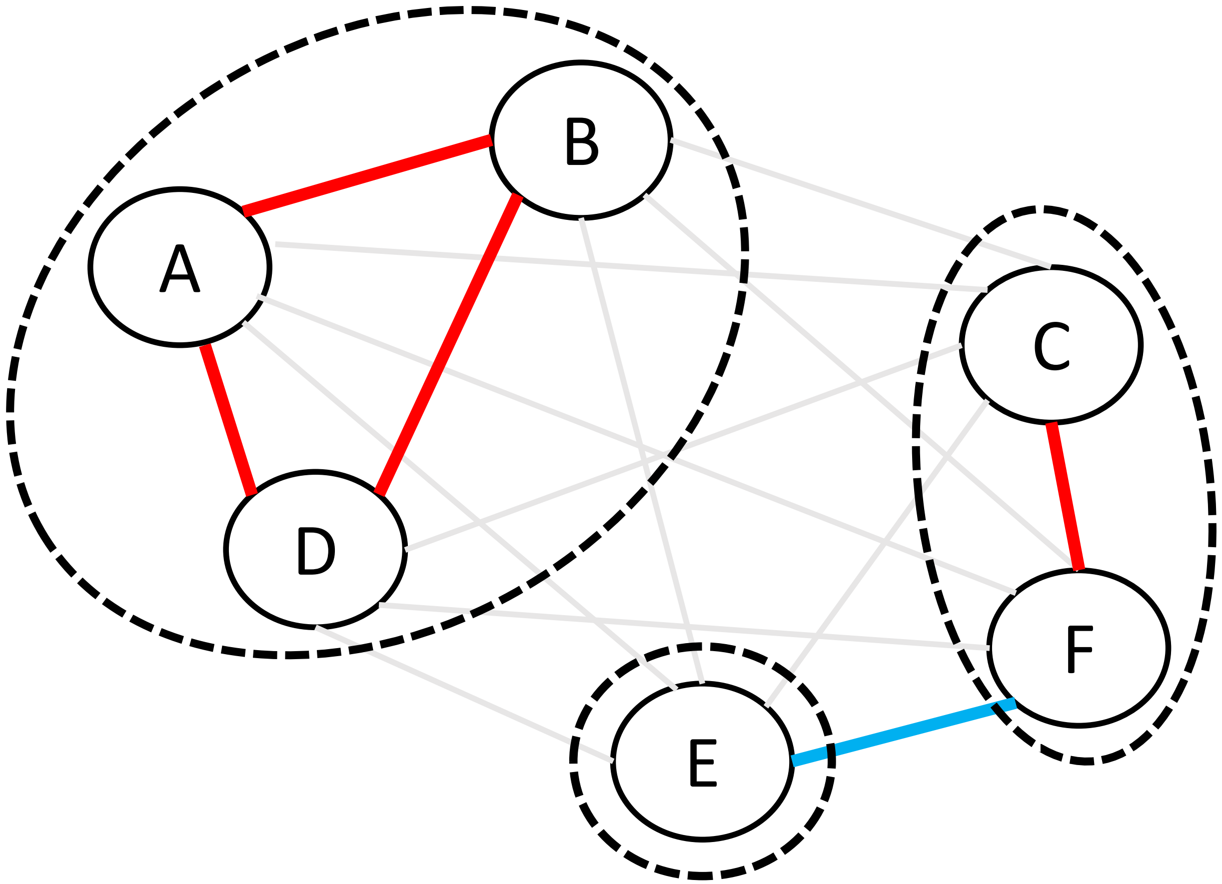


...Then $\mathcal{O}$ can't have **extra** intra-category edges that were inter-category in $\mathcal{G}$.

Set of red edges = E' (picked in step 4(a))

The blue edge defines the cost of our greedy solution

Case 2: all edges of E' are intra-category in $\mathcal{O}$

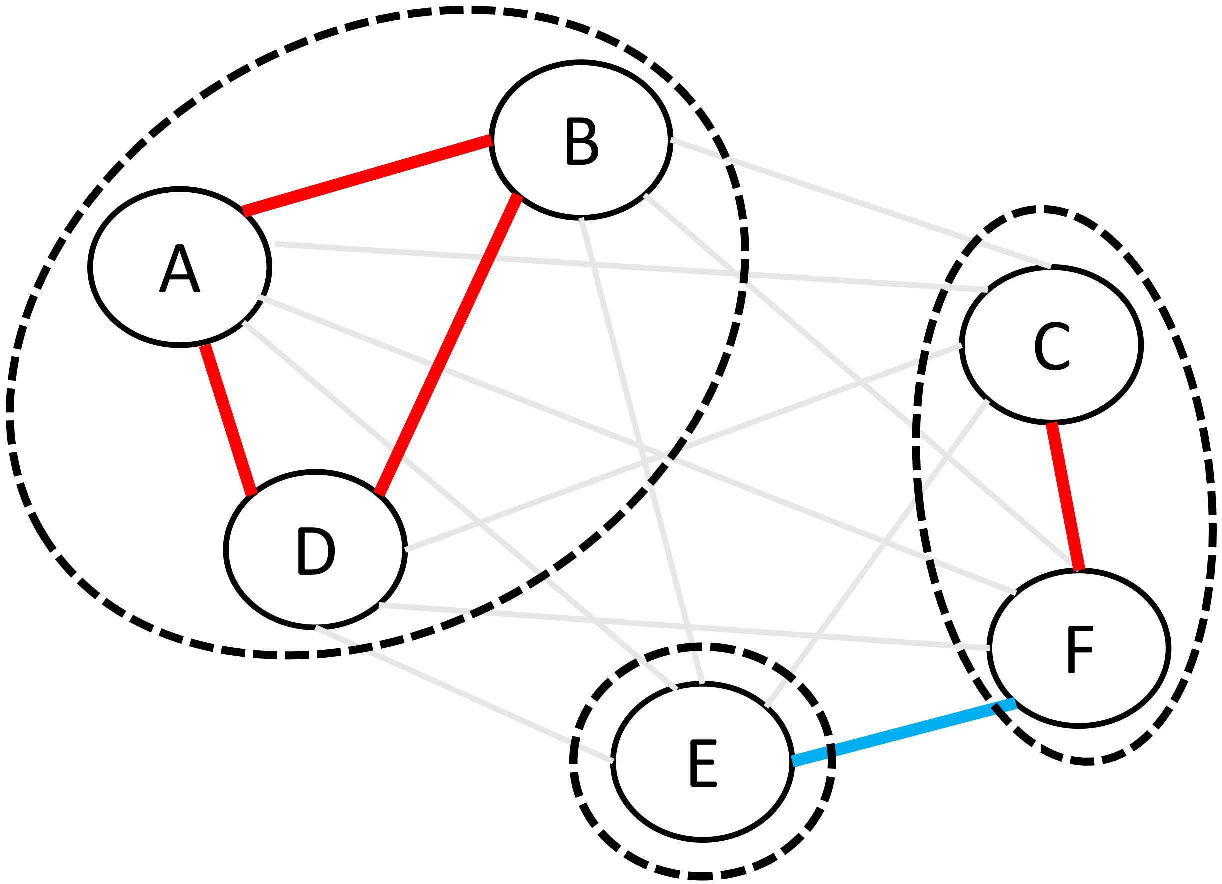


...Then $\mathcal{O}$ can't have **extra** intra-category edges that were inter-category in $\mathcal{G}$.
→ all inter-category edges in $\mathcal{G}$ are inter-category in $\mathcal{O}$

Set of red edges = E' (picked in step 4(a))

The blue edge defines the cost of our greedy solution

Case 2: all edges of E' are intra-category in $\mathcal{O}$

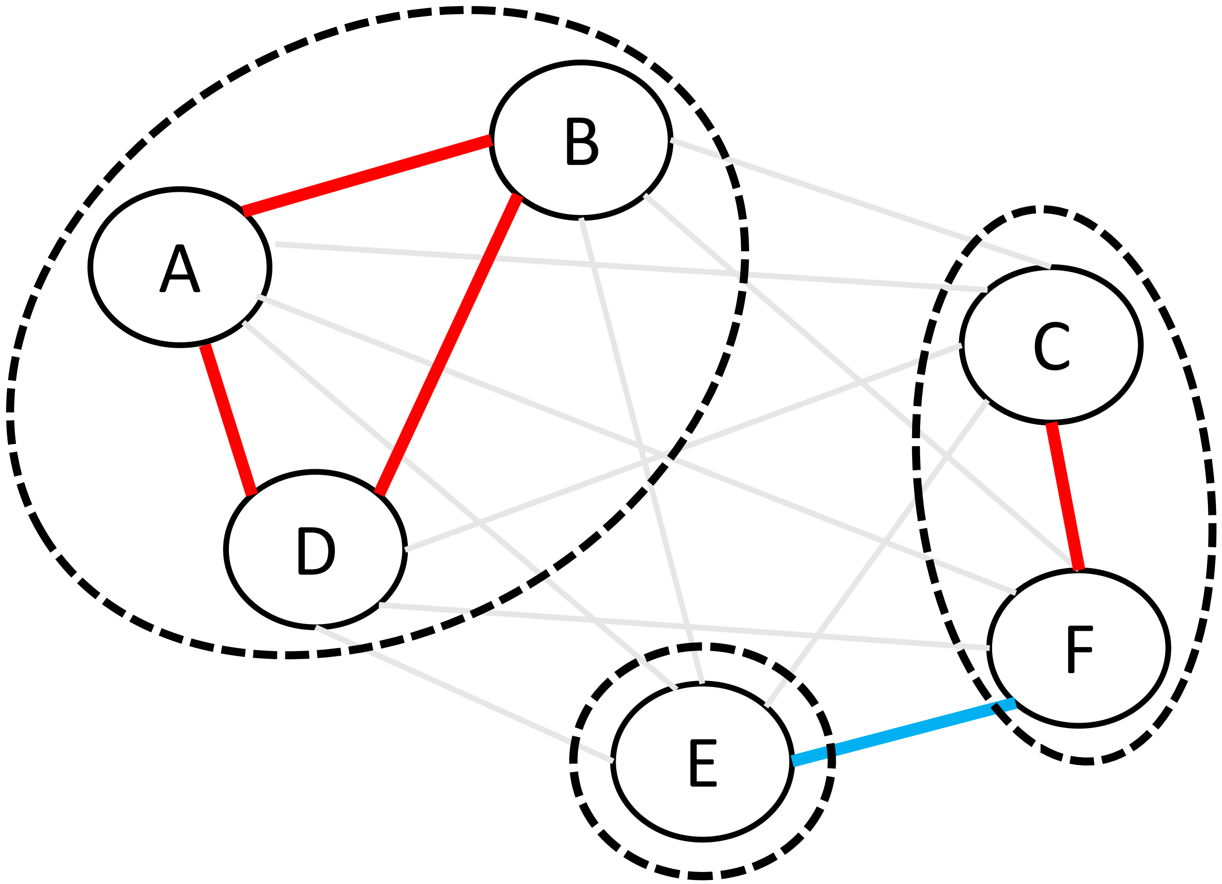


…Then $\mathcal{O}$ can't have **extra** intra-category edges that were inter-category in $\mathcal{G}$.
→ all inter-category edges in $\mathcal{G}$ are inter-category in $\mathcal{O}$
→ Cost($\mathcal{G}$) $\leq$ Cost($\mathcal{O}$)

Set of red edges = E' (picked in step 4(a))

The blue edge defines the cost of our greedy solution

Summary

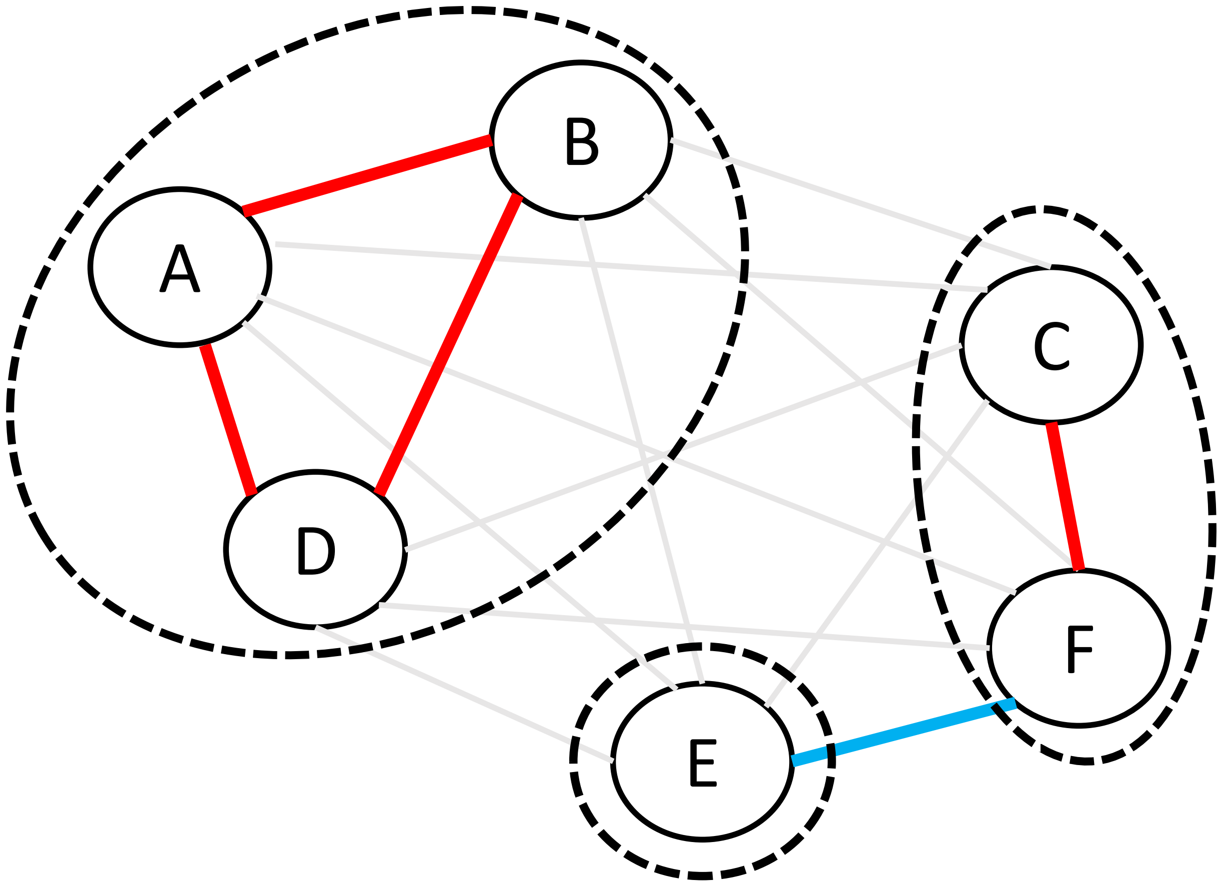


When some edges of E' are inter-category in $\mathcal{O}$:
$\text{Cost}(\mathcal{G}) \leq \text{Cost}(\mathcal{O})$

Set of red edges = E' (picked in step 4(a))

The blue edge defines the cost of our greedy solution

Summary



When **some** edges of E' are inter-category in $\mathcal{O}$:

$\text{Cost}(\mathcal{G}) \leq \text{Cost}(\mathcal{O})$

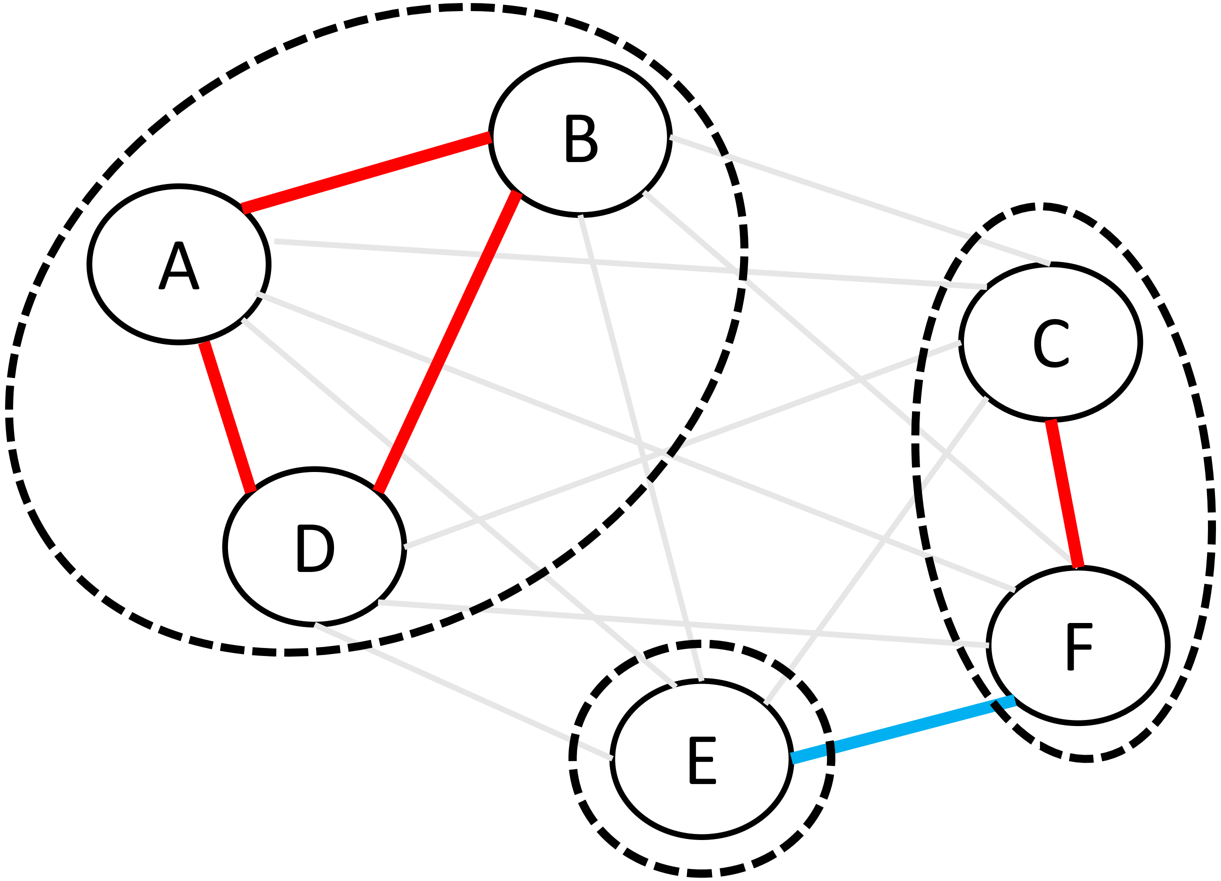
When **no** edges of E' are inter-category in $\mathcal{O}$:

$\text{Cost}(\mathcal{G}) \leq \text{Cost}(\mathcal{O})$

Set of red edges = E' (picked in step 4(a))

The blue edge defines the cost of our greedy solution

Summary



When **some** edges of E' are inter-category in $\mathcal{O}$:

$\text{Cost}(\mathcal{G}) \leq \text{Cost}(\mathcal{O})$

When **no** edges of E' are inter-category in $\mathcal{O}$:

$\text{Cost}(\mathcal{G}) \leq \text{Cost}(\mathcal{O})$

....QED, $\text{Cost}(\mathcal{G}) \leq \text{Cost}(\mathcal{O})$, which means $\mathcal{G}$ is optimal.

Set of red edges = E' (picked in step 4(a))

The blue edge defines the cost of our greedy solution