Definition: A function $f$ of 2 variables with domain $D \subseteq \mathbb{R}^2$ is a rule that assigns to each $(x,y) \in D$ a unique number $f(x,y)$.

Examples of domains:

a) $f(x,y) = \sqrt{1-x^2-y^2}$
   $$D = \{ (x,y) : x^2+y^2 \leq 1 \}$$
   (you cannot have $x^2+y^2 = 1$)

b) $g(x,y) = \ln(x+y)$
   $$D = \{ (x,y) : x+y > 0 \}$$

Office hours: 1-2 pm 
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Important:
- Check the MLC
- Homework 1 is due.
- Office hours: 1-2 pm
- Come ask questions!
\[ h(x, y) = \frac{1}{x^2 + y^2} \]
\[ D = \{ (x, y) : (x, y) \neq (0, 0) \} \]

\[ b(x, y) = e^{x^2 + y^2} + \sin(y^4 + 3x) \quad D = \mathbb{R}^2 \]

d) Graphs of functions

Just like the one-variable case, we use graphs to understand functions.

=> Function of one variable:

\[ f(x) \]

=>

\[ y = f(x) \]
two-variables case

$\begin{align*}
&\text{graph is a surface} \\
&z = f(x, y)
\end{align*}$

examples: $f(x, y) = \sqrt{1-x^2-y^2}$

$\Rightarrow$ write $z = \sqrt{1-x^2-y^2}$

$D = \{ (x, y) : x^2 + y^2 \leq 1 \}$

then $z^2 = 1 - x^2 - y^2$

$\Rightarrow$ $x^2 + y^2 + z^2 = 1$

This is a unit sphere.
we also use contour plots: we draw the contours \( f(x, y) = k \) in the \( xy \) plane.

\[
\begin{align*}
k &= \sqrt{1 - x^2 - y^2} \\
k^2 &= 1 - x^2 - y^2 \\
x^2 + y^2 &= (1 - k^2)
\end{align*}
\]

Rightarrow radius of the circle is \( \sqrt{1 - k^2} \)

\Rightarrow \ k = 0 \text{ radius is } 1

\Rightarrow \ k = \frac{1}{2} \text{ radius is } 1 - \frac{1}{4} = 1 - \frac{1}{4} = \sqrt{3}/2 \approx 0.866

you have seen this on contour maps.

Visualizing functions of 3 variables is harder: \( f(x, y, z) \), the "graph" would be in 4-space.

\( w = f(x, y, z) \). We can draw the contour surfaces \( f(x, y, z) = k \) but it is hard to see all the surfaces.
Example: $p(x,y,z) = x^2 + y^2 + z^2 = k$

These are contour surfaces of a sphere of radius $\sqrt{k}$.