Gate AB is a quarter circle or radius $R = 2\text{m}$ and depth $D = 5\text{m}$ (in the $3^{rd}$ dimension). It has weight $W = 5\text{kN}$ which acts vertically as shown (the distance marked indicates the position of the centroid of a thin quarter circle shell).

**Question 1. (8 marks)**

a) [2 marks] Calculate the horizontal force acting on the gate AB due to the water and specify the depth at which it acts.

b) [3 marks] Calculate the vertical force acting on the gate AB due to the water and specify the horizontal position at which it acts.

c) [3 marks] Calculate the moments around the hinge at B and in this way determine the vertical force $F_F$, exerted at A, needed to keep the gate in position.

\[
F_H = Pca\left|A\right| = \frac{pg R^2}{2} RD = \frac{10^3 \times 9.81 \times 4 \times 5}{2} = 98.1 \text{ kN} \quad (pg \frac{R^2}{2}D) \quad 1
\]

\[
\text{Displaced volume is in shaded. } F_V \text{ acts upwards through centroid of } V_D. \quad \frac{V_D}{V_D} = \frac{V_D}{V_D} + \frac{V_W}{V_W} = \frac{V_D}{V_D} + \frac{V_W}{V_W}
\]

\[
\Rightarrow x_{cc}V_D (R^2 - \frac{\pi R^2}{4}) = \frac{R^2}{2} R^2 W - \left(1 - \frac{R}{3\pi} \right) \frac{\pi R^2 W}{4} \quad 1.5
\]

\[
x_{cc}V_D = R \left( \frac{1}{2} + \frac{1}{3} - \frac{\pi}{4} \right) = 0.2233 \text{ R from B}
\]
\[ \Sigma M_B = 0 \implies FR + W(1 - \frac{2}{\pi})R - F_H \frac{R}{3} - F_V 0.2233R = 0 \]

\[ F = \frac{F_H}{3} + 0.2233F_V - (1 - \frac{2}{\pi})W \]

\[ = \frac{98.1 + 0.2233 \times 42.16}{3} - (1 - \frac{2}{\pi})5 \]

\[ = 40.28 \text{ kN} \]

\text{Answer: 40.28 kN}
Question 2. (4 marks – see page 1 for data)
a) [2 marks] A narrow tube is inserted vertically into a bath of fluid which rises up the tube under capillary action. Which of the following situations gives the largest rise \( h \)?

(i) Glycerin in a 3mm radius tube with contact angle 25° (\( \rho = 1260 \text{ kg/m}^3 \))

(ii) Water at 20°C in a 4mm tube with contact angle 22°

(iii) SAE 30 oil in a 3mm radius tube with contact angle 15° (\( \rho = 860 \text{ kg/m}^3 \))

\[
\text{Force balance:} \quad \pi R^2 h \rho g = 2 \pi R \sigma_s \cos \phi
\]
\[
\Rightarrow h = \frac{2 \sigma_s \cos \phi}{\rho g R} \quad \text{(0.5)}
\]

(i) \( h = 3.08 \text{ mm} \) (0.5)

(ii) \( h = 3.45 \text{ mm} \) (0.5)

(iii) \( h = 2.67 \text{ mm} \) (0.5)

\[\therefore \text{ water has highest rise}\]
b) [2 marks] Contrary to what you might expect, a solid steel ball can float on water due to surface tension. Determine the maximum diameter of a steel ball that would float on water at 20°C. What would your answer be for an aluminum ball? Take the densities of steel and aluminum to be 7800 kg/m³ and 2700 kg/m³, respectively.

That could be revision on this, depending on your sketch, but this seems to be the best possible.

\[ W = \rho g V = 7800 \times 9.81 \times \frac{4}{3} \pi R^3 \]

\[ F_{\sigma_s} = 2\pi R \sigma_s = 2\pi \times 0.073 \times R \]

\[ F_B = 1000 \times 9.81 \times \frac{4}{3} \pi R^3 \]

(or you could have \( \frac{1}{2} ( \frac{4}{3} \pi R^3 ) \))

\[ \Rightarrow 0 = (\rho - \rho_{\text{water}}) g \frac{4}{3} \pi R^3 - 2\pi \sigma_s R \]

\[ \Rightarrow R = \sqrt{\frac{3 \times 7800 \sigma_s}{2g(\rho - \rho_{\text{water}})}} = 1.28 \text{ mm (steel)} \]

\[ 2.56 \text{ mm (aluminum)} \]

The End