Solution 1:

**Solution:** The test section wall area is \((\pi)(0.8 \text{ m})(4 \text{ m}) = 10.053 \text{ m}^2\), hence the total number of holes is \((1200)(10.053) = 12064\) holes. The total suction flow leaving is

\[
Q_{\text{suction}} = NQ_{\text{hole}} = (12064)(\pi/4)(0.005 \text{ m})^2(8 \text{ m/s}) \approx 1.895 \text{ m}^3/\text{s}
\]

(a) Find \(V_0\): \(Q_o = Q_1\) or \(V_o \frac{\pi}{4}(2.5)^2 = (35) \frac{\pi}{4}(0.8)^2\),

solve for \(V_o \approx 3.58 \frac{\text{m}}{\text{s}}\) \text{ Ans. (a)}

(b) \(Q_2 = Q_1 - Q_{\text{suction}} = (35) \frac{\pi}{4}(0.8)^2 - 1.895 = V_2 \frac{\pi}{4}(0.8)^2\),

or: \(V_2 \approx 31.2 \frac{\text{m}}{\text{s}}\) \text{ Ans. (b)}

(c) Find \(V_f\): \(Q_f = Q_2\) or \(V_f \frac{\pi}{4}(2.2)^2 = (31.2) \frac{\pi}{4}(0.8)^2\),

solve for \(V_f \approx 4.13 \frac{\text{m}}{\text{s}}\) \text{ Ans. (c)}
Solution 2:

**Given:** Small lawn sprinkler as shown.

**Find:**
(a) Jet speed relative to each nozzle.
(b) Friction torque at pivot.

**Solution:**
Apply continuity and angular momentum equations using fixed control volume enclosing sprinkler arms.

Governing equations:
\[
\frac{\partial}{\partial t} \int_{CV} \rho \, dV + \int_{CS} \rho \mathbf{V} \cdot d\mathbf{A} = 0
\]
\[
\mathbf{r} \times \mathbf{F} + \int_{CV} \mathbf{r} \times \mathbf{g} \, dV + \mathbf{T}_{shaft} = \frac{\partial}{\partial t} \int_{CV} \mathbf{r} \times \mathbf{V} \, dV + \int_{CS} \mathbf{r} \times \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A}
\]

where all velocities are measured relative to the inertial coordinates XYZ.

Assumptions:
(1) Incompressible flow.
(2) Uniform flow at each section.
(3) \(\omega = \) constant.

From continuity, the jet speed relative to the nozzle is given by
\[
V_{rel} = \frac{Q}{2A_{jet}} = \frac{Q}{2 \frac{4}{\pi} D_{jet}^2} = \frac{1}{2} \times \frac{7.5}{\text{min}} \times \frac{4}{\pi} \frac{1}{(4)^2} \frac{\text{mm}^2 \times 10^6 \text{mm}^2 \times 10^{-6} \text{min}}{1000 \text{L} \times \text{m}^2 \times 60 \text{s}} = 4.97 \text{ m/s}
\]

Consider terms in the angular momentum equation separately. Since atmospheric pressure acts on the entire control surface, and the pressure force at the inlet causes no moment about \(O\), \(\mathbf{r} \times \mathbf{F} = 0\). The moments of the body (i.e., gravity) forces in the two arms are equal and opposite and hence the second term on the left side of the equation is zero. The only external torque acting on the CV is friction in the pivot. It opposes the motion, so
\[
\mathbf{T}_{shaft} = -T_f \hat{K}
\]

Our next task is to determine the two angular momentum terms on the right side of Eq. 1. Consider the unsteady term: This is the rate of change of angular momentum in the control volume. It is clear that although the position \(\mathbf{r}\) and velocity \(\mathbf{V}\) of fluid particles are functions of time in XYZ coordinates, because
the sprinkler rotates at constant speed the control volume angular momentum is constant in XYZ coordinates, so this term is zero; however, as an exercise in manipulating vector quantities, let us derive this result. Before we can evaluate the control volume integral, we need to develop expressions for the instantaneous position vector, \( \vec{r} \), and velocity vector, \( \vec{V} \) (measured relative to the fixed coordinate system XYZ) of each element of fluid in the control volume.

\[ \vec{V} = \hat{i}(V_t \cos \theta - r \omega \sin \theta) + \hat{j}(V_t \sin \theta + r \omega \cos \theta) \]

(Note that \( \theta \) is a function of time.) The position vector is

\[ \vec{r} = \hat{i}r \cos \theta + \hat{j}r \sin \theta \]

and

\[ \vec{r} \times \vec{V} = \hat{k}(r^2 \omega \cos^2 \theta + r^2 \omega \sin^2 \theta) = \hat{k}r^2 \omega \]

Then

\[ \int_{\nu_{OA}} \vec{r} \times \vec{V} \rho d\nu = \int_{0}^{R} \hat{k}r^2 \omega \rho A dr = \hat{k} \frac{R^3 \omega}{3} \rho A \]

and

\[ \frac{\partial}{\partial t} \int_{\nu_{OA}} \vec{r} \times \vec{V} \rho d\nu = \frac{\partial}{\partial t} \left[ \hat{k} \frac{R^3 \omega}{3} \rho A \right] = 0 \]

where \( A \) is the cross-sectional area of the horizontal tube. Identical results are obtained for the other horizontal tube in the control volume. We have confirmed our insight that the angular momentum within the control volume does not change with time.

Now we need to evaluate the second term on the right, the flux of momentum across the control surface. There are three surfaces through which we have mass and therefore momentum flux: the supply line (for which \( \vec{r} \times \vec{V} = 0 \) because \( \vec{r} = 0 \)) and the two nozzles. Consider the nozzle at the end of branch \( OAB \). For \( L \ll R \), we have
\[ \vec{r}_{\text{jet}} = \vec{r}_{B} = \vec{r} \bigg|_{r=R} = (\hat{I}r \cos \theta + \hat{J}r \sin \theta) \bigg|_{r=R} = \hat{I}R \cos \theta + \hat{J}R \sin \theta \]

and for the instantaneous jet velocity \( \vec{V}_j \) we have

\[
\vec{V}_j = \vec{V}_{\text{rel}} + \vec{V}_{\text{tip}} = \hat{I}V_{\text{rel}} \cos \alpha \sin \theta - \hat{J}V_{\text{rel}} \cos \alpha \cos \theta + \hat{K}V_{\text{rel}} \sin \alpha - \hat{I} \omega R \sin \theta + \hat{J} \omega R \cos \theta
\]

\[
\vec{V}_j = \hat{I}(V_{\text{rel}} \cos \alpha - \omega R) \sin \theta - \hat{J}(V_{\text{rel}} \cos \alpha - \omega R) \cos \theta + \hat{K}V_{\text{rel}} \sin \alpha
\]

\[
\vec{r}_B \times \vec{V}_j = \hat{I} RV_{\text{rel}} \sin \alpha \sin \theta - \hat{J} RV_{\text{rel}} \sin \alpha \cos \theta - \hat{K} R(V_{\text{rel}} \cos \alpha - \omega R)(\sin^2 \theta + \cos^2 \theta)
\]

\[
\vec{r}_B \times \vec{V}_j = \hat{I} RV_{\text{rel}} \sin \alpha \sin \theta - \hat{J} RV_{\text{rel}} \sin \alpha \cos \theta - \hat{K} R(V_{\text{rel}} \cos \alpha - \omega R)
\]

The flux integral evaluated for flow crossing the control surface at location B is then

\[
\int_{CS} \vec{r} \times \vec{V}_j \rho \vec{v} \cdot d\vec{A} = \left[ \hat{I} RV_{\text{rel}} \sin \alpha \sin \theta - \hat{J} RV_{\text{rel}} \sin \alpha \cos \theta - \hat{K} R(V_{\text{rel}} \cos \alpha - \omega R) \right] \rho Q \frac{Q}{2}
\]

The velocity and radius vectors for flow in the left arm must be described in terms of the same unit vectors used for the right arm. In the left arm the \( I \) and \( J \) components of the cross product are of opposite sign, since \( \sin (\theta + \pi) = -\sin \theta \) and \( \cos (\theta + \pi) = -\cos \theta \). Thus for the complete CV,

\[
\int_{CS} \vec{r} \times \vec{V}_j \rho \vec{v} \cdot d\vec{A} = -\hat{K} R(V_{\text{rel}} \cos \alpha - \omega R) \rho Q
\]

Substituting terms (2), (3), and (4) into Eq. 1, we obtain

\[-T_f \hat{K} = -\hat{K} R(V_{\text{rel}} \cos \alpha - \omega R) \rho Q
\]

\[T_f = R(V_{\text{rel}} \cos \alpha - \omega R) \rho Q\]

This expression indicates that when the sprinkler runs at constant speed the friction torque at the sprinkler pivot just balances the torque generated by the angular momentum of the two jets.

From the data given,

\[\omega R = \frac{30 \text{ rev}}{\text{min}} \times \frac{150 \text{ mm}}{\text{rev}} \times \frac{2\pi \text{ rad}}{\text{rev}} \times \frac{\text{min}}{60 \text{ s}} \times \frac{\text{m}}{1000 \text{ mm}} = 0.471 \text{ m/s}\]

Substituting gives

\[T_f = \frac{150 \text{ mm} \times \left(4.97 \text{ m/s} \times \cos 30^\circ - 0.471 \text{ m/s} \right) \times 999 \text{ kg} \times 7.5 \text{ L}}{1000 \text{ L} \times 60 \text{ s} \times \frac{N \cdot s^3}{\text{kg} \cdot \text{m} \times 1000 \text{ mm}}} \]

\[T_f = 0.0718 \text{ N} \cdot \text{m} \]
Solution 3:

Consider uniform flow past a cylinder with a \( V \)-shaped wake, as shown. Pressures at (1) and (2) are equal. Let \( b \) be the width into the paper. Find a formula for the force \( F \) on the cylinder due to the flow. Also compute \( C_D = F/(\rho U^2 L b) \).

**Solution:** The proper CV is the entrance (1) and exit (2) plus streamlines above and below which hit the top and bottom of the wake, as shown. Then steady-flow continuity yields,

\[
0 = \int_2^1 \rho u \, dA - \int_1^0 \rho u \, dA = 2 \int_0^L \rho \frac{U}{2} \left( 1 + \frac{y}{L} \right) b \, dy - 2 \rho U b H.
\]

where \( 2H \) is the inlet height. Solve for \( H = 3L/4 \).

Now the linear momentum relation is used. Note that the drag force \( F \) is to the right (force of the fluid on the body) thus the force \( F \) of the body on fluid is to the left. We obtain,

\[
\sum F_x = 0 = \int_2^1 u \rho u \, dA - \int_1^0 u \rho u \, dA = 2 \int_0^L \frac{U}{2} \left( 1 + \frac{y}{L} \right) \rho \frac{U}{2} \left( 1 + \frac{y}{L} \right) b \, dy - 2H \rho U^2 b = -F_{\text{drag}}
\]

Use \( H = \frac{3L}{4} \), then \( F_{\text{drag}} = \frac{3}{2} \rho U^2 L b - \frac{7}{6} \rho U^2 L b = \frac{1}{3} \rho U^2 L b \quad \text{Ans.} \)

The dimensionless force, or drag coefficient \( F/(\rho U^2 L b) \), equals \( C_D = 1/3 \). \quad \text{Ans.}
Solution 4:

Assumptions 1 The flow is steady and incompressible. 2 The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. 3 Effects of water falling down during upward discharge is disregarded. 4 Pipe outlet diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be 1000 kg/m³.

Analysis We take the entire pipe as the control volume, and designate the inlet by 1 and the outlet by 2. We also take the x and y coordinates as shown. The control volume and the reference frame are fixed.

The conservation of mass equation for this one-inlet one-outlet steady flow system is \( \dot{m}_1 = \dot{m}_2 = \dot{m}_o \), and \( V_1 = V_2 = V \) since \( A_c = \) constant. The mass flow rate and the weight of the horizontal section of the pipe are

\[
\dot{m} = \rho A_c V = 1000 \text{ kg/m}^3 \cdot \left( \pi \left( \frac{0.12}{2} \text{ m} \right)^2 / 4 \right) (4 \text{ m/s}) = 45.24 \text{ kg/s}
\]

\[
W = mg = 15 \text{ kg} / \text{m} \cdot (2 \text{ m}) \cdot 9.81 \text{ m/s}^2 \cdot \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 294.3 \text{ N/m}
\]

(a) Downward discharge: To determine the moment acting on the pipe at point \( A \), we need to take the moment of all forces and momentum flows about that point. This is a steady and uniform flow problem, and all forces and momentum flows are in the same plane. Therefore, the angular momentum equation in this case can be expressed as \( \sum M = \sum_{\text{out}} r \dot{m}V - \sum_{\text{in}} r \dot{m}V \) where \( r \) is the moment arm, all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative.

The free body diagram of the pipe section is given in the figure. Noting that the moments of all forces and momentum flows passing through point \( A \) are zero, the only force that will yield a moment about point \( A \) is the weight \( W \) of the horizontal pipe section, and the only momentum flow that will yield a moment is the outlet stream (both are negative since both moments are in the clockwise direction). Then the angular momentum equation about point \( A \) becomes

\[
M_A = r_2 W - r_2 \dot{m}V_2
\]

Solving for \( M_A \) and substituting,

\[
M_A = r_1 W - r_2 \dot{m}V_2 = (1 \text{ m})(294.3 \text{ N}) - (2 \text{ m})(45.54 \text{ kg/s})(4 \text{ m/s}) \cdot \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = -70.0 \text{ N·m}
\]

The negative sign indicates that the assumed direction for \( M_A \) is wrong, and should be reversed. Therefore, a moment of 70 N·m acts at the stem of the pipe in the clockwise direction.

(b) Upward discharge: The moment due to discharge stream is positive in this case, and the moment acting on the pipe at point \( A \) is

\[
M_A = r_1 W + r_2 \dot{m}V_2 = (1 \text{ m})(294.3 \text{ N}) + (2 \text{ m})(45.54 \text{ kg/s})(4 \text{ m/s}) \cdot \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 659 \text{ N·m}
\]

Discussion Note direction of discharge can make a big difference in the moments applied on a piping system. This problem also shows the importance of accounting for the moments of momentum of flow streams when performing evaluating the stresses in pipe materials at critical cross-sections.
Solution 5:

Given: Tank driven by jet along horizontal track. Neglect resistance. Acceleration is from rest. Initial mass is $M_0$. Track horizontal.

Find: (a) Apply continuity and momentum to show $M = M_0 V/(V-U)$
(b) General expression for $U/V$ as a function of time.

Solution: Apply continuity and $x$ component of momentum equation to linearly accelerating CV shown.

Basic equations: 

$$0 = \frac{d}{dt} \int_{CV} \rho v_x \, dV + \int_{CV} \rho v_y v_z \, dA$$

Case (1): $t = 0$

$$F_{Sx} - \int_{CV} \sigma_n v_x \, dV = \frac{d}{dt} \int_{CV} \rho v_x v_x \, dV + \int_{CV} u_y v_z \rho v_y v_z \, dA$$

Assumptions: (1) $F_{Sx} = 0$
(2) $F_{Sy} = 0$
(3) Neglect $U$ within CV
(4) Uniform flow in jet

From continuity

$$0 = \frac{d}{dt} M_{CV} + \left\{- \frac{1}{2} \rho (V-U) A \right\} \text{ or } \frac{dM}{dt} = \rho (V-U) A$$

From momentum

$$- \sigma_n \frac{d}{dt} M = \frac{dU}{dt} M = \frac{1}{2} (V-U) \rho (V-U) A \text{; } U = V-U$$

But from continuity, $\rho (V-U) A = \frac{dM}{dt}$, and $dU = -d(V-U)$, so

$$- \frac{dU}{dt} M = \frac{d(V-U)}{dt} M = -(V-U) \frac{dM}{dt} \text{ or } M(V-U) = \text{constant} = M_0 V$$

Thus $M = M_0 V/(V-U)$

Substituting into momentum, $- \frac{dU}{dt} M = \frac{d(V-U)}{dt} \frac{M_0 V}{(V-U)} = -\rho (V-U)^2 A$, or

$$\frac{d(V-U)}{(V-U)^3} = -\frac{\rho A}{V M_0} \frac{dt}{dt}$$

Integrating,

$$\int_{V-U}^{U} \frac{d(V-U)}{(V-U)^3} = \frac{1}{2} \left[ \frac{1}{(V-U)^2} - \frac{1}{V^2} \right] = -\int_0^t \frac{\rho A}{V M_0} \, dt = -\frac{\rho A}{V M_0} t$$

Solving

$$\frac{U}{V} = \left\{ 1 - \left[ 1 + 2 \frac{\rho V A}{M_0} t \right] \right\}^{1/2}$$
Thus \[ \frac{gD_2}{2} \left( 1 + \frac{D_2}{D_1} \right) = V_1 \frac{D_2}{D_1} \] or \[ \frac{D_1}{D_1} \left( 1 + \frac{D_2}{D_2} \right) = \frac{2V_1^2}{gD_1} \] or \[ \frac{D_1}{D_1} + \frac{D_2}{D_2} - \frac{2V_1^2}{gD_1} = 0 \]

Using the quadratic equation,
\[
\frac{D_1}{D_1} = \frac{1}{2} \left[ -1 \pm \sqrt{1 + \frac{8V_1^2}{gD_1}} \right] \quad \text{or} \quad D_2 = \frac{D_1}{2} \left[ \sqrt{1 + \frac{8V_1^2}{gD_1}} - 1 \right]
\]

Solving for \( D_2 \)

\[
D_2 = \frac{1}{2} \times 0.6m \left[ \sqrt{1 + \frac{8 \times (5 \times 10^{-3})^2}{0.6m}} - \frac{1}{0.6m} \right] = 1.47 m
\]

\[
V_k = \frac{D_2}{V_k} \times \frac{5 \text{m}}{s} = 2.04 \text{ m/s}
\]

From the energy equation, with \( \epsilon_{\text{mach}} = \frac{V_1^2}{2} + g \frac{x}{2} + \frac{P}{\rho} \), and \( dA = w \, dx \), the mechanical energy fluxes are

\[
\text{mef}_1 = \int_0^{D_2} \left[ \frac{V_1^2}{2} + g \frac{x}{2} + \frac{P}{\rho} \right] \rho V_1 \, w \, dx = \left( \frac{V_1^2}{2} + g D_2 \right) \rho V_1 \, w \, D_2
\]

\[
\text{mef}_2 = \int_0^{D_2} \left[ \frac{V_2^2}{2} + g \frac{x}{2} + \frac{P}{\rho} \right] \rho V_2 \, w \, dx = \left( \frac{V_2^2}{2} + g D_2 \right) \rho V_2 \, w \, D_2
\]

and

\[
\Delta \text{mef} = \text{mef}_2 - \text{mef}_1 = \left[ \frac{V_2^2}{2} - \frac{V_1^2}{2} + g (D_2 - D_1) \right] \rho V_1 \, w \, D_2, \quad \text{since } V_1 D_1 = V_2 D_2
\]

Thus \( \frac{\Delta \text{mef}}{\Delta \text{m}} = \frac{1}{2} \left[ \frac{V_2^2}{2} - \frac{V_1^2}{2} + g (D_2 - D_1) \right] \rho V_1 \, w \, D_2 \)

\[
\Delta \text{mef} = \frac{1}{2} \left[ \frac{V_2^2}{2} - \frac{V_1^2}{2} + g (D_2 - D_1) \right] \rho V_1 \, w \, D_2 = -1.88 \text{ Nm/kg}
\]

From the energy equation,

\[
0 = \left[ u_1 + \frac{V_1^2}{2} + g \frac{x}{2} + \frac{P}{\rho} \right] \left\{ - \frac{\rho V_1 \, w \, D_2}{1} \right\}
\]

or

\[
0 = (u_2 - u_1) \Delta \text{m} + \Delta \text{mef}
\]

Thus

\[
u_2 - u_1 = C_\text{v} \left( T_e - T_i \right) = -\frac{\Delta \text{mef}}{\Delta \text{m}}
\]

\[
\Delta T = T_e - T_i = -\frac{\Delta \text{mef}}{\Delta \text{m} C_\text{v}} = -\frac{1.88 \text{ Nm/kg}}{1 \text{kcal/kg}} \times \frac{1 \text{kcal}}{4184 \text{ J}} = 4.49 \times 10^{-4} \text{ K}
\]

\{ This small temperature change would be almost impossible to measure. \}
Solution 6:

Water flows through the pipe contraction shown in Fig. P3.30. For the given 0.2-m difference in manometer level, determine the flow-rate as a function of the diameter of the small pipe, \(D\).

\[
\begin{align*}
\frac{F_1}{g} + \frac{V_1^2}{2g} + Z_1 &= \frac{F_2}{g} + \frac{V_2^2}{2g} + Z_2 \quad \text{or with } Z_1 = Z_2 \text{ and } V_1 = 0 \\
V_2 &= \sqrt{2g \left( \frac{p_1 - p_2}{\rho_g} \right)} \\
\text{but } \rho_1 &= \delta h_1 \text{ and } \rho_2 = \delta h_2 \text{ so that } p_1 - p_2 = \delta (h_1 - h_2) = 0.2 \delta \\
\text{Thus,} \\
V_2 &= \sqrt{2g \left( \frac{0.2 \delta}{\delta} \right)} = \sqrt{2g (0.2)} \\
\text{or} \\
Q &= A_2 V_2 = \frac{\pi}{4} D^2 V_2 = \frac{\pi}{4} D^2 \sqrt{2 (9.81) (0.2)} = 1.56 D^2 \frac{m^3}{s} \text{ when } D \sim m
\end{align*}
\]