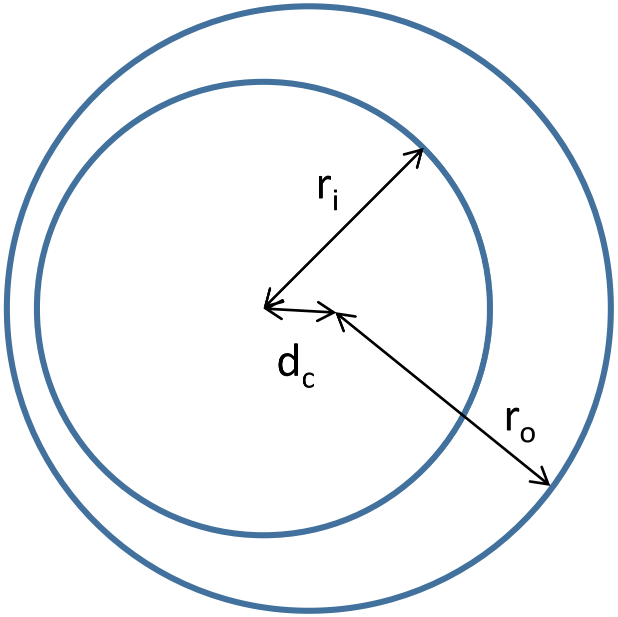

Aspect ratio: $\delta = (r_o - r_i)/r_i$

Eccentricity: $e = d_c/(r_o - r_i)$

$$\dot{\gamma} = [\dot{\gamma}_{xz}^2 + \dot{\gamma}_{\theta z}^2]^{\frac{1}{2}}. \tag{28}$$

For $\delta \ll 1$ it is natural to seek a solution by expanding in powers of δ. Thus, if w is expanded as

$$w = w_0 + \delta w_1 + O(\delta^2), \tag{29}$$

then it follows that the components of the rate-of-strain tensors take the form

$$\dot{\gamma}_{xz} = \dot{\gamma}_{xz0} + \delta \dot{\gamma}_{xz1} + O(\delta^2), \tag{30}$$

$$\dot{\gamma}_{\theta z} = \delta \dot{\gamma}_{\theta z1} + O(\delta^2), \tag{31}$$

where
$$\dot{\gamma}_{xz0} = \frac{\partial w_0}{\partial x}, \quad \dot{\gamma}_{xz1} = \frac{\partial w_1}{\partial x}, \quad \dot{\gamma}_{\theta z1} = \frac{\partial w_0}{\partial \theta}. \tag{32}$$

The components of the stress tensor are also expanded in powers of δ as follows:

$$\tau_{xz} = \tau_{xz0} + \delta \tau_{xz1} + O(\delta^2), \tag{33}$$

$$\tau_{\theta z} = \delta \tau_{\theta z1} + O(\delta^2). \tag{34}$$

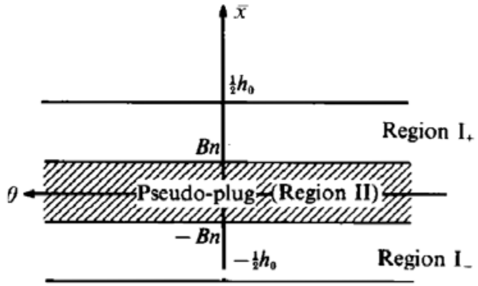

FIGURE 3. Flow regions in the central section of the annulus.

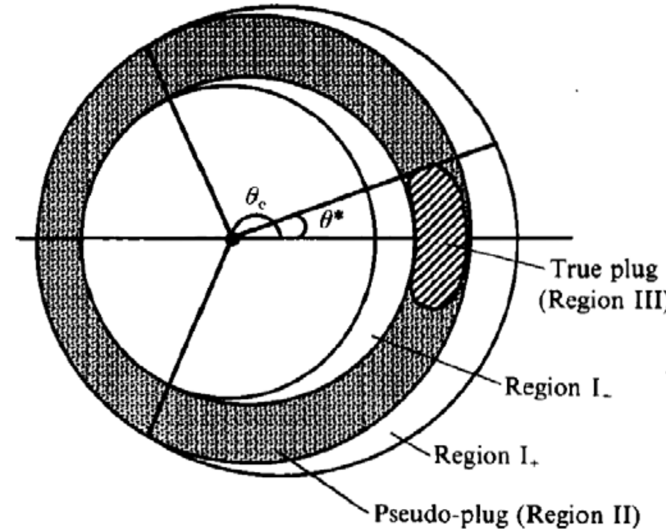

FIGURE 2. Flow regions in the annulus.

$$\frac{\partial}{\partial x}\tau_{xz0} = -1.$$

On integration and after appealing to symmetry about $x = 0$ this

$$\tau_{xz0} = -\bar{x},$$

where $\bar{x} = x - \frac{1}{2}h_0$. Since the yield surfaces are now defined by $\tau = \pm Bn$, it follows that they are located at $\bar{x} = \mp Bn$. The region within these surfaces will be designated Region II and the regions outside will be denoted by Regions $\mathrm{I}_\pm$ for $\bar{x} > Bn$ and $\bar{x} < -Bn$, respectively, as shown in figures 2, 3. In Regions $\mathrm{I}_\pm$ equation (8) applies; to leading order it gives

$$\dot{\gamma}_{xz0} + \operatorname{sgn}(\dot{\gamma}_{xz0})\, Bn = -\bar{x}, \tag{37}$$

i.e.

$$\dot{\gamma}_{xz0} = -(\bar{x} \pm Bn). \tag{38}$$

The solution that satisfies the no-slip boundary conditions

$$w_0 = 0 \quad \text{at} \quad \bar{x} = \pm\tfrac{1}{2}h_0 \tag{39}$$

is

$$w_0 = -\tfrac{1}{2}[(\bar{x} \mp Bn)^2 - (\tfrac{1}{2}h_0 - Bn)^2]. \tag{40}$$

Inside the yield surfaces $\dot{\gamma}_{xz0} = 0$, from which it follows that w_0, does not change across this region and is equal to its value on the yield surfaces, i.e.

$$w_0 = w_{p0} = \tfrac{1}{2}(\tfrac{1}{2}h_0 - Bn)^2. \tag{41}$$

A balance between these terms of apparently different orders of magnitude is achieved if θ is chosen to be $O(\delta^{\frac{1}{3}})$. On writing $\theta = \delta^{\frac{1}{3}}\phi$, $\theta^* = \delta^{\frac{1}{3}}\phi^*$ and integrating, this becomes

$$\tau_{\theta z} = -\frac{1}{2Bn} h_{01}(\tfrac{1}{3}\phi^3 - \phi^{*2}) + \dots, \tag{69}$$

where the constant of integration has been chosen so that $\tau_{\theta z} = 0$ at $\phi = 0$. The extent of the modified Region II (and hence of the true plug, Region III) is now determined by the requirement that $\tau_{\theta z} = -Bn$ at $\phi = \phi^*$, i.e.

$$\phi^* = \left(-\frac{3Bn^2}{h_{01}}\right)^{\frac{1}{3}}. \tag{70}$$

Note that $\tau_{\theta z} = Bn$ at $\phi = -\phi^*$ as required. In terms of e and θ^*, (70) becomes

$$\theta^* = \left(\frac{6Bn^2\delta}{e}\right)^{\frac{1}{3}}. \tag{71}$$

The yield surfaces are determined by the requirement that

$$\tau_{xz}^2 + \tau_{\theta z}^2 = Bn^2, \tag{72}$$

which gives, to leading order,

$$\bar{x}^2 + \left(\frac{e}{4Bn}\right)^2 (\tfrac{1}{3}\phi^3 - \phi^{*2}\phi)^2 = Bn^2. \tag{73}$$

We shall refer to the region within the yield surface as the true plug, even though strictly speaking we need to calculate the stress field in this region to confirm that yielding does not occur; this requires specification of the material properties at stresses below the yield stress and will not be examined in this paper. These results confirm that the true plug disappears for a Newtonian fluid, for which $Bn = 0$. For a Bingham plastic the extent of the true plug increases as the offset of centres decreases. When θ^* is $O(1)$, corresponding to $e \sim \delta$, the theory presented here breaks down and new scalings are required. This will not be investigated here, but we note that the growth of the true plug as e decreases is consistent with the fact that for a concentric annulus ($e = 0$) the true plug extends all round the annulus.

Similar remarks apply to the region near $\theta = \pi$ if Bn is sufficiently small for the pseudo-plug to extend all round the annulus. The inner edge of the pseudo-plug again runs parallel to the circular boundaries, but this time it narrows towards $\theta = \pi$. It can be shown that the solution is entirely analogous to that given above for the region near $\theta = 0$ and, in particular, that the azimuthal widths of the plugs are identical to leading order.

Maybe not true: see e.g. Szabo & Hassager 1992?