Review of Mathematical Proof Techniques

- **Direct proof:** start with the assumptions or definitions and apply some reasoning to obtain the conclusion. May involve breaking the proof into cases, or proving a logically equivalent statement.
  - **Exercise:** Prove that the square of any odd number is odd.

- **Proof by contradiction:** assume that the conclusion is false and obtain a contradiction with an assumption or some known facts. In CPSC 320, this is frequently (but not always!) the easiest way to prove a statement.
  - **Exercise:** Let a and b be two real numbers. Prove that if a is rational and ab is irrational, then b is irrational.

- **Proof by contrapositive:** Instead of proving the statement "if A, then B", we prove the logically equivalent *contrapositive* statement "if not B, then not A." In CPSC 320, this is often useful in proving the correctness of a reduction (as we saw in the RHP example).
  - **Exercise:** Prove that if x^2 is even, then x must be even.

- **Proof by induction:** used to prove that a statement holds for every natural number ($n = 0, 1, \ldots$). There are three components: first, the *base case*, in which you establish that the statement holds for some small, trivial case $n = n_0$ (usually $n_0 = 0$ or $n_0 = 1$). Then, the *inductive hypothesis* **assumes** that the statement holds for $n = k$ (weak induction) or for all $n \leq k$ (strong induction) and we show in the *inductive step* that, given the inductive hypothesis, the statement must hold for $n = k + 1$. We can then conclude that it's true for all natural numbers n.
  - **Exercise:** Prove that the first n odd integers sum to n^2.