Introduction to non-Newtonian Fluid Mechanics and Industrial Applications
• Introductory
  – Main ideas of non-Newtonian fluids
  – Not particularly mathematical
  – Continuum approach
• Introduction and some oilfield flows
  – Ian Frigaard
• Injection molding
  – Bart Buffel
• Assessment
  – Projects/assignments

Course outline:

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From Cauchy to Navier-Stokes

- Continuum mechanics perspective, Cauchy:
  \[ \rho a = \nabla \cdot \sigma + \rho g \]
  - \( \rho \) = density; \( a \) = acceleration; \( \sigma \) = stress tensor; \( g \) = gravitational acceleration

- For convective accelerations & incompressible flows \( \nabla \cdot \mathbf{u} = 0 \)
  \[ \rho \frac{d\mathbf{u}}{dt} = -\nabla p + \nabla \cdot \tau + \rho g \]
  - \( p \) = pressure; \( \mathbf{u} \) = velocity; \( \tau \) = deviatoric stress tensor

- Material (convective) derivative and outer product
  \[ \frac{d\mathbf{u}}{dt} = \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}), \quad \mathbf{u} \otimes \mathbf{v} = \mathbf{u} \mathbf{v}^T \]
Navier-Stokes

- Euler's equations: 1750’s (no deviatoric stress)
- Cauchy's equation 1820’s (Cauchy stress tensor)
  \[
  \mathbf{\sigma} = -p\mathbf{I} + \mathbf{\tau}, \quad \sigma_{ij} = -p\delta_{ij} + \tau_{ij}
  \]
  \[
  p = -\frac{1}{3}\sigma_{ii}, \quad \tau_{ii} = 0
  \]
- Navier/Stokes: specific form of $\mathbf{\tau}$ for viscous fluid
  - Galilean invariant: does not depend directly on velocity
  - Depends only on local variations in velocity, i.e. assumed linear dependence on velocity gradients
  - Fluid is isotropic, hence $\mathbf{\tau}$ is an isotropic tensor
- Stokes constitutive law for Newtonian fluid
  \[
  \mathbf{\tau} = \mu\left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right], \quad \tau_{ij} = \mu \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right]
  \]
Usual simplifications for Newtonian fluids

- Viscosity is constant
  \[ \nabla \cdot \tau = \nabla \cdot \left( \mu \left[ \nabla u + \nabla u^T \right] \right) = \mu \nabla^2 u \]

- Flow incompressible
  \[ \frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho} \nabla p + \nu \nabla^2 u + g \]
  \[ \nabla \cdot u = 0 \]
  \[ \nu = \mu / \rho = \text{the kinematic viscosity (a diffusivity)} \]

- Navier-Stokes equations:

- Non-Newtonian fluid? A fluid that does not satisfy:
  \[ \tau = \mu \left[ \nabla u + \nabla u^T \right] \]
  - Mathematically, fluid needs closure law (9 unknowns, 4 equations)
  - Physical and mathematical constraints on \( \tau \), e.g. frame invariance
  - **Constitutive law** for \( \tau \) must reflect actual mechanical behaviour
Notational issues

• We’ll use both vector and Einstein indicial notation

• Stress tensor: \( \sigma = \sigma_{ij} \)
  
  - \( \sigma_{ij} \) is the \( j \) component of the traction on a surface with unit normal in direction \( i \). Traction for a general \( \mathbf{n} \) is the vector \( \mathbf{n} \cdot \sigma = n_i \sigma_{ij} \), which has both normal and shear components.
  
  - The normal direction \( \mathbf{n} \) points from the inside to the outside and the traction is the force per unit area exerted on the fluid inside by the fluid on the outside.
    - Normal stresses are negative in compression for us – the usual fluids convention
    - Other disciplines use - \( \sigma \), i.e. the force exerted on the outside by the inside, which is common e.g. in geo-mechanics.
    - Due to the bridge with other mechanics disciplines, some authors also adopt this convention for non-Newtonian fluids.

• Rate of strain: 3 different notations are common

  - \( e_{ij} \) Newtonian fluids, e.g. splitting velocity gradients into strain and rotation
  
  - \( D_{ij} (= e_{ij} ) \) common in mathematical texts
  
  - \( \dot{\gamma}_{ij} \) used in non-Newtonian fluid mechanics

\[
e_{ij} = \frac{1}{2} \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] = D_{ij}
\]

\[
\dot{\gamma}_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} = 2e_{ij}
\]
Vets & doctors

- Polymer solutions, e.g. HDPE, LDPE
  - Concentrated or dilute
  - Linear or branched, end groups, cross-linked, networked
  - Synthetic or biological
    - Molecular weights: water $\approx 18$ g/mol; synthetic polymers $10^4$-$10^6$ g/mol; biological polymers up to $10^8$ g/mol
  - Often modeled as polymer phase + solvent phase.
    Mechanical properties of polymer that lead to non-Newtonian behaviours, e.g. memory, normal stresses, nonlinearity

Suspensions

• Classifications:
  – Concentrated or dilute
  – Monodisperse or polydisperse
    ▪ Particle shape & size, e.g. fibers to spheres
    ▪ Particle mechanics: flexible, elastic, hard
  – Brownian, Stokesian, inertial
  – Colloidal or non-colloidal
  – Active and passive, smart

• Different origins, e.g.
  – Mined suspensions
  – Geophysical (mud slides, avalanches...)
  – Polymer solutions, food & drink
  – Pulp fiber suspensions, oilfield fluids
  – ER/MR fluids, biological, effluent
  – Model laboratory suspensions

Other

- **Emulsions**
  - Dispersed & continuous phase; liquid in liquid

- **Liquid foams**
  - Gas bubbles bordered by metastable lamellar liquid films
  - Typically high void fractions, e.g. 85-95%

- **Bubbly liquids**
  - Gas in liquid, separated by bulk liquid

- **Granular flows**
  - Solids in gas at high volume fraction
  - E.g. dense inertial granular media modelled with $\mu(I)$ “rheology”
    - Mathematical analogy
Non-Newtonian phenomena 1

- Viscosity varies with shear rate
- Shear-thinning is most common
  - In transporting fluids, pressure drop increases less than linearly with flow rate
  - Newtonian: linear variation
- Experiment: 2 fluids of same density

A: small particle settling

B: draining of tube under gravity
Non-Newtonian phenomena 2

• Shear generates normal stress **differences**
  - The normal stress differences generate observable flow effects
    - Viscoelastic effects
    - Note: isotropic normal stresses (pressure) only generate motion via gradients

• Examples:
  - Rod climbing (Weisssenberg effect)
  - Die swell
  - Tanner’s tilted trough
  - Hole pressure effect

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Bird, Armstrong, Hassager, "Dynamics of Polymeric Liquids, Vol. 1"
Non-Newtonian phenomena 3

• Extensional and memory effects
  – Elastic recoil
  – Tubeless siphon
  – Relaxation timescales in rheometry

• Examples
  http://web.mit.edu/nnf/
Non-Newtonian phenomena 4

- Negative wakes & velocity jumps/hysteresis, e.g.
  - Fraggedakis et al 2016

**Figure 22.** (Colour online) (a) Rise velocity for varying Deborah number and \( (\text{Ar, Bo, } \varepsilon, \beta) = (0, 0.4, 0.05, 10^{-3}) \), (b) shear rate, \( ||\dot{\gamma}|| \) for \( De = 6 \) along the hysteresis region.

**Figure 8.** (Colour online) (a,b) Flow field around the bubble before (blue) and after (red) the velocity jump. (c) Axial velocity along the axis of symmetry, the bubble axis is in \(-1 < Z < 1\). Three cases are considered for \( (R_0, \text{Ar, De, Bo}) = (2.03, 0.019, 4.95, 0.35) \).

**Figure 23.** Experimental versus predicted bubble shapes for J-100, Astarita & Apuzzo (1965) under the conditions given in table 3. (a) Experiments, (b) simulation.
Non-Newtonian phenomena 5

- Viscoelastic turbulence
- Yielding and plug zones
- Secondary flows in rotating spheres & lid-driven cavities
- Filament formation and instability – beads on a string
- Bubbles shapes
- Drag reduction in turbulent flows