Week 12 - On Taylor polynomials

Dec. 1 & 3, 2015

True/false questions

a. Only even powers of x appear in the Taylor polynomilas for $f(x) = e^{-2x}$ centered at $a = 0$.

b. Let $f(x) = x^5 - 1$. The Taylor polynomial for f of order 10 centered at 0 is itself.

c. Only even powers of x appear in the n^{th}–order Taylor polynomial for $f(x) = \sqrt{1+x^2}$

d. Suppose f'' is continuous on an interval that contains a, where f has an inflection point at a. Then the second order Taylor polynomial for f at a is linear.

e. Let $f(x) = x^3 - 2x + 1$. The Taylor polynomial for f of order 3 centered at 2 is itself.

Notes on odd and even functions

- If $f(-x) = f(x)$, then f is an even function.
- If $f(-x) = -f(x)$, then f is an odd function.

  Note:

  $$f(-x) = -f(x) \Rightarrow f(-0) = -f(0) \Rightarrow f(-0) + f(0) = 0 \Rightarrow 2f(0) = 0 \Rightarrow f(0) = 0$$

  So when $f(x)$ is an odd function and $x = 0$ is in its domain, $f(0) = 0$.

- If $f(x)$ is an even function, then $f'(x)$ is an odd function.

  Here's why:

  $$f \text{ is even} \Rightarrow f(x) = f(-x) \Rightarrow \frac{d}{dx}(f(x)) = \frac{d}{dx}(f(-x))$$
  $$\text{use chain rule} \Rightarrow f'(x) = -f'(-x) \Rightarrow f'(x) \text{ is an odd function}$$

- If $f(x)$ is an odd function, then $f'(x)$ is an even function.

  Here's why:

  $$f \text{ is odd} \Rightarrow f(x) = -f(-x) \Rightarrow \frac{d}{dx}(f(x)) = \frac{d}{dx}(-f(-x))$$
  $$\text{use chain rule} \Rightarrow f'(x) = -(-1 \times f'(-x)) = f'(-x) \Rightarrow f'(x) \text{ is an even function}$$

  Note:

  If f is even $\Rightarrow f'$ is odd $\Rightarrow f''$ is even $\Rightarrow f'''$ is odd $\Rightarrow f^{(4)}$ is even $\Rightarrow \ldots$

  If f is odd $\Rightarrow f'$ is even $\Rightarrow f''$ is odd $\Rightarrow f'''$ is even $\Rightarrow f^{(4)}$ is odd $\Rightarrow \ldots$