Linear stability equations:

$$\nabla\cdot\boldsymbol{u}=0, \tag{37}$$

$$u_t+vU_y+Uu_x=-p_x+\frac{1}{R}\nabla^2u+\frac{B}{R}\left\{\frac{\nabla^2u-u_{yy}-v_{ux}}{\dot\gamma(U)}\right\}, \tag{38}$$

$$v_t+Uv_x=-p_y+\frac{1}{R}\nabla^2v+\frac{B}{R}\left\{2v_y\frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{1}{\dot\gamma(U)}\right]+\frac{\nabla^2v-v_{xx}-u_{yx}}{\dot\gamma(U)}\right\}, \tag{39}$$

$$w_t+Uw_x=-p_z+\frac{1}{R}\nabla^2w+\frac{B}{R}\left\{(v_z+w_y)\frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{1}{\dot\gamma(U)}\right]+\frac{\nabla^2w}{\dot\gamma(U)}\right\}. \tag{40}$$

Boundary conditions:

$$\boldsymbol{u}=0 \quad \text{on} \quad y=\pm 1. \tag{41}$$

Yield surface perturbation:

$$y=\pm\tau_0/\tau_w\pm\epsilon h_{\pm}(x,z,t), \tag{42}$$

Velocity continuous and

$$u_x(x,\pm\tau_0/\tau_w,z,t)=0, \tag{43}$$

$$v_y(x,\pm\tau_0/\tau_w,z,t)=0, \tag{44}$$

$$w_z(x,\pm\tau_0/\tau_w,z,t)=0, \tag{45}$$

$$u_z(x,\pm\tau_0/\tau_w,z,t)+w_x(x,\pm\tau_0/\tau_w,z,t)=0, \tag{46}$$

$$w_y(x,\pm\tau_0/\tau_w,z,t)+v_z(x,\pm\tau_0/\tau_w,z,t)=0, \tag{47}$$

$$v_x(x,\pm\tau_0/\tau_w,z,t)+u_y(x,\pm\tau_0/\tau_w,z,t)=\frac{\pm 2h_{\pm}(x,z,t)}{(1-\tau_0/\tau_w)^2}, \tag{48}$$

Linear acceleration of plug:

$$u_t(x,\pm\tau_0/\tau_w,t)=\frac{B\tau_w^2}{R\tau_0^2}\frac{1}{8XZ}\int_{-X}^{X}\int_{-Z}^{Z}[h_+(x,z,t)+h_-(x,z,t)]\,\mathrm{d}x\,\mathrm{d}z, \tag{49}$$

$$v_t(x,\pm\tau_0/\tau_w,t)=\frac{\tau_w}{\tau_0}\frac{1}{8XZ}\int_{-X}^{X}\int_{-Z}^{Z}[p(x,-\tau_0/\tau_w,z,t)-p(x,\tau_0/\tau_w,z,t)]\,\mathrm{d}x\,\mathrm{d}z, \tag{50}$$

$$w_t(x,\pm\tau_0/\tau_w,t)=0. \tag{51}$$

Main points: plane Poiseuille flow of generalised Newtonian fluids

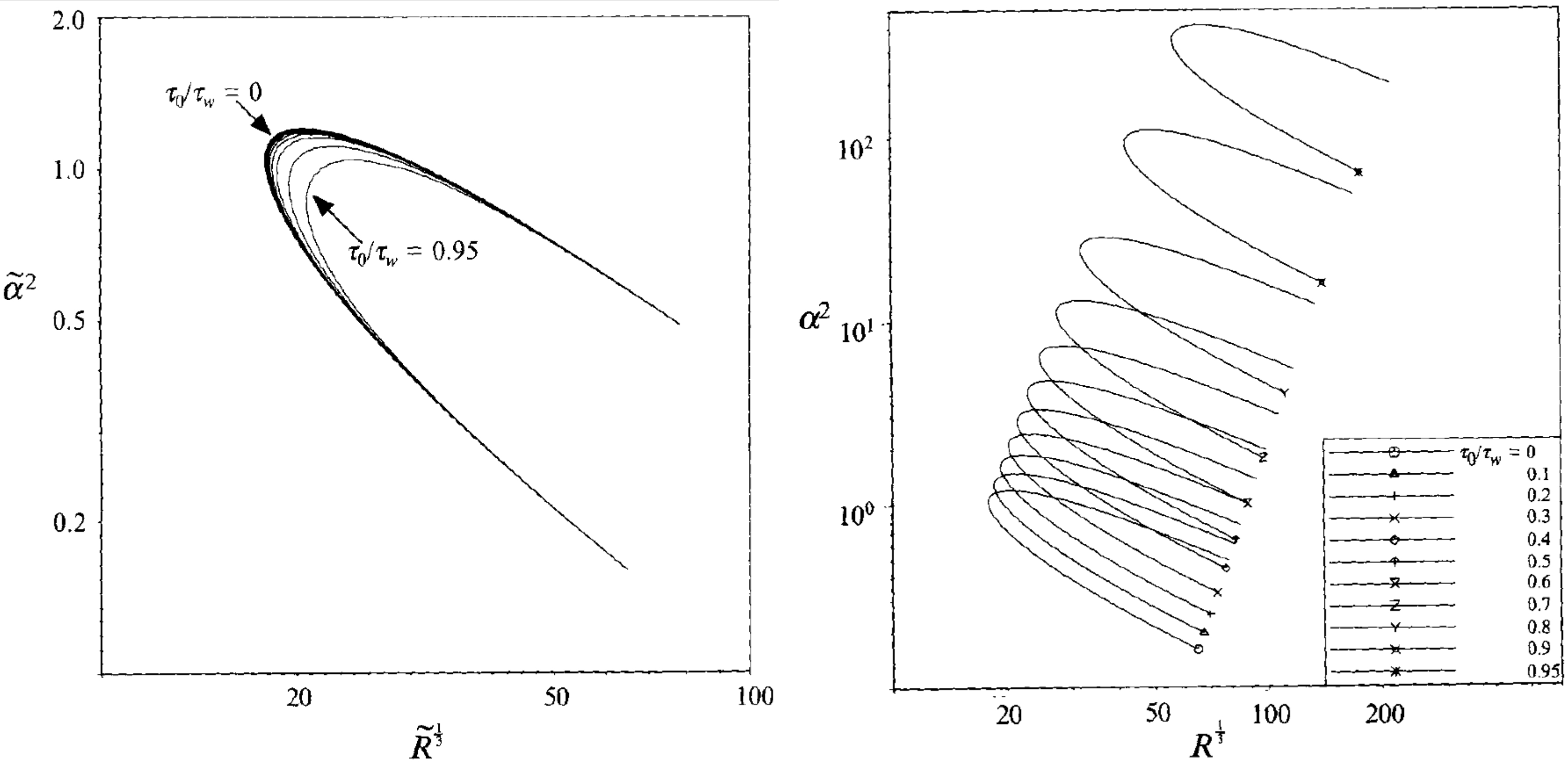
- Linearization of effective viscosity results in an anisotropic perturbation problem
- There is no Squire's theorem
- For YSF we need to perturb the boundary conditions at the yield surface and the yield surface
- We need to solve the linearised stability equations only in the yielded part of the base flow
- We also need to derive a linear perturbation equation for the perturbed plug motion

$y \in [\tau_0/\tau_w, 1]$. By putting $y = \tau_0/\tau_w + \xi(1-\tau_0/\tau_w)$, and defining

$$\tilde{R} = (1-\tau_0/\tau_w)\,R,$$
$$\tilde{B} = (1-\tau_0/\tau_w)\,R,$$
$$\tilde{\alpha} = (1-\tau_0/\tau_w)\,\alpha,$$

(61) transforms into

$$\mathrm{i}\tilde{\alpha}\tilde{R}[(1-\xi^2-c)(f''(\xi)-\tilde{\alpha}^2 f)+2f(\xi)] = \left(\frac{\mathrm{d}^2}{\mathrm{d}\xi^2}-\tilde{\alpha}^2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}\xi^2}-\tilde{\alpha}^2\right)f(\xi) - 4\tilde{\alpha}^2\tilde{B}\frac{\mathrm{d}}{\mathrm{d}\xi}\left[\frac{f'(\xi)}{2|\xi|}\right].$$



Frigaard et al, JFM 1994

YSF Poiseuille Flow is essentially a Poiseuille-Couette flow

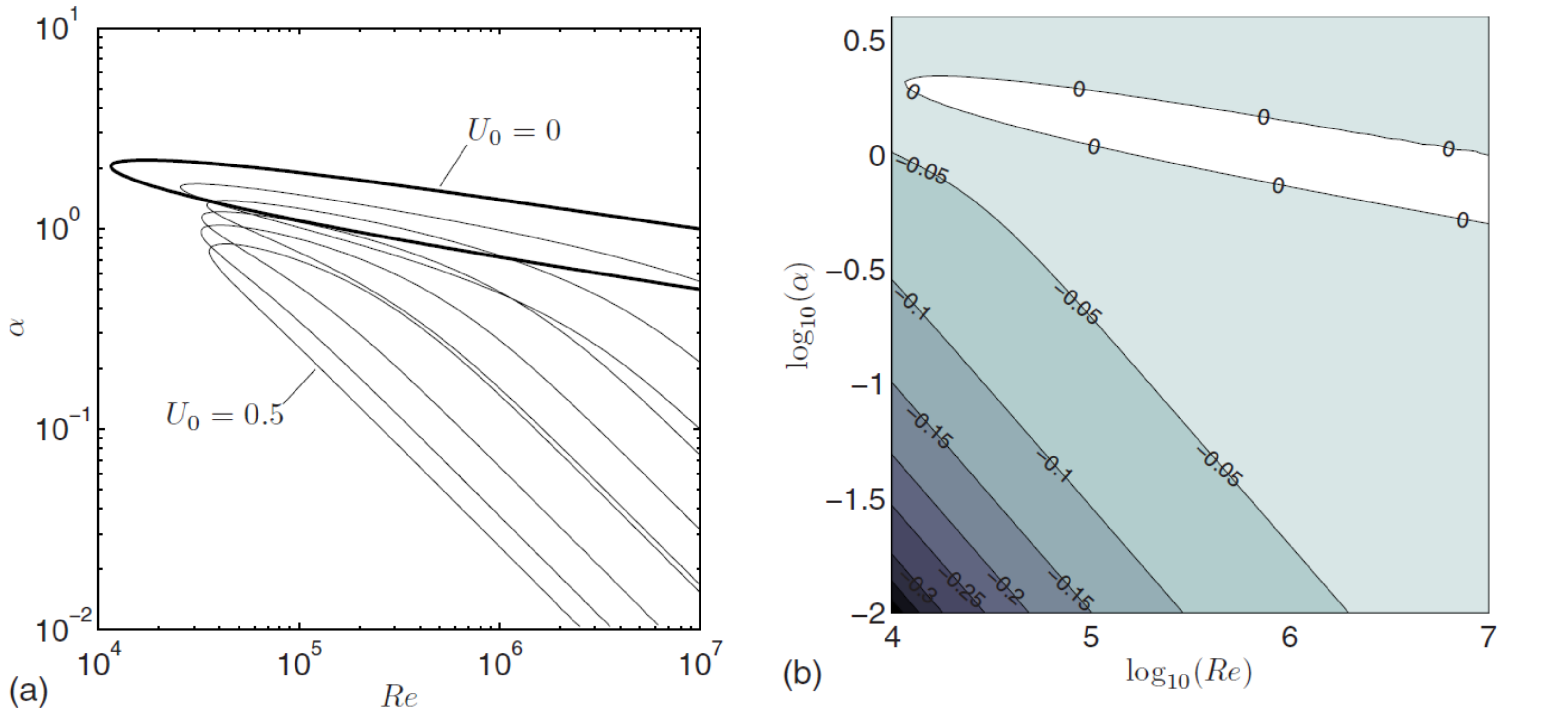


FIG. 8. (Color online) (a) Marginal stability curves for a Newtonian fluid at various wall velocities: $U_0=0,0.1,0.2,0.3,0.4,0.5$. (b) Contours of $C_I=\omega_I/\alpha$ for PPF, $U_0=0$.