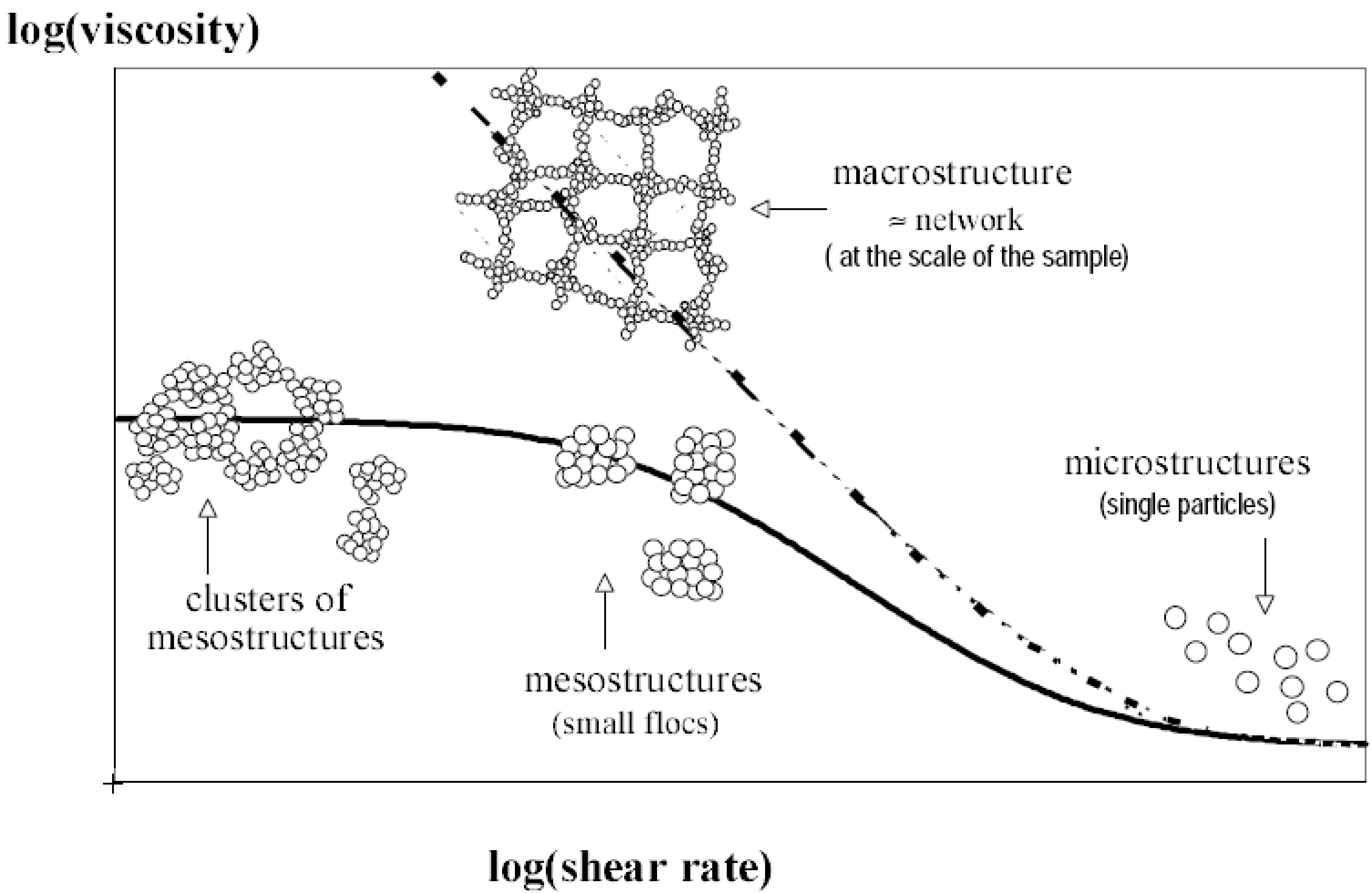
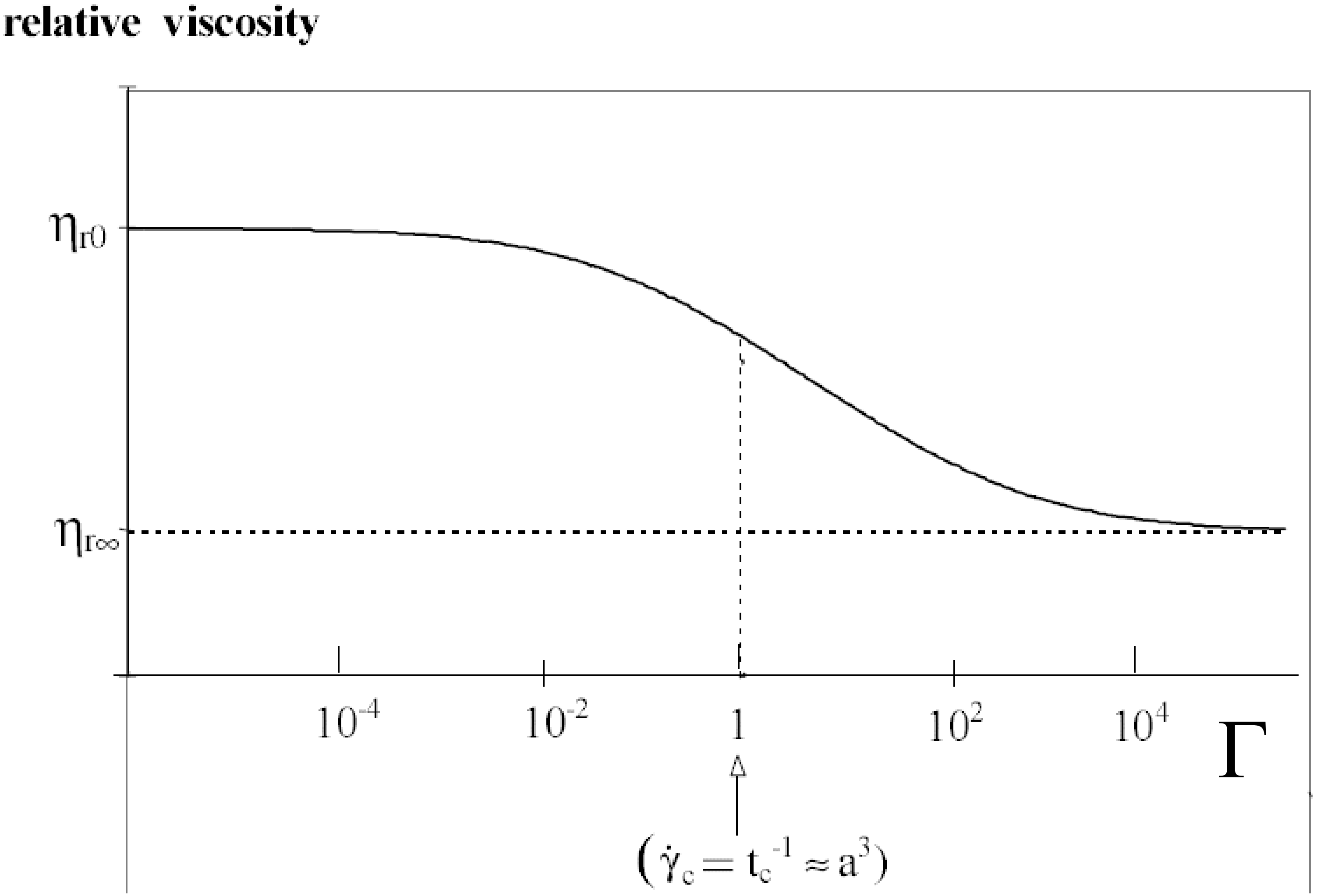
Quemada, Eur Phys J. AP 1, pp 119-127 (1998)

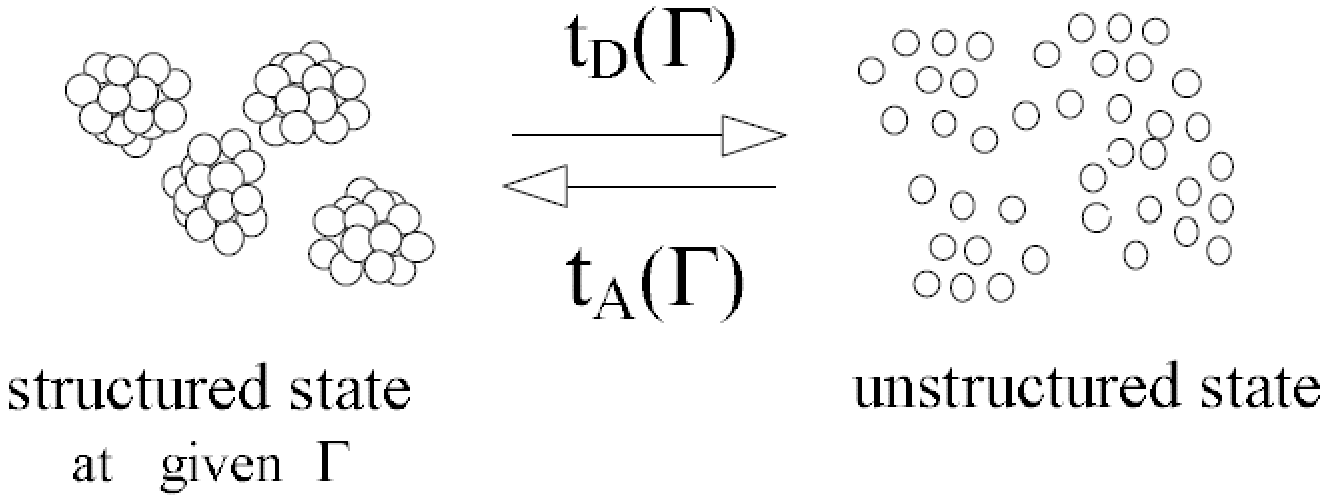
Yield stress phenomenon coming from an effective volume fraction in a classic suspension model


log(viscosity)
macrostructure
≃ network
(at the scale of the sample)
microstructures
(single particles)
clusters of
mesostructures
mesostructures
(small flocs)
log(shear rate)

Fixed ϕ *(S)*



SU’s = structural units IF’s = individual flocs



t_D = destruction timescale

t_A = aggregation timescale

Table 1. Different rheological behaviors, associated with the value of the rheological (structural) index.

rheological index	rheological behavior	range of volume fraction
$0 < \chi < 1$	PSEUDO-PLASTIC	$\phi < \phi_0 < \phi_\infty$
$1 < \chi < \infty$	DILATANT	$\phi < \phi_\infty < \phi_0$
$\chi = 0$	PLASTIC (CASSON)	$\phi = \phi_0$
$-\infty < \chi < 0$	DISCONTINUOUS VISCOSITY	$\phi_\infty < \phi < \phi_0$
$\chi = 1$	(NEWTONIAN)	$\phi_0 = \phi_\infty$

Steady 1D Viscoplastic Flows

Yield stress models in Navier-Stokes equations

$$\tau_{ij} = \eta(\dot{\gamma})\dot{\gamma}_{ij} \quad \Leftrightarrow \quad \tau > \tau_0$$

$$\dot{\gamma}_{ij} = 0 \quad \Leftrightarrow \quad \tau \le \tau_0$$

$$\dot{\gamma}_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} = 2e_{ij}$$

- Bingham: $$\eta = \eta_\infty + \frac{\tau_0}{\dot{\gamma}}, \qquad \tau = \eta_\infty \dot{\gamma} + \tau_0$$
- Casson: $$\eta = \eta_\infty + 2\sqrt{\frac{\eta_\infty \tau_0}{\dot{\gamma}}} + \frac{\tau_0}{\dot{\gamma}}, \qquad \tau = \eta_\infty \dot{\gamma} + 2\sqrt{\eta_\infty \dot{\gamma} \tau_0} + \tau_0$$
- Herschel-Bulkley: $$\eta = \kappa \dot{\gamma}^{n-1} + \frac{\tau_0}{\dot{\gamma}}, \qquad \tau = \kappa \dot{\gamma}^n + \tau_0$$

- τ_0 = yield stress
- η_∞ = high shear (plastic) viscosity
- κ = consistency
- n = power law index

Plane Couette flow (simple shear)

- 2 parallel plates, separated by distance D
- Flow is 2D, steady and fully developed
- No driving pressure gradient
- Two versions:
  - Impose velocity U on top plate
  - Impose shear stress τ_w at top plate

Plane Poiseuille flow

- 2 stationary parallel plates, separated by distance D
- Flow is 2D, steady and fully developed
- Two versions:
  - Impose pressure gradient along channel
  - Impose a mean velocity U_0 along the channel

Cylindrical coordinates

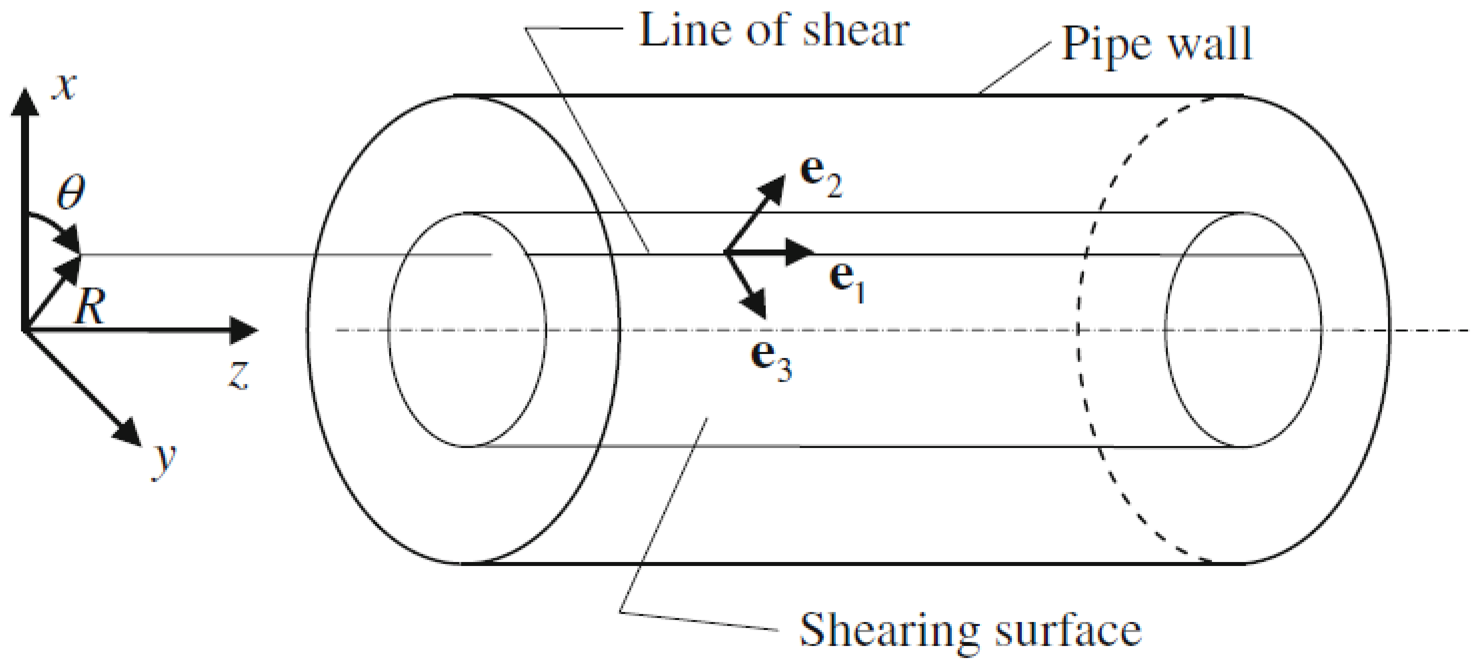
$$\nabla \cdot \boldsymbol{\sigma} = \begin{pmatrix} -\dfrac{\partial p}{\partial r} + \dfrac{1}{r}\dfrac{\partial}{\partial r}\left[r\tau_{rr}\right] + \dfrac{1}{r}\dfrac{\partial \tau_{r\theta}}{\partial \theta} + \dfrac{\partial \tau_{rz}}{\partial z} - \dfrac{\tau_{\theta\theta}}{r} \\ -\dfrac{1}{r}\dfrac{\partial p}{\partial \theta} + \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left[r^2\tau_{\theta r}\right] + \dfrac{1}{r}\dfrac{\partial \tau_{\theta\theta}}{\partial \theta} + \dfrac{\partial \tau_{\theta z}}{\partial z} \\ -\dfrac{\partial p}{\partial z} + \dfrac{1}{r}\dfrac{\partial}{\partial r}\left[r\tau_{zr}\right] + \dfrac{1}{r}\dfrac{\partial \tau_{z\theta}}{\partial \theta} + \dfrac{\partial \tau_{zz}}{\partial z} \end{pmatrix}$$

$$\mathbf{u} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \qquad \dot{\gamma}_{rr} = 2\frac{\partial u}{\partial r}, \quad \dot{\gamma}_{\theta\theta} = \frac{2}{r}\frac{\partial v}{\partial \theta} + \frac{2u}{r}, \quad \dot{\gamma}_{zz} = 2\frac{\partial w}{\partial z}$$

$$\dot{\gamma}_{r\theta} = \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} = \dot{\gamma}_{\theta r}, \qquad \dot{\gamma}_{\theta z} = \frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta} = \dot{\gamma}_{z\theta},$$

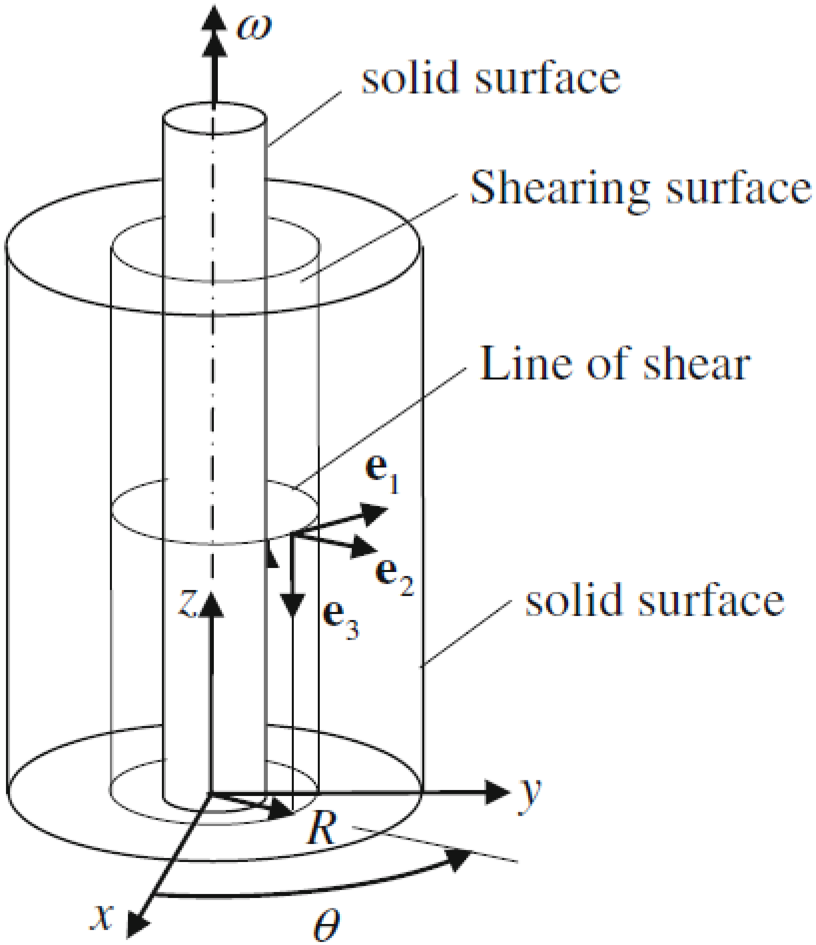
$$\dot{\gamma}_{zr} = \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} = \dot{\gamma}_{rz},$$

Pipe flow



- Flow is axisymmetric, steady and fully developed
- Two versions:
  - Impose pressure gradient along pipe
  - Impose a flow rate along the pipe

Cylindrical Taylor-Couette flow



- Flow is axisymmetric, steady and fully developed
- Various versions:
  - Impose shear stresses at walls
  - Impose angular velocity at walls
  - Some combination of above