Linear stability equations:

\[ \nabla \cdot \mathbf{u} = 0, \tag{37} \]

\[ u_t + v U_y + U u_x = -p_x + \frac{1}{R} \nabla^2 u + \frac{B}{R} \left\{ \frac{\nabla^2 u - u_{yy} - v_{ux}}{\gamma(U)} \right\}, \tag{38} \]

\[ v_t + U v_x = -p_y + \frac{1}{R} \nabla^2 v + \frac{B}{R} \left\{ 2v_y \frac{d}{dy} \left[ \frac{1}{\gamma(U)} \right] + \frac{\nabla^2 v - v_{xx} - u_{yx}}{\gamma(U)} \right\}, \tag{39} \]

\[ w_t + U w_x = -p_z + \frac{1}{R} \nabla^2 w + \frac{B}{R} \left\{ (v_z + w_y) \frac{d}{dy} \left[ \frac{1}{\gamma(U)} \right] + \frac{\nabla^2 w}{\gamma(U)} \right\}. \tag{40} \]

Boundary conditions:

\[ \mathbf{u} = 0 \quad \text{on} \quad y = \pm 1. \tag{41} \]

Yield surface perturbation:

\[ y = \pm \tau_0 / \tau_w \pm \epsilon h_\pm(x, z, t), \tag{42} \]

Velocity continuous and

\[ u_x(x, \pm \tau_0 / \tau_w, z, t) = 0, \tag{43} \]

\[ v_y(x, \pm \tau_0 / \tau_w, z, t) = 0, \tag{44} \]

\[ w_z(x, \pm \tau_0 / \tau_w, z, t) = 0, \tag{45} \]

\[ u_z(x, \pm \tau_0 / \tau_w, z, t) + w_x(x, \pm \tau_0 / \tau_w, z, t) = 0, \tag{46} \]

\[ w_y(x, \pm \tau_0 / \tau_w, z, t) + v_z(x, \pm \tau_0 / \tau_w, z, t) = 0, \tag{47} \]

\[ v_x(x, \pm \tau_0 / \tau_w, z, t) + u_y(x, \pm \tau_0 / \tau_w, z, t) = \frac{\pm 2 h_\pm(x, z, t)}{(1 - \tau_0 / \tau_w)^2}, \tag{48} \]

Linear acceleration of plug:

\[ u_t(x, \pm \tau_0 / \tau_w, t) = \frac{B \tau_w^2}{R \tau_0^2} \frac{1}{8 X Z} \int_{-X}^{X} \int_{-Z}^{Z} [h_+(x, z, t) + h_-(x, z, t)] \, dx \, dz, \tag{49} \]

\[ v_t(x, \pm \tau_0 / \tau_w, t) = \frac{\tau w}{\tau_0} \frac{1}{8 X Z} \int_{-X}^{X} \int_{-Z}^{Z} [p(x, -\tau_0 / \tau_w, z, t) - p(x, \tau_0 / \tau_w, z, t)] \, dx \, dz, \tag{50} \]

\[ w_t(x, \pm \tau_0 / \tau_w, t) = 0. \tag{51} \]
Main points: plane Poiseuille flow of generalised Newtonian fluids

• Linearization of effective viscosity results in an anisotropic perturbation problem
• There is no Squire’s theorem
• For YSF we need to perturb the boundary conditions at the yield surface and the yield surface
• We need to solve the linearised stability equations only in the yielded part of the base flow
• We also need to derive a linear perturbation equation for the perturbed plug motion
\( y \in [\tau_0/\tau_w, 1] \). By putting

\[
R = (1 - \tau_0/\tau_w) R,
\]

\[
B = (1 - \tau_0/\tau_w) R,
\]

\[
\tilde{\alpha} = (1 - \tau_0/\tau_w) \alpha,
\]

(61) transforms into

\[
i \bar{\alpha} \tilde{R} \left[ (1 - \xi^2) (f''(\xi) - \bar{\alpha}^2 f) + 2f(\xi) \right]
\]

\[
= \left( \frac{d^2}{d\xi^2} - \bar{\alpha}^2 \right) \left( \frac{d^2}{d\xi^2} - \bar{\alpha}^2 \right) f(\xi) - 4\bar{\alpha}^2 \tilde{B} \frac{d}{d\xi} \left[ \frac{f'(\xi)}{2|\xi|} \right]
\]
YSF Poiseuille Flow is essentially a Poiseuille-Couette flow.

FIG. 8. (Color online) (a) Marginal stability curves for a Newtonian fluid at various wall velocities: $U_0=0, 0.1, 0.2, 0.3, 0.4, 0.5$. (b) Contours of $C_f=\omega_f/\alpha$ for PPF, $U_0=0$. 