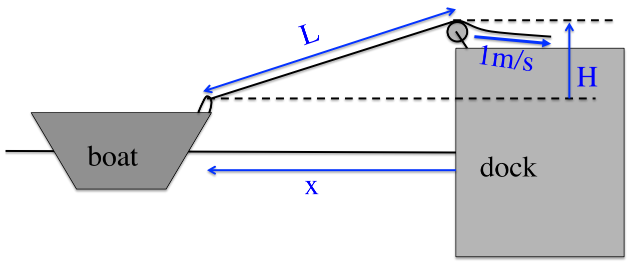
Problem 1.

A boat is being pulled into a dock by a rope attached to the bow of the boat and passing through a pully on the dock that is $3m$ higher than the bow of the boat. If the rope is pulled in at a rate of $1m/s$, how fast is the boat approaching the dock when it is $4m$ from the dock?

Step 0. See the schematic below.



Step 1. $\frac{dL}{dt} = -1m/s, \quad x = 4m, \quad H = 3m$

Step 2. $\frac{dx}{dt} = ?$

Step 3. $x^2 + H^2 = L^2$

Step 4. x and L change with time $\Rightarrow x(t), L(t)$

H is not changing with time $\Rightarrow H$ is a constant and $\frac{dH}{dt} = 0$.

Step 5.

$$\frac{d}{dt}(x(t)^2 + H^2) = \frac{d}{dt}(L(t)^2)$$
$$\frac{dx^2}{dt} + \frac{dH^2}{dt} = \frac{dL^2}{dt}$$
$$\frac{dx^2}{dx}\frac{dx}{dt} + 0 = \frac{dL^2}{dL}\frac{dL}{dt}$$
$$2x\frac{dx}{dt} = 2L\frac{dL}{dt}$$
$$\frac{dx}{dt} = \frac{L}{x}\frac{dL}{dt}$$

Step 6.

$$H = 3m, \quad x = 4m, \quad \frac{dL}{dt} = -1m/s, \quad L = ?$$
$$L = \sqrt{x^2 + H^2} = \sqrt{4^2 + 3^2} = 5$$
$$\frac{dx}{dt} = \frac{5m}{4m} \times (-1)m/s = -\frac{5}{4}m/s = -1.25m/s$$