In the $xy$ plane, a curve given by an equation involving $x^2, xy, y^2, x, y$ or const are given by circles, ellipses, parabolas or hyperbolas. These are quadric curves.

**Example:** \( \frac{x^2 + y^2}{4} = \frac{9}{9} \)  

\[ \Rightarrow \text{ellipse} \]

or the circle defined by \( x^2 + y^2 + 2x - 6y = 0 \)  

\[ \Rightarrow (x+1)^2 + (y-3)^2 = (\sqrt{10})^2 \]

or \( x^2 - y^2 = 1 \)
\[ y^2 = 1 - \frac{1}{x^2} \text{ and as } x \to \infty \quad y^2 \to 1 \]

so \( y \sim \frac{1}{x} \) are the two asymptotes.

or \( xy = 1 \)

Here the asymptotes are \( x = 0 \) and \( y = 0 \)

\[ \Rightarrow \text{ in 3D, a quadratic surface is a surface given by an equation involving } x^2, y^2, z^2, xy, yz, \]
\[ ax, x, y, z, \text{ const.} \]

Example: Sphere \((x-a)^2 + (y-b)^2 + (z-c)^2 = r^2\)

* Ellipsoid \(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1\)
If the equation involves only 2 variables instead of three, then the surface is a 3
\[ x^2 + \frac{y^2}{a^2} = 1 \]

\[ y^2 + z^2 = 1 \]

\[ z = \pi c \]
to sketch more general surfaces we use trace curves that we get when we cut the surface by planes parallel to coordinate planes.

**Example**: use of trace curves

$z = y^2 - x^2$

$x = 0 \Rightarrow z = y^2$

$z = 1 \Rightarrow 1 = y^2 - x^2 \Rightarrow y = \sqrt{1 + x^2}$

$z = 0 \Rightarrow y^2 - x^2 = 0 \Rightarrow (y - x)(y + x)$

$y = 0 \Rightarrow z = -x^2$

$z = -1 \Rightarrow y = \sqrt{1 - x^2}$
horizontal traces $\Rightarrow$ hyperbolas
vertical traces $\Rightarrow$ parabolas

$\Rightarrow$ cutting by horizontal planes give us conhar curves