we have seen parametric equations for lines \( \mathbf{r}(t) = \mathbf{r}_0 + t \mathbf{v} \). More generally, \( \mathbf{r}(t) \) can describe a curve in \( \mathbb{R}^3 \).

\[
\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle
\]

**Example** (i) equation of a line

(ii) \( \mathbf{r}(t) = \langle \cos t, \sin t, t/4\pi \rangle \)

\[
\frac{d}{dt} \mathbf{r}(t) = \mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle
\]

The derivative of a parametrized curve is the velocity of a particle following \( \mathbf{r}(t) \).
From this, we can get the tangent line to \( \mathbf{r}(t) \) at the point \( \mathbf{r}(c) \).

\[
\mathbf{r}(s) = \mathbf{r}(c) + s \mathbf{r}'(c)
\]

Fixed point \( \mathbf{r}(c) \)
Fixed vector \( \mathbf{r}'(c) \)

New parameter.

(!) This is not present in the Apex book at definition 71)

Tangent line

Eg \( s = t - c \)
where does this line intersects the y-z plane?

\( y = -8 \\ z = +9 \)

This was done in lecture 4

The point where the line intersects the y-z plane is \((0, -8, 9)\).

Is there a similar way to determine a plane (with vectors)?

\[ \hat{n} = \langle a, b, c \rangle \]

A plane is determined by a point on it \((x_0, y_0, z_0)\) and a normal to this point.

This vector is \(\langle x-x_0, y-y_0, z-z_0\rangle\).
\( (x, y, z) \) is on the plane determined by \( \vec{n} \) and \( (x_0, y_0, z_0) \) if and only if the vector \( \langle x-x_0, y-y_0, z-z_0 \rangle \cdot \langle a, b, c \rangle = 0 \)

\[ \implies \quad ax + by + cz + d = 0 \]

with \( d = -ax_0 - by_0 - cz_0 \)

**Example:** Find the equation of the plane passing through the points \((0,0,3)\), \((0,2,0)\) and \((1,0,0)\)

\[ \vec{a} = (-1, 2, 0), \quad \vec{b} = (-1, 0, 3) \]

\[ \vec{n} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 6\hat{i} + 3\hat{j} + 2\hat{k} \]
any point \((x,y,z)\) on the plane satisfies

\[ 6(x-x_0) + 3(y-y_0) + 2(z-z_0) = 0 \]

pick \(x_0 = 0\) or \(x_0 = 0\) or \(x_0 = 1\)
\(y_0 = 0\) or \(y_0 = 2\) or \(y_0 = 0\)
\(z_0 = 3\) or \(z_0 = 0\) or \(z_0 = 0\)

\[ \Rightarrow 6x - 6 + 3y + 2z = 0 \]
Problem: Find the distance between the two parallel planes: \( x+y-2z=2 \) and \( x+y-2z=4 \).

\[ \text{Plane 1: } x+y-2z=2 \]
\[ \text{Plane 2: } x+y-2z=4 \]

Choose any point on the plane for \( P \). We choose \( \vec{OP} = \langle 2,0,0 \rangle \). The desired distance is \( |\vec{PA}| \).

The line through \( P \) and \( Q \) is:

\[ \vec{r}(t) = \vec{OP} + t \vec{v} \]

with \( \vec{v} = \langle 1,1,-2 \rangle \) (from the equations of the planes)

\[ \vec{r}(t) = \langle 2,0,0 \rangle + t \langle 1,1,-2 \rangle \]

Any point \( x,y,z \) on the line is:

\[ \vec{r}(t) = \langle x,y,z \rangle = \langle 2+t, t, -2t \rangle \]
Now find the value $t = t_q$ which corresponds to the point $Q$.

$$\vec{r}(t_q) = (x_q, y_q, z_q) = (2 + t_q, t_q, -2t_q)$$

As the point $Q$ is on the plane, it satisfies:

$$x_q + y_q - 2z_q = 4$$

So it comes: (inject into the equation above)

$$2 + t_q + t_q - 2(-2t_q) = 4 \Rightarrow 6t_q = 2$$

$$\Rightarrow t_q = \frac{1}{3}$$

$$|\vec{PQ}| = |\vec{r}(t_q) - \vec{r}(\infty)| = |\vec{r}(t = 0) + t_q\vec{e}(1, 1, -2) - \vec{r}(\infty)|$$

$$|\vec{PQ}| = \sqrt{(\frac{1}{3})^2 + (\frac{1}{3})^2 + (-2)^2} = \sqrt{\frac{4}{3}} = \sqrt{\frac{2}{3}}$$

$$|\vec{PQ}| = \sqrt{\frac{2}{3}}$$