**Question 1:**

![Diagram](image)

**Solution:** (a) Considering the right side of the liquid column, the surface tension acts tangent to the local surface, that is, along the dashed line at right. This force has magnitude \( F = Yb \), as shown. Its vertical component is \( F \cos(\theta - \alpha) \), as shown. There are two plates. Therefore, the total z-directed force on the liquid column is

\[
F_{\text{vertical}} = 2Yb \cos(\theta - \alpha) \quad \text{Ans. (a)}
\]

(b) The vertical force in (a) above holds up the entire weight of the liquid column between plates, which is \( W = \rho g \{bh(L - h \tan \alpha)\} \). Set \( W \) equal to \( F \) and solve for

\[
U = \frac{\rho g bh (L - h \tan \alpha)}{2 \cos(\theta - \alpha)} \quad \text{Ans. (b)}
\]
Question 2:

For the cone-plate viscometer in Fig. P1.56, the angle is very small, and the gap is filled with test liquid \( \mu \). Assuming a linear velocity profile, derive a formula for the viscosity \( \mu \) in terms of the torque \( M \) and cone parameters.

Solution: For any radius \( r \leq R \), the liquid gap is \( h = r \tan \theta \). Then

\[
\begin{align*}
\text{d(Torque)} &= \text{d}M = \tau \text{d}A_{\omega} \, r = \left( \mu \frac{\Omega r}{r \tan \theta} \right) \left( 2 \pi r \frac{dr}{\cos \theta} \right) r, \quad \text{or} \\
M &= \frac{2 \pi \Omega \mu}{\sin \theta} \int_0^R r^2 \, dr = \frac{2 \pi \Omega \mu R^3}{3 \sin \theta}, \quad \text{or} \quad \mu = \frac{3 M \sin \theta}{2 \pi \Omega R^3} \quad \text{Ans.}
\end{align*}
\]
Question 3:

Given: Block of mass \( M \) slides on thin film of oil of thickness \( h \). Contact area of block is \( A \). At time \( t = 0 \), mass \( m \) is released from rest.

Find: (a) Expression for viscous force on block when moving at speed \( v \)
(b) Differential equation governing block speed as a function of time
(c) Expression for block speed \( v = v(t) \); sketch.

Solution:

Basic equations: \( 
\Sigma F = \mu \frac{dv}{dt} \quad \Sigma F = m a
\)

Assumptions: (1) Newtonian fluid
(2) Linear velocity profile in oil film

Then, \( F_v = \tau A = \mu \frac{dv}{dt} A = \mu \frac{dv}{dt} h A \)

For the block, \( \Sigma F_e = F_e - F_v = M \frac{dv}{dt} \) \( \tag{1} \)

For the falling mass \( \Sigma F_g = mg - F_e = m \frac{dv}{dt} \), or
\( F_e = mg - m \frac{dv}{dt} \) \( \tag{2} \)

Since \( v_b = v_c = v \), then substituting from Eq. (2) into 1 gives
\( mg - m \frac{dv}{dt} = m \frac{dv}{dt} = \mu \frac{dv}{dt} h A \)

Finally, \( mg - \mu \frac{dv}{dt} h A = (M + m) \frac{dv}{dt} \) \( \frac{dt}{dt} \) \( \tag{3} \)

To solve we separate variables and integrate
\( \int \frac{1}{m + 1} \frac{dt}{dt} = \int \left( \frac{(M + m)\frac{dv}{dt}}{m} \right) \frac{dt}{dt} = -(M + m) \frac{1}{m} \ln \left( \frac{1 - \mu v}{1 - \mu v} \right) \)

Taking antilogarithms,
\( 1 - \frac{\mu v}{1 - \mu v} = e^{-\frac{t}{m}} \)

Solving for \( v \),
\( v = \frac{m}{\mu} \ln \left( \frac{1 - \mu v}{1 - \mu v} \right) \)
The velocity increases exponentially to \( v_{max} = \frac{mg}{\mu A} \).
Question 4:

A thin moving plate is separated from two fixed plates by two fluids of unequal viscosity and unequal spacing, as shown below. The contact area is $A$. Determine (a) the force required, and (b) is there a necessary relation between the two viscosity values?

![Diagram of the setup](image)

Solution: (a) Assuming a linear velocity distribution on each side of the plate, we obtain

$$F = \tau_1 A + \tau_2 A = \left( \frac{\mu_1 V}{h_1} + \frac{\mu_2 V}{h_2} \right) A \quad Ans. \ (a)$$

The formula is of course valid only for laminar (nonturbulent) steady viscous flow.
Question 5:

\[ F_h = P_{cg} A = \rho g \frac{h}{2} b h = \rho g \frac{b h^2}{2} \]

\[ F_v = W_p = \rho g \cdot b \cdot \frac{h}{2} \cdot \frac{h}{2 \tan \theta} = \rho g \frac{b h^2}{4 \tan \theta} \]

\[ \sum M_c = 0 = Ph - \rho g b h^2 \cdot \frac{h}{3} - \rho g b h^2 \cdot \frac{h}{6 \tan \theta} \]

\[ \Rightarrow p = \frac{\rho g b h^2}{2h^2} \left( \frac{4 \tan^2 \theta + 1}{\tan^2 \theta} \right) = \frac{\rho g b h^2}{2h} \left( \frac{1}{\tan \theta} \right) \]
Question 6:

The tank in Fig. P2.63 has a 4-cm-diameter plug which will pop out if the hydrostatic force on it reaches 25 N. For 20°C fluids, what will be the reading \( h \) on the manometer when this happens?

**Solution:** The water depth when the plug pops out is

\[
F = 25 \text{ N} = \gamma h_{CG} A = (9790)h_{CG} \frac{\pi(0.04)^2}{4}
\]

or \( h_{CG} = 2.032 \text{ m} \)

It makes little numerical difference, but the mercury-water interface is a little deeper than this, by the amount \( (0.02 \sin 50^\circ) \) of plug-depth, plus 2 cm of tube length. Thus

\[
p_{atm} + (9790)(2.032 + 0.02 \sin 50^\circ + 0.02) - (133100)h = p_{atm},
\]

or: \( h = 0.152 \text{ m} \quad \text{Ans.} \)