Lecture 1-4: Pipe flows

- Introduce friction factor – estimating head losses in pipe and ducts
- Laminar and turbulent flows
- Introduce Moody chart
- Minor losses
- Hydraulic diameter, non-circular ducts, non-Newtonian fluids

Motivations:
- In networks of pipes, in systems with long sections of pipe, with small pipes, viscous fluids, etc, frictional head losses become significant and must be modeled
- How do we account for them in the energy balance, so we can properly size pumps and deal with other design aspects?
Fully Developed Pipe Flow

- Comparison of laminar and turbulent flow

There are some major differences between laminar and turbulent fully developed pipe flows

**Laminar**
- Can solve exactly
- Flow is steady
- Velocity profile is parabolic
- Pipe roughness not important

It turns out that: \[ u_{avg} = \frac{1}{2} u_{max} \quad \text{and} \quad u(r) = 2 u_{avg}(1 - r^2/R^2) \]
Fully Developed Pipe Flow

Turbulent

- Flow is unsteady (3D swirling eddies), but steady in the mean
- Mean velocity profile is fuller, with very sharp slope at the wall
- $u_{avg}$ roughly 85% of $u_{max}$ (depends on Re a bit)

- Cannot solve exactly (flow is too complex and is unsteady)
  - No analytical solution, but there are some good semi-empirical expressions that approximate the velocity profile shape.
  - Logarithmic law & power law expressions covered earlier
  - Pipe roughness is very important – disturbs the wall layer
Fully Developed Pipe Flow Wall-shear stress

- For simple shear flows \( u = u(y) \), we had \( \tau_{xy} = \mu \frac{du}{dy} \)
- Axisymmetric flows \( u = u(r) \), becomes \( \tau_{xr} = \mu \frac{du}{dr} \)
- Wall shear stress \( \tau_w \) is the value of \( |\tau| \) at wall
  - If we know \( u(r) \), we can simply evaluate this

\[
\tau_{w,turb} > \tau_{w,lam}
\]
How do we analyze pipe flow?

Let’s apply conservation of mass, momentum, and energy to this CV

Conservation of mass:

\[ \dot{m}_1 = \dot{m}_2 = \dot{m} \]

\[ \rho Q_1 = \rho Q_2 \implies Q = \text{constant} \]

\[ V_1 \frac{\pi D^2}{4} = V_2 \frac{\pi D^2}{4} \implies V_1 = V_2 \]
Momentum and energy balances:

- Conservation of x-momentum

\[ \sum F_x = \sum F_{x,\text{grav}} + \sum F_{x,\text{press}} + \sum F_{x,\text{visc}} + \sum F_{x,\text{other}} = \sum \beta mV - \sum \beta mV \]

\[ (P_1 - P_2) \frac{\pi D^2}{4} = \tau_w \pi D L \]

\[ P_1 - P_2 = 4 \tau_w \frac{L}{D} \]

Terms cancel since \( \beta_1 = \beta_2 \) and \( V_1 = V_2 \)

- Energy equation (in head form)

\[ \frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L \]

Terms cancel \( V_1 = V_2, \alpha_1 = \alpha_2 \) (shape not changing), and \( z_1 = z_2 \)

- \( h_L = \) irreversible head loss, felt as a pressure drop

\[ P_1 - P_2 = \rho g h_L \]
Summary: Fully Developed Pipe Flow

- Momentum CV analysis:
  \[ P_1 - P_2 = 4\tau_w \frac{L}{D} \]

- Energy CV analysis:
  \[ P_1 - P_2 = \rho g h_L \]

- Equating the two gives:
  \[ h_L = \frac{4\tau_w}{\rho g} \frac{L}{D} \]

- To predict head loss, we need to be able to calculate \( \tau_w \).
  - Laminar flow: solve exactly in simple geometries
  - Turbulent flow: rely on empirical data (experiments)

- In either case, we can benefit from dimensional analysis
Dimensional analysis for $\tau_w$

- Starting point: $\tau_w$ should depend on fluid properties, velocity, pipe diameter & roughness (if turbulent)

$$\Pi_1 = \frac{8\tau_w}{\rho V^2}, \quad \Pi_2 = \frac{\rho V D}{\mu} = \text{Re}, \quad \Pi_3 = \frac{\varepsilon}{D} \quad \Rightarrow \quad \frac{8\tau_w}{\rho V^2} = f\left(\text{Re}, \frac{\varepsilon}{D}\right)$$
Now go back to equation for $h_L$ and substitute $f$ for $\tau_w$

$$h_L = \frac{4\tau_w L}{\rho g D} \quad f = \frac{8\tau_w}{\rho V^2} \rightarrow \tau_w = f \frac{\rho V^2}{8}$$

Our problem is now reduced to solving for Darcy-Weisbach friction factor $f=f(Re, \varepsilon/D)$

- Laminar flow: $f = 64/Re$ (exact). For laminar flow roughness does not affect $h_L$ except in extreme cases
- Turbulent flow: Use charts or empirical equations (Moody Chart, a famous plot of $f$ vs. $Re$ and $\varepsilon/D$, See Fig. A-12, p. 934 in text)
Laminar flow $\tau_w$

\[ f = \frac{64}{\text{Re}} \]
Pipe Roughness $\varepsilon$

- Absolute roughness $\varepsilon$ is an average value
- Commercial products typically have a range of roughness, described in charts & tables
  - Uncertainty means there can be a need to repeat calculations for different $\varepsilon$.
  - Depends on application and on $\varepsilon/D$
  - In some situations actual and nominal diameters can vary considerably

<table>
<thead>
<tr>
<th>Surface</th>
<th>Absolute Roughness - $\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10^{-3}$ (m)</td>
</tr>
<tr>
<td></td>
<td>(feet)</td>
</tr>
<tr>
<td>Copper, Lead, Brass, Aluminum (new)</td>
<td>0.001 - 0.002</td>
</tr>
<tr>
<td>PVC and Plastic Pipes</td>
<td>0.0015 - 0.007</td>
</tr>
<tr>
<td>Epoxy, Vinyl Ester and Isophthalic pipe</td>
<td>0.005</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>0.015</td>
</tr>
<tr>
<td>Steel commercial pipe</td>
<td>0.045 - 0.09</td>
</tr>
<tr>
<td>Stretched steel</td>
<td>0.015</td>
</tr>
<tr>
<td>Weld steel</td>
<td>0.045</td>
</tr>
<tr>
<td>Galvanized steel</td>
<td>0.15</td>
</tr>
<tr>
<td>Rusted steel (corrosion)</td>
<td>0.15 - 4</td>
</tr>
<tr>
<td>New cast iron</td>
<td>0.25 - 0.8</td>
</tr>
<tr>
<td>Worn cast iron</td>
<td>0.8 - 1.5</td>
</tr>
<tr>
<td>Rusty cast iron</td>
<td>1.5 - 2.5</td>
</tr>
<tr>
<td>Sheet or asphalted cast iron</td>
<td>0.01 - 0.015</td>
</tr>
<tr>
<td>Smoothed cement</td>
<td>0.3</td>
</tr>
<tr>
<td>Ordinary concrete</td>
<td>0.3 - 1</td>
</tr>
<tr>
<td>Coarse concrete</td>
<td>0.3 - 5</td>
</tr>
<tr>
<td>Well planed wood</td>
<td>0.18 - 0.9</td>
</tr>
<tr>
<td>Ordinary wood</td>
<td>5</td>
</tr>
</tbody>
</table>
Moody chart features

- Fully rough flows for large Re
  - Physically the viscous sublayer becomes smaller than roughness
- Moody can be used for non-circular pipes using hydraulic diameter
- Colebrook equation is a curve-fit of data which is convenient for computations (e.g., using Matlab)
  \[
  \frac{1}{\sqrt{f}} = -2.0 \log \left( \frac{\varepsilon / D}{3.7} + \frac{2.51}{\sqrt{f}} \frac{1}{Re} \right)
  \]
- Limiting cases
  - Prandtl equation (smooth pipes)
    \[
    \frac{1}{\sqrt{f}} = 2 \log(Re \sqrt{f}) - 0.8
    \]
  - von Karman equation (fully rough)
    \[
    \frac{1}{\sqrt{f}} = -2.0 \log \left( \frac{\varepsilon}{3.7D} \right)
    \]
- To avoid iteration on \( f \) in Colebrook equation, explicit approximations (valid for \( Re > 2100 \)) have been developed:
  - Chen equation:
    \[
    f = \left[ -2.0 \log \left( \frac{\varepsilon}{3.7065D} - \frac{5.0452}{Re} \log \left( \frac{1}{2.8257} \left[ \frac{\varepsilon}{D} \right]^{0.1098} + \frac{5.8506}{Re^{0.8981}} \right) \right) \right]^2
    \]
  - Swamee-Jain equation
    \[
    f = 0.250 \left[ \log \left( \frac{\varepsilon}{3.7D} + \frac{5.74}{Re^{0.9}} \right) \right]^2
    \]
- Others (Churchill, Haaland)
- Both Moody chart and Colebrook equation are accurate to ±15% due to roughness size, experimental error, curve fitting of data, etc.
- Additional errors of ±2% can be assumed with approximations as above
What we covered:

- Used Momentum and Energy conservation to relate pressure drop to shear stress at the wall.

\[
\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine,e} + h_L
\]

\[P_1 - P_2 = 4\tau_w \frac{L}{D}\]

\[h_L = \frac{4\tau_w}{\rho g} \frac{L}{D}\]

- Used Dimensional analysis and experimental data (Moody chart) to find the head loss in a pipe

\[h_L = f \frac{L V^2}{D \cdot 2g}\]

\[f = f(Re, \varepsilon/D)\]

\[f = \frac{64}{Re} \quad Re < 2300\]
Today's lecture: pipe flow examples

- Last time: used momentum and energy balances to relate pressure drop to shear stress at the wall
- Introduced concept of friction factor to express:

\[ h_L = f \frac{L}{D} \frac{V^2}{2g} \quad f = f(Re, \varepsilon/D): \text{Moody chart} \]

\[ f = \frac{64}{Re} \quad \text{Re} < 2300 \]

- Today: use expressions for \( h_L \) in computing losses, so that we can use the energy equation

\[
\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L
\]
Different types of computation:

- Three essential problems in Pipe Flow
  - \( V \) and \( D \) are given, but the driving force, \((P, gh, h_p)\), unknown
  - \( D \) and driving force are given, \( V \) unknown
    - Iterative solution needed
  - \( V \) and driving force are given, \( D \) unknown
    - Double iterative solution is needed

- Each type of problem could involve flow which is
  - Laminar (here all 3 problems are straightforward)
  - Turbulent

- But, knowing if laminar or turbulent depends on the \( Re \), which may be part of the solution....
Laminar Example

\[ \frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine,e} + h_L \]

Water

1.2 m/s

\[ D = 0.2 \text{ cm} \]

\[ L = 15 \text{ m} \]
Turbulent example, $V$ given

\[
\frac{P_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_{turbine,e} + h_L
\]
Turbulent Example, $V$ unknown

\[
\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L
\]
Turbulent example, $D$ unknown?

\[
\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine,e} + h_L
\]
Example:

A cast iron pipe of diameter 14.64cm and length 3.434km is to convey octane. The available pump can provide a pressure drop of 172.32 kPa. Determine the expected flow rate of octane, assuming no change in elevation along the pipe.
What we covered:

- Three essential problems in Pipe Flow
  - $V$ and $D$ are given, but the driving force, $(P, gh, h_p)$, unknown (easy)
  - $D$ and driving force are given, $V$ unknown
    - Iterative solution needed (OK)
  - $V$ and driving force are given, $D$ unknown
    - Double iterative solution is needed (horrible)

- Each type of problem could involve flow which is
  - Laminar (easy)
  - Turbulent

- Examples of each
Today’s lecture

- Last lectures analyzed pipe flow using conservation laws
  - Energy equation: \( \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_{	ext{pump},u} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{	ext{turbine},e} + h_L \)
  - Head loss due to friction: \( h_L = f\left(\frac{Re, \varepsilon}{D}\right) \frac{L}{D} \frac{V^2}{2g} \)
    - Laminar Flow (\( Re < 2200 \)), \( f=64/Re \); turbulent flow – Moody chart

- Three essential problems in Pipe Flow
  - \( V \) and \( D \) are given, but the driving force, \( (P, gh, h_p) \), unknown
  - \( D \) and driving force are given, \( V \) unknown
    - Iterative solution needed
  - \( V \) and driving force are given, \( D \) unknown
    - Double iterative solution is needed

- Today: Minor losses = other contributions to \( h_L \)
Fully developed flows in pipes only arise a distance $L_h$ from the entrance.

- Dimensional analysis shows that $L_h/D$ is a function of $Re$.
- $\tau_w$ larger in entrance region than in fully developed flow $\rightarrow h_L$.

Piping systems include, joints, valves, junctions...

- Each non-uniformity disturbs flow. Re-establishing fully developed flow results in increased $\tau_w$ and hence contributes to $h_L$. 
Minor Losses

- Each component in a piping systems (fittings, valves, bends, elbows, tees, inlets, exits, enlargements, contractions) that interrupts the smooth flow creates a minor loss of head.

- We introduce a relation for the minor losses associated with each of these components.

- Usual approach:
  - $K_L$ is the loss coefficient.
  - Is different for each component.
  - Is assumed to be independent of $Re$.
  - Typically provided by manufacturer or generic table (e.g., Table 8-4 in text).

\[
h_L = K_L \frac{V^2}{2g}
\]

Other methods also exist (see e.g. in MECH 386)
Minor Losses

- Total head loss in a system is comprised of major losses, in the pipe sections, and the minor losses in the components

\[ h_L = h_{L,major} + h_{L,minor} \]

\[
h_L = \sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} + \sum_j K_{L,j} \frac{V_j^2}{2g}
\]

- i pipe sections, j components

- If the piping system has constant diameter

\[ h_L = \left( f \frac{L}{D} + \sum K_L \right) \frac{V^2}{2g} \]
Typical minor loss coefficients:

Loss coefficients $K_L$ of various pipe components for turbulent flow (for use in the relation $h_L = K_L V^2/(2g)$, where $V$ is the average velocity in the pipe that contains the component):

**Pipe Inlet**
- Reentrant: $K_L = 0.80$ (if $t \ll D$ and $t \approx 0.1D$)
- Sharp-edged: $K_L = 0.50$
- Well-rounded ($r/D > 0.2$): $K_L = 0.03$
- Slightly rounded ($r/D = 0.1$): $K_L = 0.12$

**Pipe Exit**
- Reentrant: $K_L = \alpha$
- Sharp-edged: $K_L = \alpha$
- Rounded: $K_L = \alpha$

Note: The kinetic energy correction factor is $\alpha = 2$ for fully developed laminar flow, and $\alpha = 1.05$ for fully developed turbulent flow.
$\alpha = 2$ (laminar); $\alpha = 1.05$ (fully turbulent)
### Bends and Branches

<table>
<thead>
<tr>
<th>Type</th>
<th>Flanged ( K_L )</th>
<th>Threaded ( K_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90° smooth bend</td>
<td>0.3</td>
<td>0.9</td>
</tr>
<tr>
<td>90° miter bend (without vanes)</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>90° miter bend (with vanes)</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>45° threaded elbow</td>
<td>0.4</td>
<td></td>
</tr>
</tbody>
</table>

### 180° return bend

<table>
<thead>
<tr>
<th>Type</th>
<th>Flanged ( K_L )</th>
<th>Threaded ( K_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>180° return bend</td>
<td>0.2</td>
<td>1.5</td>
</tr>
<tr>
<td>Tee (branch flow):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flanged: ( K_L )</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Threaded: ( K_L )</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Tee (line flow):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flanged: ( K_L )</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Threaded: ( K_L )</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

### Threaded union

<table>
<thead>
<tr>
<th>Type</th>
<th>Flanged ( K_L )</th>
<th>Threaded ( K_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threaded union</td>
<td>0.08</td>
<td></td>
</tr>
</tbody>
</table>

### Valves

<table>
<thead>
<tr>
<th>Valve Type</th>
<th>Fully Open ( K_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Globe valve</td>
<td>10</td>
</tr>
<tr>
<td>Angle valve</td>
<td>5</td>
</tr>
<tr>
<td>Ball valve</td>
<td>0.05</td>
</tr>
<tr>
<td>Swing check valve</td>
<td>2</td>
</tr>
<tr>
<td>Gate valve, fully open</td>
<td>0.2</td>
</tr>
<tr>
<td>1/2 closed</td>
<td>0.3</td>
</tr>
<tr>
<td>3/4 closed</td>
<td>2.1</td>
</tr>
<tr>
<td>1 closed</td>
<td>17</td>
</tr>
</tbody>
</table>

*These are representative values for loss coefficients. Actual values strongly depend on the design and manufacture of the components and may differ from the given values considerably (especially for valves). Actual manufacturer’s data should be used in the final design.*
General principles

- Inlet losses sensitive to geometry
  - Rounding of corners reduces losses
- Exit losses are insensitive to geometry
- Threaded components have larger losses than flanged
- Use generic tables/charts in pre-design, but manufacturers values for final design

![Diagram of flow loss](image)

<table>
<thead>
<tr>
<th>Nominal diameter, in</th>
<th>Screwed</th>
<th>Flanged</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 4</td>
<td>1 2 4 8 20</td>
</tr>
<tr>
<td>Valves (fully open):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Globe</td>
<td>1.4 8.2 6.9 5.7 13</td>
<td>8.5 6.0 5.8 5.5</td>
</tr>
<tr>
<td>Gate</td>
<td>0.3 0.24 0.16 0.11 0.08</td>
<td>0.35 0.16 0.07 0.03</td>
</tr>
<tr>
<td>Swing check</td>
<td>5.1 2.9 2.1 2.0 2.0</td>
<td>2.0 2.0 2.0 2.0</td>
</tr>
<tr>
<td>Angle</td>
<td>9.0 4.7 2.0 1.0 4.5</td>
<td>2.4 2.0 2.0 2.0</td>
</tr>
<tr>
<td>Elbows:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45° regular</td>
<td>0.39 0.32 0.30 0.29</td>
<td>0.21 0.20 0.19 0.16</td>
</tr>
<tr>
<td>45° long radius</td>
<td>2.0 1.5 0.95 0.64 0.23</td>
<td>0.40 0.30 0.19 0.15 0.10</td>
</tr>
<tr>
<td>90° regular</td>
<td>1.0 0.72 0.41 0.23 0.41</td>
<td>0.41 0.35 0.30 0.25 0.20</td>
</tr>
<tr>
<td>90° long radius</td>
<td>2.0 1.5 0.95 0.64 0.23</td>
<td>0.40 0.30 0.21 0.15 0.10</td>
</tr>
<tr>
<td>180° regular</td>
<td>0.40 0.30 0.21 0.15 0.10</td>
<td></td>
</tr>
<tr>
<td>180° long radius</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tees:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line flow</td>
<td>0.9 0.9 0.9 0.9 0.24</td>
<td>0.19 0.14 0.10 0.07 0.00</td>
</tr>
<tr>
<td>Branch flow</td>
<td>2.4 1.8 1.4 1.1 1.0</td>
<td>0.8 0.64 0.58 0.41</td>
</tr>
</tbody>
</table>
Example 1:

A 6cm horizontal water pipe expands gradually to a 9cm pipe. The walls of the expansion are angled at 20° from the axis. The average velocity and pressure of the water before the expansion are 7m/s and 150kPa, respectively. Determine the head loss in the expansion section and the pressure in the larger-diameter pipe.
Example 2:

The system shown consists of 1200m of 5cm cast iron pipe, two 45° and four 90° flanged long radius elbows, a fully open flanged globe valve and a sharp exit into a reservoir. If the elevation at point 1 is 400m, what gage pressure is required at point 1 to deliver 0.005 m³/s of water at 20°C into the reservoir?
Example 3:

Water at 60 °F flows from the basement to the second floor through the 0.75-in. (0.0625-ft)-diameter copper pipe (a drawn tubing) at a rate of \( Q = 12 \text{ gal/min} \) and exits through a faucet of diameter 0.50 in. as shown.

Q. Determine the pressure at point (1) if (a) all losses are neglected, (b) the only major losses are included, (c) all losses are included?
Rectangular and annular ducts are fairly common.

For non-round pipes, define the hydraulic diameter

\[ D_h = \frac{4A_c}{P} \]

- \( A_c \) = cross-section area
- \( P \) = wetted perimeter

For turbulent flows we compute friction factors based on mean flow velocity and hydraulic diameter:

Circular tube:

\[ D_h = \frac{4(\pi D^2/4)}{\pi D} = D \]

Square duct:

\[ D_h = \frac{4a^2}{4a} = a \]

Rectangular duct:

\[ D_h = \frac{4ab}{2(a + b)} = \frac{2ab}{a + b} \]
Example 3:

Compute the hydraulic diameter of the open channel illustrated and hence the frictional pressure drop in fully developed turbulent flow of water along a 100m channel at flow rate 0.2 m$^3$/s.
Laminar flows

- For laminar flows the relation: $fRe = C$ holds
- Constant $C$ depends on geometry considered
- Tabulated for many common geometries

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>$fRe$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (channel)</td>
<td>96.00</td>
</tr>
<tr>
<td>0.05</td>
<td>89.81</td>
</tr>
<tr>
<td>0.1</td>
<td>84.68</td>
</tr>
<tr>
<td>0.125</td>
<td>82.34</td>
</tr>
<tr>
<td>0.167</td>
<td>78.81</td>
</tr>
<tr>
<td>0.25</td>
<td>72.93</td>
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<tr>
<td>0.4</td>
<td>65.47</td>
</tr>
<tr>
<td>0.5</td>
<td>62.19</td>
</tr>
<tr>
<td>0.75</td>
<td>57.89</td>
</tr>
<tr>
<td>1 (square)</td>
<td>56.91</td>
</tr>
</tbody>
</table>

$A_c = ab$

$P = 2a + 2b$
Example 4:

Air flows along a horizontal duct that is 3m long. The duct has dimensions 30cm x 15cm and the air velocity is 10cm/s. Determine the pressure drop along the duct.
What we covered

- Simple method to handle minor losses
  - $K_L$ determined by experiment
  - Assumed to be $Re$ independent ….
  - Add all minor and pipe losses together

- Non-round duct flow

\[
h_L = K_L \frac{V^2}{2g}
\]

\[
h_L = \sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} + \sum_j K_{L,j} \frac{V_j^2}{2g}
\]

- $D_h = 4A_c/P$
- $A_c = \text{cross-section area}$
- $P = \text{wetted perimeter}$
- Use hydraulic diameter and everything is same for turbulent flows
- Laminar flows require special charts/tables
Introduction to piping networks

- Pipes in series
- Pipes in parallel
- Networks of pipes
  - General form of equations
  - Discussion of solution methods and difficulties
Pipes in series

- Volume flow rate constant (steady incompressible)
- Head loss is the summation of parts
  - Plus possible minor losses at connections

$$Q_A = Q_B$$

$$h_{L,A} = \frac{L_A}{D_A} f_A \frac{V_A^2}{2g}$$,  \quad $$h_{L,B} = \frac{L_B}{D_B} f_B \frac{V_B^2}{2g}$$

$$h_L = \frac{L_A}{D_A} f_A \frac{V_A^2}{2g} + \frac{L_B}{D_B} f_B \frac{V_B^2}{2g} + K_{AB} \frac{V_B^2}{2g}$$

Example 1: Water is pumped along a 225m long 300mm diameter concrete pipe connected in series to a 400m long 500mm diameter pipe. Calculate the pressure drop at a flow rate of 0.1m$^3$/s
Parallel pipes

- Perform CV analysis between points A and B

\[
\frac{P_A}{\rho g} + \alpha_1 \frac{V_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \alpha_2 \frac{V_B^2}{2g} + z_B + h_L
\]

\[Q_A = Q_B \Rightarrow A_A V_A = A_B V_B\]

\[h_L = \frac{P_A - P_B}{\rho g} + \frac{\alpha_1 V_A^2 - \alpha_2 V_B^2}{2g} + z_A - z_B\]

- Since \(\Delta P\) is the same for all branches, head loss in all branches is the same

\[h_{L,1} = h_{L,2}\]

\[f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}\]

- Finally summing between branches:

\[Q_A = Q_2 + Q_1 = A_1 V_1 + A_2 V_2\]
Parallel pipes, continued

- Head loss relationship between branches allows the following ratios to be developed

\[
\frac{V_1}{V_2} = \left( \frac{f_2}{f_1} \frac{L_2}{L_1} \frac{D_1}{D_2} \right)^{1/2} \quad \frac{Q_1}{Q_2} = \left( \frac{D_1}{D_2} \right)^2 \left( \frac{f_2 D_1 L_2}{f_1 D_2 L_1} \right)^{1/2}
\]

- Note that this extends to N branches:
  - N+1 variables: \(Q_1, ..., Q_N, (P_A - P_B)\)
  - N+1 equations: \(Q = Q_1 + Q_2 + ... + Q_N\); and N equations for \(h_L\) on each branch

- Two generic problems:
  - Total pressure drop specified, compute \(Q\) and individual \(Q_i\)
  - Total flow rate specified, compute: \(Q_1, ..., Q_N, (P_A - P_B)\)

- Note: the analogy with electrical circuits should be obvious
  - Flow rate (VA) : current (I)
  - Pressure difference (\(\Delta p\)) : electrical potential (V)
  - Head loss (\(h_L\)) : resistance (R), however \(h_L\) is very nonlinear
Example 2:

The pressure head at A is 40m and that at B is 24m; both A and B are at the same elevation. Branch 1 consists of 4km of 300mm diameter pipe and branch 2 consists of 1.25km of 200mm pipe (both cast iron $\varepsilon = 0.26\text{mm}$). Calculate the total flow rate between A and B.
General Pipe Networks

- Analysis involves setting as unknowns the pressure (or head) at interior junctions in the network and setting the velocity (or flow rate) along each branch of the network as additional unknowns.

- We simply have to apply:
  - Continuity at all interior junctions, i.e. the net flow rate into a junction is zero
  - Energy equation with appropriate losses (or pump head rises) applied along each branch of the network

- To close system of equations we need extra conditions at exterior junctions

- These conditions are exactly analogous to an electronic circuit, where at each node (remember Kirchoff’s laws) the net current flowing into the node must be zero, and the voltage at the node computed from all directions must be the same.
Example network:

- This network has 12 pipe legs and 5 interior junctions (B,F,D,E,H). The velocity and/or pressure is assumed known approaching (or exiting) junctions A,C,G,I. For the sake of argument, let’s assume the pressure is known at A,C,G,I (equally, we could specify the velocity and pressure at A, and, for example, the pressures at C,G,I).
- There are therefore 12 unknown velocities and 5 unknown pressures.
- Writing the Energy equation with losses for each of the legs of the network will give us 12 equations, 1 for each leg, relating pressure and velocity.
- The continuity equation written at each interior pipe junction will give us 5 equations, 1 for each junction, relating velocities in the pipe legs. So, for example, at point B we have \( Q_1 = Q_2 + Q_4 \rightarrow V_{1A1} = V_{2A2} + V_{4A4} \).