Math 605 Exercises 2: due November 20th

Problem 1:

Consider the **axial** flow of a Bingham fluid along a rectangular duct of area $\hat{H}$ x $\hat{L}$, driven by a constant pressure gradient $\hat{p}_z = -\hat{G}$, in the z-direction.

a) Scale the equations using $\hat{H}$ as a length-scale, $\hat{G}\hat{H}$ as the stress scale, $\hat{G}\hat{H}^2/\hat{\mu}$ as the velocity scale. Look for a solution ***u****=(0,0,W(x,y))* for steady fully developed flow in the z-direction. Show that the problem is governed by 2 dimensionless groups: a Bingham number ($B = \hat{\tau}_Y / \hat{G}\hat{H}$) and an aspect ratio: $L = \hat{L}/\hat{H}$.
b) Using the Matlab code distributed, read through and comment as needed – if you can make it more efficient do so (and give it back….). Implement a convergence error tolerance criterion on the Uzawa algorithm loop and experiment with different r and ρ satisfying the convergence criterion, or not. Is Glowinski's bound "sharp"?
c) Solve this problem numerically (=run the code) for: L=2 and for B=0, 0.1, 0.2, 0.3, 0.4 – what happens?
d) Using down the variational formulation as in class, you know that there will be a critical value of B (depending only on L) above which the velocity is zero. You can find this in either the papers by Mosolov & Miasnikov, or by Huilgol 2004. Write down the exact critical B as a function of L.
e) Determine the critical B numerically, using the code in any way you feel is justified, for L=1 (square), L=2, L=5. Is your estimate affected by mesh size? Explain how close your estimate is to the exact critical B.

Problem 2:

Consider the Stokes flow of a Bingham fluid in a closed 3D region Ω with **u**=0 on the smooth boundary, driven by a body force ρ**f**, i.e. as we have done in class.

a) Write down the <u>dimensional</u> form of the velocity minimization and stress maximization principles.
b) Show what happens to <**f**.**u**> as the density ρ is increased.
c) Show that $<\dot{\gamma}(\mathbf{u})>$ decreases as τ_Y increases and strictly, unless **u**=0.
d) Show what happens to: $\mu_\infty<\dot{\gamma}^2(\mathbf{u})>$ as the plastic viscosity μ_∞ is increased.

For b-d, no strict proof is needed – just use the variational methods as in class and any inequalities we have used.