Abstract

Annuities, long-term care insurance (LTCI), and reverse mortgages appear to offer important consumption smoothing benefits to the elderly, yet private markets for these products are small. A prominent idea is to combine LTCI and annuities to alleviate both supply (selection) and demand (liquidity) problems in these markets. This paper shows that if consumers typically liquidate home equity only in the event of illness or very old age, then LTCI and annuities become less attractive and may become substitutes rather than complements. The reason is that the marginal utility of wealth drops when an otherwise illiquid home is sold, an event correlated with the payouts of both annuities and LTCI. Simulations confirm that demand for LTCI and annuities is highly sensitive to the liquidity and magnitude of home equity.

1 Introduction

Among the most significant difficulties in financial planning for the elderly are these: length of life, health status, and medical expenditures are stochastic; a large fraction of wealth is typically tied up in a home; and moving out of the home generates psychic and financial costs. These considerations are related: poor health is associated with old age, exit from the home, and rapid mortality.

Home equity products, long-term care insurance (LTCI), and annuities hold the promise of consumption smoothing benefits for the elderly by addressing these issues separately.
Home equity loans, reverse mortgages, and sale-leasebacks allow homeowners to consume housing wealth without moving out of the home. LTCI spreads large medical expenses across states of the world. Annuities transfer wealth from those who die young to those who live a long time. Relatively weak demand for these products among the elderly has spawned a large literature evaluating the strength of different explanations.

Bequest motives, adverse selection, moral hazard, and partial public provision of LTCI and annuities through Medicaid and Social Security may well dampen demand for private actuarial products. However, these factors do not easily explain away the smallness of private markets. Several papers have shown that with empirical pricing and plausible bequest strength, annuities remain attractive.¹ Finkelstein and McGarry (2003) and Davidoff and Welke (2006) argue that selection may be favorable, rather than adverse, in the markets for LTCI and reverse mortgages. Brown and Finkelstein (2007) show that women face much better pricing for LTCI than men, but do not have much stronger demand, calling into question supply side problems as the dominant market failure. Ameriks et al. (2007a) show that long term care expenditures are a major concern for wealthier households for whom Medicaid is unlikely to be an attractive alternative to LTCI.

Given the correlations among illness, mortality, and exit from the home, it is not surprising that a growing literature considers the interactions among demands for these actuarial products. Pauly (1990) shows that demand for LTCI may be weakened by the absence of a market for annuities because life after long-term care is typically short and there is public insurance against extremely high expenditures. In this case, LTCI serves primarily to transfer wealth from relatively young and healthy states to the public provider and to estates after death. If annuities are unavailable, then the marginal utility of wealth after death may be much lower than during life, and LTCI exacerbates this problem. By the same logic, demand for annuities should be increased by the presence of LTCI.

Annuities and LTCI may exhibit demand complementarity not only through the trade-off

¹See Brown (2007) for a summary.
between consumption while alive and the size of the estate passed on after death, but also through a liquidity channel. Turra and Mitchell (2004), Sinclair and Smetters (2004), and Ameriks et al. (2007a) show that absence of LTCI may weaken demand for illiquid annuities. The intuition for this result is that tying up wealth in an illiquid asset exposes households to very low consumption in the event of expenditure shocks. In this way, annuities may be attractive only in the presence of LTCI, and annuities will make LTCI coverage more important.

Spillman et al. (2001) and Webb (2006) observe that the risks of long-term medical care need and long life are likely negatively correlated, creating a selection-based supply side complementarity that reinforces the demand argument for combining the two products. Combining these considerations, a rising consensus among economists is that bundling LTCI with illiquid annuities may broaden the appeal of both. According to Lysiak (2007), partly in response to tax incentives, most of the major US annuity providers now offer, or are planning to offer, a bundled annuity and long term care product.

This paper shows that if homeowners typically sell their homes only in the event of illness or extreme old age, and if home equity is illiquid before sale, then optimal demand for both annuities and LTCI may be weak, even in the absence of bequest motives and abstracting from the other risk. Moreover, illiquid home equity tends to reverse conclusions about the complementarity between annuities and LTCI that arise when home equity is ignored or assumed liquid. That illiquid home equity would serve as a substitute for either annuities or LTCI for risk averse consumers is straightforward. Home equity spent only in the event of illness serves the same role as insurance against illness (LTCI). If home equity is spent primarily in old age, then it serves as pre-existing insurance against old age, as do annuities.

A graphical intuition for these possibilities is that consumer's utility is given by the upper envelope of two intersecting concave utility functions, one that includes a lump sum of sales proceeds and another that does not. The resultant kink around the point of choosing to move or not may lead to risk seeking behavior. This intuition has been applied to other portfolio

The key to home equity’s role in undoing the complementarity between LTCI and annuities lies in the role of LTCI in providing liquidity. Absent home equity, long term care expenses that arise early in life, while relatively unlikely, may render locking away savings in annuitized form unattractive. LTCI eases this liquidity problem. But if home equity is large relative to potential medical expenses, then homeowners may have a greater need for liquidity while healthy and holding home equity than when sick and having sold the home. In this case, the liquidity problem of annuities that LTCI solves disappears. Instead, early retirement, when the home is unlikely to have been sold, and when net proceeds from annuities and LTCI are most negative, may be the time of greatest marginal utility of liquid assets. In this case, LTCI and annuities are likelier to be substitutes.

Before describing the formal evaluation of these insights into home equity’s role in retirement planning, it is worth reflecting on how reasonable the implied assumptions are, and whether or not the results are likely to be robust to relaxing which assumptions. Most important is to establish two facts about the home equity of the elderly. The first fact is that that home equity is a large fraction of wealth for the elderly. The second is that home equity tends to be liquidated only late in life or when in long term care. I establish both with data from the Health and Retirement Study that confirms numerous earlier findings, such as those of Venti and Wise (2000) and Walker (2004).

The ratio of home equity to total asset wealth is a natural measure of the significance of home equity in the portfolio. Among all singles or married couples whose youngest member was over 62 in the 2004 wave of the HRS, the median ratio of home equity to all assets was .38. Among the roughly three quarters of respondents who were homeowners, the median was .56. For 46 percent of homeowners, home equity was more than double non-housing wealth.

Reflecting the general illiquidity of home equity among the elderly, the mean ratio of
home equity to home value in that sample was .90. What mortgage debt exists among the elderly appears to be carried over from working years, rather than extracted during retirement. The mean ratio of equity to value rose with age in the HRS, from .84 among homeowners between 60 and 70 in 2004 to .96 among owners in their 90s.

The correlation between mobility and long term care is strong in the HRS data. Only 15% of homeowners in the 1998 wave who did not live in a nursing home in 1998 or 2004 transitioned out of homeownership between 1998 and 2004. For those households with no members in a nursing home in 1998, but at least one member who did live in a nursing home in 2004, however, the transition rate was 69%. There is also a correlation between age and mobility. Among homeowners in their 60s in 1998 and still alive in 2004, only 8% moved between 1998 and 2004. By contrast, 41% of homeowners in their 90s in 1998, and still alive in 2004, moved in the intervening years.

The argument that annuities are less beneficial for home owners or those facing medical expenditure risk presumes that annuities cannot be made liquid or with payment streams that are highly front-loaded. Annuities certainly can be designed to be front-loaded (e.g. nominally constant annuities in high inflation economies). However, consumers who want front-loaded consumption are unlikely to gain as much from annuitization as those who can tolerate a smoother stream of payments. This is because the gain from annuitization comes from transferring money from after death to during life.\(^2\) These transfers are largest late in life, when the consumer is less likely to be alive. One might design a liquid annuity to overcome the problem of stochastic cash needs, but if consumers learn about mortality, the number of annuitants dying with assets to share among survivors will be smaller than when annuities are illiquid, so some of the benefit of annuitization is lost.\(^3\)

Institutionally, I ignore public provision of long term care insurance (e.g. Medicaid in the US, and public health systems in most developed countries) and annuities (through public pensions). Medicaid would complicate matters considerably, because the program is means

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\(^2\)See Bernheim (1987b) and Davidoff et al. (2005).

\(^3\)See Dierer (2007).
tested, but treats home equity generously. This friendly treatment of home equity reinforces the prospect that home equity would serve as a substitute for LTCI.

I assume in some simulations that the elderly cannot borrow at all against home equity. This is not literally true, but as shown above, home equity liquidation is very highly correlated with entry into long term care. Given the large utility gain to reverse mortgages or sale-leasebacks shown in the simulations, it is well worth asking why there is not a larger market for home equity debt for the elderly.

Optimal supply and demand of a joint product that simultaneously insures long term care expense and longevity while providing home equity liquidity would be very difficult to characterize, particularly in a competitive environment. This exercise is left for future research. I will take the prices of annuities and LTCI as fixed in the simulations, but the measured willingness to pay for fairly priced insurance in the simulations provides a way to measure how much pricing would have to be improved by bundling LTCI and annuities to make the combination worthwhile from the consumer’s perspective.

One reasonable explanation for both weak annuity demand and thinness of the home equity market is strong bequest motives. If the optimal bequest is larger than home equity in most states of nature, then the intuitions above disappear. In that case, though, the absence of a market for annuities is not particularly interesting. The present line of research can be justified based on uncertainty among economists that bequests are sufficiently strong among a sufficiently large swath of the elderly to rationalize near absence of demand for these products, by the perennial interest in annuities among economists, and by considerable growth in the last few years in the market for reverse mortgages.

The next section of this paper lays out conditions under which home equity crowds out demand for LTCI and annuities separately, and then shows how home equity affects the complementarity between LTCI and annuities. This discussion takes a similar model without home equity in Davidoff et al. (2005) as a starting point. Under a strong assumption on consumer utility, it is possible to characterize optimal LTCI and annuity demand for different
degrees of home equity liquidity. Richer comparative statics in other settings are not feasible and are left to simulations.

Section 3 provides calibrated examples of a single 62 year old male’s consumption and savings problem. The numerical examples verify the results of the simple analytical model and show surprisingly large effects of home equity on the complementarity between LTCI and a constant real annuity. A large number of recent papers have employed similar simulations to those in section 3; the only novelty is the introduction of home equity. Examples with discussions of results from other, similar, simulations are provided by Mitchell et al. (1999) and Brown and Finkelstein (2007) in the areas of annuities and LTCI, respectively.

2 A two period problem

To analyze the joint demand for LTCI and annuities, and the role of home equity in determining demand, this section lays out a stylized retiree’s consumption and insurance problem. Simulations in the next section allow a somewhat more realistic sequence of events, at the cost of specifying the problem’s parameters.

A consumer currently living in a home with cash value $h$ faces uncertainty about the length of her life and the need for costly medical care. She derives utility from the consumption of housing and another good in each of two future periods. In the first future period, she will be alive for sure, but may be either sick or healthy. In the second future period, she will be dead, sick, or healthy. There is no possibility of survival past the second period. As long as she is healthy, she has a sufficiently strong preference for remaining in her home that she does not sell the home. However, if she becomes ill in either period, there is no utility cost to moving, and the parameters are such that she chooses to sell the home.

Before any consumption takes place, the consumer must allocate wealth $w$ between liquid savings $b$ and an illiquid annuity $a$. Liquid savings may be spent in the first period or carried over to the second period, but the annuity can only be consumed in the second period. The
consumer also chooses insurance coverage $i$ that is paid in the event of illness in either period. In exchange, the consumer pays a constant premium if healthy in periods one and two.

After the annuitization and insurance decisions, the consumer learns whether she is healthy or sick in the first period. If she is ill (i.e. in long-term care), she sells the home, incurs a medical expense $x$, and allocates all remaining wealth between rental housing (if any is needed during or after care) and other consumption, giving rise to indirect utility $v_1$, and then dies before the start of period two. If the consumer is healthy in the first period, she lives to the second period after enjoying utility $u_1$ over first period expenditures. $u_t$ and $v_t$ are concave functions of expenditures. Home rental prices are constant so $v_t$ implicitly takes this price as an argument. If alive in period two, the consumer learns again whether she is in good or ill health, and again remains in the home if and only if healthy.

The consumer is exogenously endowed with, and forced to hold, a quantity $m$ of a reverse mortgage. The interest rate in the economy is zero and the actuarial products are fairly priced, so the consumer must repay $m$ out of the sales proceeds if and when she moves. The probabilities of ill health are $q_1$ and $q_2$, so an actuarially fair LTCI premium $z$ per unit of coverage $i$ satisfies the zero profit condition:

$$z = \frac{q_1 + [1 - q_1] q_2}{[1 - q_1] [1 + [1 - q_2]]}.$$  \hspace{1cm} (1)

Because the interest rate is zero, an annuity paying $a$ in the second period costs $a [1 - q_1]$ in either state in the first period. By contrast, liquid savings $b$ carried over to the second period cost $b$ only if healthy in the first period (liquid savings carried from the ill state in period one to either state in period two are logically impossible). As a matter of interpretation, from a period zero perspective, the entire quantity $w - a$ represents liquid savings. A decrease in the quantity $b$ need not imply an increase in annuitization because there is the alternative of greater period one consumption.

Conditional on a reverse mortgage $m$, the consumer’s problem is:
\[ \max_{a,b,i} U = q_1 v_1 (w - a [1 - q_1] - x + i + h) \]  
\[ + [1 - q_1] u_1 \left( w - a [1 - q_1] - i - \frac{q_1 [1 - q_1] q_2}{1 - q_1 [1 + [1 - q_2]]} - b + m \right) \]  
\[ + [1 - q_1] \left[ q_2 v_2 (a + b - x + i + h - m) + [1 - q_2] u_2 \left( a + b - i \frac{q_1 [1 - q_1] q_2}{1 - q_1 [1 + [1 - q_2]]} \right) \right]. \]

Recognizing non-negativity constraints, the first order conditions for insurance, annuities, and bonds are:

\[ \frac{\partial U}{\partial i} = - u_1' + [1 - q_2] u_2' \frac{q_1 + [1 - q_1] q_2}{2 - q_2} + q_1 v_1' + [1 - q_1] q_2 v_2' \leq 0 \]  
\[ \frac{\partial U}{\partial a} = [1 - q_1] [-q_1 v_1' - [1 - q_1] u_1' + q_2 v_2' + [1 - q_2] u_2'] \leq 0. \]
\[ \frac{\partial U}{\partial b} = [1 - q_1] [-u_1' + q_2 v_2' + [1 - q_2] u_2'] \leq 0. \]

Conditions (4) and (5) show that consumers will be at a corner solution in the allocation of savings between bonds and annuities. Savings will be in bonds only if if marginal utility is greater if ill than healthy in period one. If marginal utility is greater if healthy than ill in the first period, then all savings are annuitized.

Utility from expenditures is naturally shaped by health status. There are reasons to suspect that the marginal utility of wealth would be higher or lower inside or outside of long term care for a fixed level of expenditures net of medical expense \( x \). Given that the difference in marginal utilities might go either way, I establish two results under the simplest assumption, that \( u \) and \( v \) are independent of time and are identical up to a constant difference:

**Assumption 1.** \( u'_t(k) = v'_s(k) \forall k, s, t \)

One clear way in which Assumption 1 would be violated would be a bequest motive. In
that case, we would expect first period marginal utility if healthy $u'_1$ to be small relative to the other marginal utilities. Naturally, a bequest would reduce demand for the annuity. The simulations presented below consider the possibility of a moderately strong bequest motive.

The standard full insurance result follows from Assumption 1 combined with conditions (3) through (5), actuarially fair pricing, and concavity of the problem in $i$, $a$, and $b$:

**Result 1.** If Assumption 1 is satisfied and $h = m$, then all savings are annuitized with $a = \frac{w}{2}$ and there is full insurance: $i = \frac{x}{1-z}$.

If housing is illiquid, but illiquid equity $h - m$ is less than the potential loss $x$, then LTCI demand is reduced, but full insurance is still attained with a simple modification:

**Result 2.** If Assumption 1 is satisfied and $h > m$ but $x > h - m$, then all savings are annuitized with $a = \frac{w}{2}$, and $i = \frac{x}{1-z} + m - h$. If $x < h - m$ then no insurance is purchased and all savings are annuitized.

Results 1 and 2 shows that with complete markets and a simple utility structure, optimal LTCI is decreasing in illiquid home equity. The proof of both results is that the proposed solutions satisfy the first order conditions, and concavity guarantees that these are unique optima. More generally, the effect of home equity on LTCI demand, holding savings in annuities (or bonds) fixed, is obtained by differentiating the first order condition (3):

$$\frac{d i}{d[h - m]}|_{a,b} = -\frac{q_1u''_1 + z[1 - q_1]u''_1 + [1 - q_1]v''_2}{[1 - q_1][u''_1 + [1 - q_2]u''_2]}z^2 + q_1v''_1 + [1 - q_1]q_2v''_2. \quad (6)$$

Concavity of $u$ and $v$ guarantees that the right hand side of (6) is negative. The critical assumption for home equity to crowd out LTCI conditional on annuity purchases is thus that home equity is most likely to be liquidated in the event of LTCI.

The effect of home equity on annuitized savings when LTCI is held constant comes from differentiating condition (4):
The denominator of expression (7) is negative (and its negative positive), so the sign of \[
\frac{da}{d(h - m)}|^{i,b} = -\frac{[1 - q_1] [-q_1 v''_1 + [1 - q_1] u''_1 + q_2 v''] \left[1 - q_1\right]^2 [q_1 v''_1 + [1 - q_1] u''_1] + [1 - q_1] [q_2 v''_2 + [1 - q_2] u''_2]}{[1 - q_1] [q_1 v''_1 + [1 - q_1] u''_1] + [1 - q_1] [q_2 v''_2 + [1 - q_2] u''_2]}
\]

(7)

The denominator of expression (7) is negative (and its negative positive), so the sign of \[
\frac{da}{d(h - m)}
\]
is the same as that of the numerator of (7). From a starting point of no mortgage debt, the effect of illiquid home equity on annuity demand depends on whether entry into long term care is likelier to occur in period one or two. This depends on both the relative probabilities of ill health conditional on being alive \(q_1\) and \(q_2\) and on the probability of survival to period two \(1 - q_1\). Under the plausible condition that expected long term care usage unconditional on survival rises with age, illiquid home equity crowds out illiquid annuities.\(^4\)

Home equity also affects the complementarity between LTCI and annuities. To see the relationship, consider the derivatives:

\[
\frac{\partial^2 U}{\partial a \partial i} = [1 - q_1] [-q_1 v''_1 + [1 - q_1] u''_1 z + q_2 v'' - [1 - q_2] u''_2 z] \quad (8)
\]

\[
\frac{\partial^3 U}{\partial a \partial i \partial h} = [1 - q_1] [-q_1 v'''_1 + q_2 v'''_2].
\]

(9)

Equation (8) shows that annuities and LTCI tend to be complements if marginal utility when sick is decreasing more rapidly in expenditures than is marginal utility in the first period. Combined with an assumption of \(v''' > 0\) and \(u''' > 0\), this is the intuition of Turra and Mitchell (2004) and others: illiquid annuities are problematic when there is a cash need due to illness in the first period. However, equation (8) shows that if the second derivative of utility in the second period is more negative when sick than healthy, there is a force towards substitution. The substitution force is that both annuities and LTCI pay late in life when ill health is likely.

\(^4\)When there is positive mortgage debt, a reduction in mortgage debt makes annuitization more painful (the \([1 - q_1] u''_1\) term). Thus net home equity may crowd out annuity demand at a decreasing rate.
It is natural to assume that $v_1'''$ and $v_2'''$ are greater than zero, as this is a necessary condition for non-increasing absolute risk aversion. In this case, equation (9) shows that illiquid housing tends to undo both the complementarity and substitution arguments laid out above. As home equity increases, the need for liquidity in the event of illness in the first period decreases, undoing the complementarity between annuities and LTCI (the $-q_1v_1'''$ term renders $\frac{\partial^3U}{\partial a \partial i \partial h}$ more negative). On the other hand, as home equity increases, the curvature of utility in the event of illness presumably falls, so that combining LTCI with annuity payouts late in life is more tolerable (the $q_2v_2'''$ term makes the third derivative more positive).

Summarizing, holding annuity purchases constant, home equity cashed out only in illness crowds out LTCI demand. Holding LTCI constant, home equity reduces annuitization if home equity is more likely to be cashed out late in life. Whatever substitution between annuities and LTCI arises in the absence of illiquid home equity tends to be reversed when home equity is illiquid.

### 3 A Numerical Example

This section evaluates the welfare consequences of annuitizing non-housing wealth and committing to an LTCI contract for a hypothetical retired 62 year old male homeowner, with and without a reverse mortgage. Given the discussion in Section 2, the interesting questions are whether or not home equity acts as a substitute for annuities and LTCI, and whether or not home equity tends to push LTCI and annuities away from complementarity and towards substitution.

To focus on the interactions of demand for the different products, I assume that each is fairly priced, although one might interpret the floor on the utility function described below as implying a government subsidy to LTCI payments for insolvent consumers. Moreover, by estimating the willingness to pay for different institutional arrangements, one can estimate how much of a load a representative consumer might be willing to enter into a single adversely
selected product rather than a pooled, fairly priced contract.

Exactly as in Ameriks et al. (2007b), the consumer can be in any of four health states at each integer age between 62 and 120: healthy, moderately ill, severely ill, or dead. Health status maps one-to-one with medical costs that cannot be changed, but can be partially offset by insurance. If healthy, there is no medical expenditure. If moderately ill, there are positive and uninsurable out of pocket costs. Expenses are greater but possibly insured if severely ill. I use the transition probabilities of moving across health states described in Ameriks et al. (2007b). The transition probabilities are designed to match both population average mortality and health status transition rates from Robinson (2002). There is no possibility of survival past 120.\textsuperscript{5} To explore the role of uncertainty over long term care costs, I compare a case in which the cost of care in the event of care is known to a different case in which the cost of care is stochastic at the date at which the consumer commits to an insurance policy.

The consumer’s taste for living at home is perfectly correlated with health status. There is no direct utility cost to moving when moderately or severely ill, but there is a large utility cost to moving while healthy. The home is thus sold only in a state of poverty or ill health. I assume an efficient rental housing market, so there is a re-optimization gain to moving whenever in ill health. I make the more or less ad hoc assumption that moving out of the home involves only a one-time utility cost (and no financial cost) if healthy. An alternative assumption, that would yield different mobility patterns, would be that every period in the home while alive generates positive utility.\textsuperscript{6} The difference arises because it is possible that ill health will give way to future good health.

The consumer maximizes the following lifetime utility function:

$$
\sum_{t=62}^{120} [1 + \delta]^{62-t} \left[ \sum_{s=1}^{3} q_{st} \left( \alpha h_{st}^1 - \gamma + \left[ 1 - \alpha \right] c_{st}^1 - \gamma \right) - L(s) \times M_{st} \right] + q_{4t} B \frac{w_{4t}^1 - \gamma}{1 - \gamma} \right].
$$ \hspace{1cm} (10)

\textsuperscript{5}The transitions imply a probability of living to 120 of less than one in one hundred thousand as of age 62.

\textsuperscript{6}It is not obvious whether the cost of moving should be one-shot or a flow cost. A referee once took exception to flow costs, so I choose one-shot, with a conjecture that the basic results are unaffected by the choice.
$q_{st}$ is the probability of being alive and in health state $s$ in period $t$, evaluated at age 62. $L(s)$ is a large number in good health ($s = 1$) and zero otherwise, and $M_{st}$ indicates that a move out of the original home occurs in state $s$, period $t$. This utility function exhibits constant relative risk aversion and an income elasticity of housing demand of one, with both risk aversion and substitution between housing and the other good governed by $\gamma$. These features may not match reality particularly closely, but are subject to debate (see, e.g. Davis and Ortalo-Magné (2007)), so this computationally convenient and commonly modeled functional form is a natural choice.

$B$ is the strength of a bequest motive in the objective (10). This value is set to zero as a baseline.

Consumption and savings are linked as follows:

$$c_{st} = \begin{cases} w_{st} - \frac{w_{s+1}}{1+r} + a - x_s + M_{st} [h - m] - z_{st} & \text{If } w_{st} > 0, \\ f(\mu) & \text{otherwise.} \end{cases}$$ (11)

$w_{st}$ is wealth carried into state $s$ at time $t$ from some state $\hat{s}$ in period $t - 1$. $w_{s+1}$ is savings carried out of state $\hat{s}$ and period $t$. $a$ is constant annuity income, $x_s$ is net medical expense in state $s$, which by the structure of the problem is constant across time. $h$ is initial housing wealth. $z$, detailed below, represents housing expenditures. $m$ is a forward, not reverse, mortgage with interest only due each period until death or a move out of the home, at which point $m$ is also due. I consider two values for $m$: the value of the home at age 62 and zero. This mortgage is equivalent to a sale-leaseback.

Medical expense $x_s$ includes insurance and out of pocket medical expenses. The cost of moderately ill health (state 2) is always borne by the consumer. A fraction $i$ of severe health costs (state 3, long term care) are covered by insurance. In exchange for the insurance coverage, in every period and every state while alive, the consumer makes a constant payment such that the expected value of this annuity to an insurer equals the age 62 expected total costs covered. Because medical expenditures are not optional, an additive disutility to poor
health would not affect the analysis at all. Similarly, the fact that dying young makes lifetime utility greater for $\gamma > 1$ does not affect the analysis. If the LTCI payment were rising over time, there would be a somewhat greater complementarity between LTCI and annuity demand, just as demand for both would be reduced absent the sale-leaseback if home prices were appreciating.

I consider the possibility that at age 62, just after committing to LTCI, the consumer learns once and for all whether the medical expense will rise or fall by 50% from the expected value. In this case, the consumer maximizes the expectation of utility given by (10) subject to the constraint (11), under the belief that the two long term care regimes are equally likely. The LTCI premium is fixed before the insurance choice is made at the same actuarially fair price as under certainty.

The consumer’s utility function is independent of medical condition, but the marginal utility of expenditures is greatest when there are large uninsured expenses. This might match an ability to pay for better care or better surroundings in the event of long-term care. A natural conjecture is that if the marginal utility of consumption were multiplicatively lower in long-term care, the results presented here would be strengthened.

For a homeowner, housing cost $z_{st}$ is equal to the initial home value $h$ times a cost of maintenance, taxes, insurance and any mortgage debt. At the time of sale, the principal amount is owed, so the cash infusion at sale is $h - m$. Normalizing the price per unit of housing to one, a consumer who has transitioned out of homeownership to renting pays the housing quantity $h_{st}$ times the interest rate plus maintenance, taxes, and insurance. Actuarially fair pricing of the mortgage (at the riskless rate) is simplified by the assumptions that home prices are constant and that there is no choice of maintenance.

If wealth is non-positive, the consumer is allowed a very modest level of expenditures such that utility $\mu$ per period is attained regardless of health status. This backstop can thus be thought of as akin to Medicaid in that for most parameters, bankruptcy only occurs after repeated spells of severely ill health. There is no minimal annuity provided by social
security in the simulation, so there is some upward bias in the estimated welfare gains to
annuitization relative to that would accrue to retirees in developed countries with otherwise
similar parameters. The ratio of the disutility from moving to the disutility from living at
the minimum consumption level importantly affects the problem, so I present results with
different ratios.

3.1 Optimization procedure

I solve the consumer’s problem backward. After age 120, the consumer is dead. At age 120,
the consumer’s terminal utility over wealth if no longer at home is determined by finding the
utility-maximizing after-medical expense allocation of starting wealth and annuity income
between housing, non-housing consumption, and a bequest if $B$ is positive.\footnote{Savings choices are in thousands of dollars, but the allocation between housing and non-housing consumption if out of the home is continuous, as there is a closed form solution to the allocation problem.} If the consumer
is at home at the start of this final period, he chooses between moving or not, with the
choice trivial if in ill health. In the event of a move, housing sale proceeds are calculated
as described above, and consumption is allocated between rental housing, a bequest, and all
other goods. If still in the original home, housing consumption is fixed at the initial level
and all wealth after medical and mortgage expenses plus any annuity income. The value of
bequests are discounted at the same rate as is utility.

At age 119, for each health state, the consumer chooses a savings level and thus implicitly
age 119 expenditures and an age 120 bequest, pursuant to equation (11). The choice of
savings maximizes the sum of age 119 utility plus discounted age 120 indirect utility as a
function of health and wealth, with each potential health state at age 120 weighted by the
formulaic transition probabilities specific to each age 119 health state. This trade-off of
utility at time $t$ and probability-weighted indirect utility over wealth by state at time $t + 1$
is repeated back for all health states and ages back to age 62, at which age the consumer is
in good health and in the initial owned home.

To determine the welfare effects of liquid home equity, a real annuity, and LTCI un-
der different parameter combinations, I solve the consumer’s problem under 121 discrete combinations of levels of these actuarial product, with 0%, 10%, … 100% of liquid wealth annuitized and 0%, 10%, … of long term care costs insured. I only report three levels of annuitization to conserve space. The consumer has $300,000 in assets divided either equally between housing and liquid assets or two-to-one housing. The annuity pays out the amount $a$ each period while alive per dollar invested, where:

$$a = \frac{1}{\sum_{t=62}^{120} q_t (1 + r)^{62-t}},$$  \hspace{1cm} (12)

$q_t$ is the probability of survival to age $t$ conditional on being alive and healthy at age 62, and $r$ is a riskless interest rate. The probabilities $q$ are taken from the Ameriks et al. (2007b) calibration of health transitions.

An LTCI policy commits the consumer to pay a constant sum every period while alive (including if institutionalized) in exchange for payment of all long-term care expenses. The constant payment is set equal to $k \sum_{t=62}^{120} q_{3t} / a$, where $q_{3t}$ is the probability that the consumer will be in the severely ill health state at age $t$ conditional on being healthy at age 62. $k$ denotes the annual cost of LTCI times the fraction of expenses covered. This product is better than actuarially fair to the consumer in the sense that benefits are available in the event of bankruptcy, but there is no net payment to the insurer; however, medical expenses are covered in the event of bankruptcy for the non-insured, too.

For each set of parameters, I estimate the welfare effects of combinations of LTCI and annuity as follows. First, I calculate utility for a wide range of liquid wealth levels with no LTCI and no annuity. For the remaining combinations of annuity and LTCI $j$, I calculate the utility level $u_j$ achieved from a starting point of the initial wealth level. I then look up the initial wealth $w_j$ required to attain utility $u_j$ in the baseline case with no annuity and no LTCI. The welfare effect of a given combination of annuity and LTCI for each set of parameters is thus the difference (essentially a compensating or equivalent variation) between $w_j$ and the initial wealth endowment assumed in the cases $j$. A positive (negative) value $w_j$
implies that annuity and insurance combination \( j \) is better (worse) than no annuity and no LTCI. When long term care fees are stochastic, \( u_j \) is an expectation.

The consumer may not annuitize, purchase LTCI, or take on mortgage debt at any time after age 62. In the context of this model, there would be a considerable gain to annuitizing or purchasing LTCI after moving if a fair annuity or LTCI were available. However, after moving, most frequently due to poor health, the consumer has health information that is likely to be private and thus may face adverse selection.

### 3.2 Parameterization

Table 1 lists parameter choices for the consumer’s savings, consumption, and mobility problem. As in Ameriks et al. (2007b), medical expenditures are zero in the healthy state, $10,000 in the moderately ill state and $50,000 in the long-term care state, as a baseline. Given that these figures apply to a group that is mostly covered by Medicare, one might interpret the $10,000 as net of any non-LTCI insurance proceeds. I assume that these costs are constant in real terms over time, sidestepping the question of whether rising costs reflect medical inflation rates or improvements in quality. The $50,000 in expected medical costs are multiplied by .5 or 1.5 when costs are stochastic. In that case, lifetime utility (10) is calculated as the expected value across cost scenarios, with each scenario featuring probability .5. The cost of LTCI is determined before the cost of care is realized, and so is constant across cost cases.

The liquidity of home equity depends both on the disutility from moving and the availability of mortgage debt. The baseline disutility from moving is the same 99 units as is generated from each period of non-positive expenditures. I also consider a case where the disutility from moving is relatively small, at 9 units of utility.

The risk aversion parameter \( \gamma \) is set at a moderate level of two as a baseline. Results are changed almost not at all in unreported simulations at the more risk averse level of 4.

The transition probabilities are such that one dollar of LTCI coverage costs approximately 5.5 cents per period. One dollar of annuity income per period while alive costs $14.50 up
The parameter choices and utility structure I use generate housing statistics that are in line with empirical reality in the US. In the baseline case, the ratio of housing wealth to liquid non-housing wealth is two to one, with the housing endowment set at $200,000. As discussed in the introductory section, this ratio is somewhat higher than the median among consumers over 62 in the seventh wave of the HRS/AHEAD survey. However, the two-to-one ratio is just below the median ratio among those households with less than average stated probabilities of leaving a large bequest and among unmarried respondents.

The mobility pattern that arises in the simulations is broadly consistent with available US panel data: as detailed above, exit from owned homes is rare among the elderly, but much less rare when in ill health and increasing in age. Moreover, the implied exit rates from home ownership by age are not far off from empirical estimates. In the simulations presented below, the probability that a 62 year old will move before death is approximately .5. Sheiner and Weil (1992) estimate that 42% of estates include a home, implying that roughly 58% of the elderly cash out home equity before death.

I set the level of total wealth higher than population averages to avoid frequent bankruptcy and thus heavier reliance on the assumed minimum utility level. In the fourth panel, liquid wealth is set equal to housing wealth, with total wealth held constant.

### 3.3 Numerical Results

Table 2 presents the optimal fraction of long term costs covered by insurance and the benefit of different insurance and annuity arrangements relative to no insurance products under a range of parameter values. The first six columns describe the scenario, varying the cost of moving, bequest strength, initial home to cash ratio, mortgage status, and stochasticity of long term care costs. For each scenario, there are three listed values of annuitization: 0, 40, and 80 percent of initial non-housing wealth. In the case with a bequest motive, I consider smaller levels of annuitization.
## Table 1: Parameterization of the Consumer's Problem

<table>
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<th>Symbol</th>
<th>Meaning</th>
<th>Base Case</th>
<th>Other values</th>
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<tr>
<td>t</td>
<td>age</td>
<td>min 62, max 120</td>
<td>None</td>
</tr>
<tr>
<td>x</td>
<td>Medical expenditures</td>
<td></td>
<td>None</td>
</tr>
<tr>
<td>x₁</td>
<td>Expenditures if healthy</td>
<td>0</td>
<td>None</td>
</tr>
<tr>
<td>x₂</td>
<td>Expenditures if moderately ill</td>
<td>10</td>
<td>None</td>
</tr>
<tr>
<td>x₃</td>
<td>Expenditures if very ill</td>
<td>50</td>
<td>25, 75</td>
</tr>
<tr>
<td>i</td>
<td>Fraction of x₃ insured</td>
<td>0</td>
<td>10%, 20%, ... 100%</td>
</tr>
<tr>
<td>h</td>
<td>Starting home value</td>
<td>200</td>
<td>150</td>
</tr>
<tr>
<td>γ</td>
<td>Risk aversion parameter</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>δ</td>
<td>Discount rate</td>
<td>.03</td>
<td>None</td>
</tr>
<tr>
<td>r</td>
<td>Interest rate</td>
<td>.03</td>
<td>None</td>
</tr>
<tr>
<td>w</td>
<td>Starting non-housing wealth</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>L</td>
<td>Disutility of moving</td>
<td>-99</td>
<td>-9</td>
</tr>
<tr>
<td>μ</td>
<td>Disutility of consumption if bankrupt</td>
<td>-99</td>
<td>None</td>
</tr>
<tr>
<td>α</td>
<td>Utility weight on housing consumption</td>
<td>.25</td>
<td>None</td>
</tr>
<tr>
<td>B</td>
<td>Weight on terminal bequest</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

Note: Dollar values in thousands.

The eighth column of Table 2 lists the choice of LTCI coverage among the choices 0, 10% ... that maximizes consumer welfare. The last three columns describe the payment the consumer would have to receive (or would pay if negative) to move from a world of no annuity and no LTCI to the level of both listed.

The first panel of Table 2 considers the standard environment in which housing wealth is liquid. Almost all LTCI costs are optimally insured in each case. We find, though, consistent with previous simulations in the literature, that the welfare gain in moving from no LTCI to full LTCI is much larger ($193,000) when 80% of liquid wealth is annuitized than when no wealth is annuitized ($96,000). In the language of Section 2, it appears that liquidity concerns early in retirement are more important to the complementarity between LTCI and annuities than risk tolerance late in retirement.

The second panel makes housing illiquid by removing the mortgage. As conjectured above, we find a smaller gain to annuitization at optimal LTCI levels and lower optimal LTCI coverage relative to the mortgage case. While positive LTCI around 50% is optimal,
moving to full LTCI is welfare destructive when there is positive annuitization. Increasing annuitization above 40% reduces welfare when long term care risk is fully insured. Moreover, we find that optimal LTCI coverage is decreasing, rather than increasing in annuitization, and that the cost of full LTCI coverage becomes greater as annuitization increases.

The third and fourth panels demonstrate the significance of a cost to moving to the analysis. Optimal LTCI coverage, the gains to annuitization, and the complementarity between LTCI and annuities are all more positive when the disutility of moving is small (-9) relative to the disutility of bankruptcy (-99). In this case, though, optimal annuity coverage and the attendant welfare gains are much smaller without the mortgage than with.

Similarly, the fifth and sixth panels of Table 2 demonstrate that reducing the share of housing in the portfolio diminishes the illiquidity effect on insurance demand. In the sixth panel, where no mortgage is present, optimal LTCI coverage given annuity status and the benefits to annuitizing wealth are now intermediate between the case with large housing wealth and a mortgage (first panel) and the case with large housing wealth and no mortgage (second panel). Moreover, optimal LTCI coverage is no longer decreasing in annuitization.

The seventh and eighth panels of Table 2 show that the results in the top two panels are robust to making long term care costs stochastic. In these cases, utility is an expected value, reflecting an even chance that medical costs if ill will be 25 or 75, rather than 50 for sure in the baseline case. As one would expect given precautionary savings motives, the gains to LTCI are somewhat greater when medical costs are stochastic and a mortgage is available. However, stochastic costs neither make insurance or annuitization significantly more attractive, nor render LTCI and annuities complements when there is no mortgage debt.

Finally, the ninth and tenth panels of Table 2 consider the case in which the consumer has a fairly strong bequest motive. In this case, annuitizing just 20% of non-housing wealth is welfare destructive even at the optimal level of LTCI coverage, with or without a mortgage. The optimal LTCI coverage is slightly lower without the mortgage (10th panel) than with
(9th panel) in the suboptimal cases of positive annuitization. The fact that the annuity is welfare destructive even at optimal LTCI holdings and fair prices implies that even if pricing were improved by a combination of an annuity with LTCI, such a consumer could not benefit from a bundled product relative to no actuarial product at all.

Figure 1 illustrates some of the results from Table 2 graphically. The top panel plots realized utility by annuity and LTCI coverage in the case of no mortgage and large housing wealth (panel 2 in Table 2). The middle panel shows the case with no mortgage and small housing wealth (corresponding to panel 6 in Table 2), and the bottom panel shows the case with large housing wealth but a mortgage in place (the top panel in Table 2).

Figure 1 shows clearly that the slope of welfare in LTCI and annuitization, and the effect of annuitization on the slope of welfare in LTCI, is heavily affected by the extent of illiquid home equity. In the case of small but illiquid housing, the gain to LTCI coverage is larger than with a large house and liquid housing, but optimal coverage is lower, and the effect of annuitization on the slope of welfare in LTCI is weaker.

4 Discussion and Limitations

A large number of US retirees can be characterized as having a large fraction of their wealth tied up in home equity that is likely to be liquidated only later in life and in a state of poor health. For such homeowners, I find that the gains to annuitization and LTCI coverage are much smaller than for homeowners with home equity that is smaller relative to wealth. In a baseline case, a consumer who would be willing to pay $200,000 for a fairly priced package of an $80,000 annuity and 90% coverage of long term care costs if home equity were fully liquid, would be willing to pay only approximately $18,000 for a package that would optimally contain only 40% coverage of long term care expenses if home equity is illiquid. This finding is robust to adding significant uncertainty over medical expenditures.

A large literature has tried to explain away the absence of demand for annuities. Other
than a strong bequest motive, the most prominent explanation for the absence of demand for has been liquidity needs associated with long term care. The results here suggest that altogether eliminating long term care expense risk need not spur demand for annuities. To the contrary, annuitizing even 40% of wealth can be welfare destructive for a homeowner forced to fully insure against long term care risk.

Likewise, large accidental bequests have been cited as a reason for the weakness of the market for LTCI. For an illiquid homeowner, though, simulations suggest that sharply reducing the financial risk of longevity through an annuity can reduce demand for LTCI.

The results presented here are driven by the possibility that long term care is a state of low marginal utility of wealth relative to being healthy and out of long term care. Under commonly parameterized health transitions and expected long term care costs, a moderate level of home equity may “overshoot” most of the distribution of lifetime long term care expenditures. Because home equity extraction is highly correlated (in the model perfectly correlated) with long term care needs, an illiquid homeowner may have more effective resources and less time to spend them if need for care arises. Thus home equity serves as a substitute for LTCI. Moreover, because LTCI and home equity extraction typically occur later in life, transferring money from early retirement to late retirement through an annuity may be most painful when such transfers are already made in expectation through both home equity and LTCI.

The results in this paper provide further evidence that important financial decisions are shaped by the liquidity of home equity and the timing of home sales. These results augment those of Chetty and Szeidl (2004) and Shore and Sinai (2005), who show that housing considerations shape risk aversion with respect to portfolio choice and housing purchases early in life. Expanding the market for home equity products for the elderly may well be a critical step towards expanding the market for both annuities and LTCI.

The results do not imply that illiquid homeowners would find a bundled LTCI and annuity product unattractive, even relative to separately optimal holdings. Indeed, in the baseline
case, we find that a high degree of annuitization becomes welfare improving, rather than welfare reducing with a moderately positive level of LTCI. Going from no insurance and an 80% annuity alone reduces welfare by $4,000. When a 40% LTCI policy is added, however, welfare increases by $18,000 relative to no annuity and no LTCI. Given that bundling is likely to improve pricing due to selection considerations, the gains to the optimal product are likely understated in the simulations, which assume that pricing is independent of the insurance regime.

The simulations demonstrate that the quantity of LTCI that must be added to annuities to significantly improve pricing may affect the appeal of a bundled product to illiquid homeowners. If a relatively small LTCI component can improve pricing, the product should be widely appealing. However, if a large quantity of LTCI is required, illiquid homeowners may be turned away on consumption smoothing grounds even if there are considerable price improvements. In some cases, simulations find that no annuity and no LTCI is preferable to too much insurance, even at fair prices.

It is very difficult to know how pricing of bundled actuarial products would change when different components are modified. A comprehensive old age security policy that converts home equity into annuities that pay for consumption and medical insurance seems attractive on consumption smoothing grounds. However, such a product might have less appealing moral hazard and selection characteristics than reverse mortgages alone (which appear to appeal to heavy discounters and the short-lived) or than only annuities and LTCI in combination (which may have offsetting selection effects that allow fair pricing).\footnote{See Spillman et al. (2001), Webb (2006), and Davidoff and Welke (2006).} Reverse mortgage contracting alone is quite complicated given endogenous maintenance and exit from the home as well as stochastic home prices. Profits from a given product offering would be immensely difficult to project in a competitive environment featuring stand-alone actuarial products, different bundles of two products, and different bundles of all three of annuities, LTCI, and reverse mortgages. As an example, could a fairly priced bundle of
LTCI and annuity break even in competition with a bundle of reverse mortgage and LTCI (as proposed by Ahlstrom et al. (2004))? If so, how much LTCI would have to be added to improve annuity pricing? It is fair to say that economists are very far from being able to answer that question with any confidence.

Some parting caveats are in order. The analysis in this paper has assumed that willingness to exit homeownership is perfectly correlated with health status. This is not a terrible approximation, but is not literally true. If long-term illness did not always entail sale of the home, then the negative relationship between LTCI and reverse mortgages presented here would be attenuated. The results suggest that a high degree of home equity to wealth is required to have meaningful effects on individual and joint LTCI and annuity demand. Thus there are a likely a large number of retirees with non-trivial asset holdings for whom the results have limited application. Nevertheless, for the large number of retirees with substantial home equity holdings and negligible mortgage debt, establishing that LTCI or annuities offer important insurance benefits will require a justification of why expected marginal utility is not plausibly highest when home equity has yet to be liquidated. In other words, if a reasonably priced reverse mortgage is welfare increasing but unavailable, then weak demand for LTCI and annuities may be perfectly rational.

Important features of retirement have been assumed away here and should be incorporated into future research. Medicaid and Social Security represent very large endowments of long-term care insurance and annuities. Medicaid not only provides a floor for consumption when ill, but also treats home equity favorably in its means tests. In the presence of Medicaid, only affluent households are likely to find LTCI attractive. Medicaid’s eligibility rules also provide an alternative reason why one would be likely to find a negative empirical relationship between home equity and LTCI and may also justify the weakness of the reverse mortgage market.

Pre-existing pensions along the lines of social security crowd out demand for private annuities in most of the developed world. Given the paper’s results, we would also expect
the home equity crowd-out of LTCI to be stronger with such pensions in place.

The analysis in this paper is most applicable to a single individual with no bequest motive. The fact that many older individuals retain life insurance past their working years (far more than take on private annuities, LTCI, or reverse mortgages) indicates that complicated bequest motives affect financial behavior. Bequest motives are likely important factors in the retention of home equity and absence of a large private market for annuities. Moreover, spouses and children frequently either perform or pay for long-term care. Strong bequest motives must undermine demand for annuities and home equity liquidation. Consistent with Pauly (1990), we find that the complementarity between annuities and LTCI is weaker in the presence of a bequest motive. However, both the nature of altruistic preferences and the availability of life insurance would matter for how home equity interacts with demand for long-term care insurance. Strategic intrafamily considerations also likely matter. As in Bernheim et al. (1986), holding home equity as opposed to an annuity or long-term care insurance may increase parents’ bargaining power in the event of illness and old age. This provides another alternative explanation for the substitution between home equity and both of LTCI and annuities.

This paper assumes constant real home prices and takes home equity at retirement as exogenous. One natural direction for future research would be to consider optimal and empirical housing investment through the lifecycle, taking into account the role of financial risks in retirement, as in Cocco (2005), but adding home equity favoritism under Medicaid, the consumption floors induced by social insurance, and perhaps intrafamily bargaining. Different Medicaid eligibility rules and treatment of home equity may provide useful variation for empirical exploration into the determinants of home equity accumulation. Such analysis should inform the theory of optimal social insurance design, and in particular how home equity should be treated in any means testing under social security or Medicaid.
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**Notes:** Dollar amounts in thousands. Parameter values as in Table 1 except where noted. Optimal LTCI is the optimal fraction of long term care costs insured given the other parameters. The value in the last three columns is the amount of money the consumer would have to be paid (or would pay if negative) to be indifferent between that annuity and LTCI combination and having no annuity or LTCI. “Stoch. Med.” refers to the case where medical expenses per year in long term care are either $25,000 or $75,000, each with 50% probability, and the true level realized only after signing insurance contracts. Otherwise, expenditures are $50,000 per year of long term care.
Figure 1: Annuitization, LTCI, and Welfare with different home equity and mortgage levels. Top panel: home is $200,000 and no mortgage. Middle panel: home is $150,000 and no mortgage. Bottom panel: home is $200,000 and $200,000 interest only mortgage. Circles are no annuity, solid line is 40% of wealth annuitized, dashed line is 80% annuitized.
References


