Introduction to Non-Newtonian Fluid Mechanics and Industrial Applications

Mech 550
UBC Vancouver
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Contents

• Introduction: viscosity of polymer melts
• Experimental viscosity measurements
• Application: Injection Moulding
• Application: Extrusion
• Application: Thermoforming
• Concluding remarks & project definition
Introduction: viscosity of polymer melts

- Viscosity vs. Shear rate
- Viscosity models
Viscosity vs. shear rate

Shear flow situation

Shear rate $\dot{\gamma} = \frac{\partial u}{\partial y}$

Shear viscosity $\eta = \frac{\tau}{\dot{\gamma}}$
Viscosity vs. shear rate

Shear flow situation

Shear rate $\dot{\gamma} = \frac{\partial u}{\partial y}$

Shear viscosity $\eta = \frac{\tau}{\dot{\gamma}}$

$\Rightarrow \tau = \eta \cdot \dot{\gamma}$
Viscosity models

Velocity and shear rate profile for pipe flow
Viscosity models

- Ostwald-Dewaele power law model

\[ \gamma = K \cdot \tau^n \quad \text{or} \quad \tau = K_R \cdot \gamma^{n_R} \]

with \( n_R = \frac{1}{n} \) and \( K_R = \left( \frac{1}{K} \right)^{\frac{1}{n}} \)

shear thinning:
\( n \) is always larger than 1
\( n_R \) is always smaller than 1

\( n = n_R = 1 \rightarrow \) newtonian fluid
Viscosity models

• Ostwald-Dewaele power law model

experimental determination of \( n \):

\[
\log(\dot{\gamma}) = \log(K) + n \cdot \log(\tau) \implies n = \frac{d \log(\dot{\gamma})}{d \log(\tau)}
\]

additional temperature dependency

\[
\tau = K_R \cdot \dot{\gamma}^{n_R} e^{-\beta T}
\]
Viscosity models

• First order model

\[ \eta = A \dot{\gamma}^B e^{CT} \]

conversion to the Ostwald-Dewaele power law model

\[ \dot{\gamma} = K \tau^n \]

\[ B = \frac{1-n}{n} \]

\[ Ae^{CT} = \left( \frac{1}{K} \right)^{\frac{1}{n}} \]
Viscosity models

• Second order model

\[ \ln(\eta) = A + B \ln(\dot{\gamma}) + C \cdot T + D \cdot (\ln(\dot{\gamma}))^2 + E \cdot T \ln(\dot{\gamma}) + F \cdot T^2 \]

• Cross WLF

\[ \eta = \frac{\eta_0}{1 + \left(\frac{\eta_0 \cdot \dot{\gamma}}{\tau^*}\right)^{1-n}} \quad \text{with} \quad \eta_0 = D_1 e^{\left(\frac{-A_1 (T-T^*)}{A_2 + (T-T^*)}\right)} \]
Viscosity models

The graph illustrates the relationship between shear rate and viscosity for different models:

- **Cross WLF**
- **second order**
- **first order**
- **power law**

Viscosity [Pa.s] is plotted on the y-axis, and shear rate [1/s] is on the x-axis. The graph compares the performance of these models over a range of shear rates.
Experimental viscosity measurements

- Capillary
- Rotational
- Inline using slit die
Capillary viscosity measurements

- Measuring method
Capillary viscosity measurements

- Measuring issues:
  - $\Delta P$ in capillary die?
  - True shear rate in non-newtonian flow
Capillary viscosity measurements

→ Bagley correction

Force balance
\[ \tau \cdot 2\pi L R = (P - P_i) \pi R^2 \]
\[ \Rightarrow \tau_t = \frac{P - P_i}{2LR} \]

If \( P_i \) is known, the true shear stress \( \tau_t \) can be calculated
Capillary viscosity measurements

→ Bagley correction / Bagley plot

\[ \frac{P_i}{c} = \tau^m \]
Capillary viscosity measurements

→ Rabinowitch correction

For a newtonian fluid in laminar flow:
\[
\dot{\gamma}_a = \frac{4. \dot{Q}}{\pi. R^3}
\]

For a non newtonian fluid in laminar flow:
\[
\dot{\gamma}_t = \frac{4. \dot{Q}}{\pi. R^3} \cdot \left(\frac{n+3}{4}\right) = \dot{\gamma}_a \cdot \left(\frac{n+3}{4}\right) \text{ with } n = \frac{d \log(\dot{\gamma})}{d \log(\tau)} \quad \text{ (power law model)}
\]
Capillary viscosity measurements

- Practical procedure:
  geometry R=1mm, L=20mm
  raw measurement data

<table>
<thead>
<tr>
<th>$\dot{\gamma}$ [s$^{-1}$]</th>
<th>$\Delta P_{tot}$ [bar]</th>
<th>$\Delta P_i$ [bar]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>100</td>
<td>42</td>
<td>11</td>
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<tr>
<td>1000</td>
<td>90</td>
<td>34</td>
</tr>
<tr>
<td>4000</td>
<td>145</td>
<td>65</td>
</tr>
</tbody>
</table>

From bagley plot (available from previous measurements with different L/R capillaries)
Capillary viscosity measurements

- Practical procedure:
  - apparent viscosity and shear stress

<table>
<thead>
<tr>
<th>$\dot{\gamma}$ [s$^{-1}$]</th>
<th>$\Delta P_{tot}$ [bar]</th>
<th>$\Delta P_i$ [bar]</th>
<th>$\Delta P_{tot} - \Delta P_i$ [bar]</th>
<th>$\dot{V}$ [mm$^3$/s]</th>
<th>$\eta$ [Pa.s]</th>
<th>$\tau$ [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>2</td>
<td>18</td>
<td>7,85</td>
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<td>65</td>
<td>80</td>
<td>3141,59</td>
<td>50</td>
<td>200000</td>
</tr>
</tbody>
</table>
Capillary viscosity measurements

- Practical procedure:
  Relationship between entrance pressure and shear stress

\[ P_i = c \cdot \tau^m \]

\[ \log(P_i) = \log(c) + m \log(\tau) \]

\[ \Rightarrow c = 229.7 \]

\[ \Rightarrow m = 0.23612 \]
Capillary viscosity measurements

- Practical procedure:

Correction for non-newtonian behaviour (Rabinowitsch)

Power law viscosity model: \( \dot{\gamma} = K \cdot \tau^n \)

\[ \log(\dot{\gamma}) = \log(K) + n \log(\tau) \]

\( \Rightarrow K = 2,40436 \cdot 10^{-18} \)

\( \Rightarrow n = 4,006 \)
Capillary viscosity measurements

- Practical procedure:
  True shear rate and viscosity

<table>
<thead>
<tr>
<th>( \dot{\gamma}_a ) [s(^{-1})]</th>
<th>( \eta_a ) [Pa.s]</th>
<th>( \tau ) [Pa]</th>
<th>( \dot{\gamma}_t ) [s(^{-1})]</th>
<th>( \eta_t ) [Pa.s]</th>
</tr>
</thead>
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<tr>
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<td>45000</td>
<td>17,5</td>
<td>2569,23</td>
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<td>175,2</td>
<td>442,48</td>
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<td>1751,5</td>
<td>79,93</td>
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<td>4000</td>
<td>50</td>
<td>200000</td>
<td>7006,0</td>
<td>28,55</td>
</tr>
</tbody>
</table>
Capillary viscosity measurements

- Practical procedure:
  True shear rate and viscosity

![Graph showing viscosity vs. shear rate]

Legend:
- Blue line: apparent data
- Red line: true data
Capillary viscosity measurements

- Extrudate quality and melt fracture

When critical shear rate or shear stress is exceeded

ok  sharkskin
Capillary viscosity measurements

• Extrudate quality and melt fracture

  o critical shear rate increases with temperature
  o $\tau_c \cdot M_w$ is constant
  o Level of branching in polymer chain does not influence $\tau_c$
  o Tapering the die entry will improve extrudate quality
  o $\tau_c$ might increase with L/D ratio of the die

  o Phenomena are still not completely understood
Rotational viscosity measurements

- Different geometries possible
  - Concentric cylinder
  - Plate-cone
  - Parallel plate

- Two different measuring methods
  - Constant shear stress
    - apply torque and measure shear rate
  - Constant shear rate
    - apply rotational speed and measure shear stress
Rotational viscosity measurements

• Constant shear rate measurements are most used for polymer melts

• Couette versus Searle set-up

  Couette: less centrifugal forces, more suited for low viscosity fluids

  Searle: easier temperature control
Rotational viscosity measurements

• Concentric cylinder geometry

\[ \dot{\gamma} = \frac{dv}{dr} = \frac{\omega r}{R - r} = \frac{\omega r}{e} \]

\[ \tau = \frac{T}{2 \pi r^2 L} \]

Newtonian flow: \( \tau = \eta \dot{\gamma} \)

\[ \tau = \eta \dot{\gamma} \Rightarrow \frac{T}{2 \pi r^2 L} = \frac{\omega r}{e} \]

\[ \Rightarrow \eta = \frac{T e}{2 \pi r^3 \omega L} \]

Assumption:
Linear velocity profile

⇒ as small as possible gap between rotor and cylinder
\( 1 < R/r < 1,1 \)
Rotational viscosity measurements

• Concentric cylinder geometry

Newtonian flow: \( \tau = \eta \cdot \dot{\gamma} \)

\[ \eta = \frac{T e}{2 \pi r^3 \omega L} \]

Viscosity links Torque T to rotational velocity \( \omega \)

For non-newtonian flow dependency of shear stress and viscosity to shear rate has to be taken into account

\( \rightarrow \) Perform measurement at different speeds and monitor \( \tau \) and \( \dot{\gamma} \)
Rotational viscosity measurements

- Concentric cylinder geometry

Assumption: torque determined by friction in the concentric gap

- Experimental correction
- Special geometries
Rotational viscosity measurements

- Cone plate geometry

\[ \dot{\gamma} = \frac{dv}{dr} = \frac{\omega r}{r \tan(\alpha)} \approx \frac{\omega}{\alpha} \]

for small angles \( \alpha \) \( \Rightarrow \) \( \dot{\gamma} \) is constant

\[ dF_T = \tau \cdot 2\pi r \cdot dr \]

\[ dT = r \cdot dF_T = \tau \cdot 2\pi r^2 \cdot dr \]

\[ T = \int_0^R dT = 2\pi \tau \int_0^R r^2 dr \]

\[ T = 2\pi \tau \left( \frac{R^3}{3} \right) \Rightarrow \tau = \frac{3T}{2\pi R^3} \]

Newtonian flow: \( \tau = \eta \cdot \dot{\gamma} \)

\[ \tau = \eta \cdot \dot{\gamma} \Rightarrow \frac{3T}{2\pi R^3} = \eta \frac{\omega}{\alpha} \]

\[ \Rightarrow \eta = \frac{3T \alpha}{2\pi R^3 \omega} \]
Rotational viscosity measurements

- Plate plate geometry

\[ \dot{\gamma} = f(r) \rightarrow \text{use "average"} \quad \dot{\gamma} = \frac{\omega r}{d} \]

\[ dF_T = \tau \cdot 2 \cdot \pi \cdot r \cdot dr \]

\[ dT = r \cdot dF_T = \tau \cdot 2 \cdot \pi \cdot r^2 \cdot dr \]

\[ T = \int_0^R dT = 2\pi \int_0^R \tau r^2 dr \]

Newtonian flow: \( \tau = \eta \cdot \dot{\gamma} \)

\[ T = 2\pi \int_0^R \eta \dot{\gamma} r^2 dr = 2\pi \eta_{avg} \int_0^R \frac{\omega r}{d} r^2 dr \]

\[ T = \frac{2 \pi \omega \eta_{avg}}{4 d} R^4 \]

\[ \Rightarrow \eta_{avg} = \frac{2d}{\pi T \omega R^4} \]
Field of application of different methods

- Rotational reometer
- Capillary rheometer
In line viscosity measurements

- Advantages:

  actual processing conditions in preheating

  applicable to high filled materials (fibers or particles)
In line viscosity measurements

- In Extrusion
  
  screw speed controls shear rate

  slit die geometry

\[ \dot{\gamma} = \frac{6Q}{wh^2} \]

\[ \tau = \frac{hw}{2(w + h)} \frac{\Delta P}{l} \]
In line viscosity measurements

- In injection molding
  
  very large measuring window: $100 \text{s}^{-1} – 1 \, 200 \, 000 \text{s}^{-1}$

  micro-slits (0.1mm)

(Friesenbichler et al. 2016)
In line viscosity measurements

- In injection molding

**Fig. 8.** (Color online) Micro rheology measurement system for capillary rheometer (above left) and injection molding machine (above right); temperature profile in the steel body and extrapolated wall temperature (below, left), cross-sectional illustration of the slit-die system (below, right), 1 die housing, 2,3 conically shaped slit-die inserts, 4 thermal insulation, $p_v$ pressure sensor at the inlet, $T_1, T_2, T_3$ temperature sensors.

(Friesenbichler et al. 2016)
In line viscosity measurements

- In injection molding

Fig. 9. (Color online) Measurement system with rheological mold (left) and the mold in cross-sectional illustration (right).

Backpressure can be controlled to determine pressure dependent viscosity

(Friesenbichler et al. 2016)
In line viscosity measurements

- In injection molding

Fig. 11. (Color online) Sectional view of a rheological split-mold with slit-die, operated on a horizontal rubber injection molding machine; a: heat flux sensors, b: pressure sensors, c: piston for applying the counter pressure.

(Friesenbichler et al. 2016)
In line viscosity measurements

- In injection molding determine pressure dependency

\[ \eta_p = \eta_{p0} \cdot e^{\beta_p (p-p_0)} \]

Fig. 10. (Color online) Pressure dependent viscosity of PS 454C (left) and of Polypropylene PP HG313MO with a calculated pressure coefficient \( \beta_p \) of 0.024 MPa\(^{-1}\).
Field of application of different methods

( Friesenbichler et al. 2016)