Problem 1.

Given that

$f'(1) = 0,\ f''(1) < 0$
$f(2) = 0,\ f''(2) > 0$
$f(3) = 0,\ f'(3) = 0$
$f'(4) < 0,\ f''(4) = 0$

fill the following table about the function $f(x)$. Your answers should be a comma separated list of x values or the word "none".

$f(x)$ must have local maximums at	$x = 1$
$f(x)$ must have local minimums at	none
$f(x)$ must have inflection points at	none
$f(x)$ cannot have a local extremum at	$x = 4$
$f(x)$ cannot have an inflection point at	$x = 1, 2$

Reminder:

- The Second Derivative Test for Extremes:
  - **a.** If $f'(c) = 0$ and $f''(c) < 0$ then f is concave down and has a local maximum at $x = c$.
  - **b.** If $f'(c) = 0$ and $f''(c) > 0$ then f is concave up and has a local minimum at $x = c$.
  - **c.** If $f'(c) = 0$ and $f''(c) = 0$ then f may have a local maximum, a minimum or neither at $x = c$.
- If $f'(a) > 0$ or $f'(a) < 0$, then $f(a)$ is not a local maximum or minimum.
- An inflection point is a point on the graph of a function where the concavity of the function changes, from concave up to down or from concave down to up.

Problem 2.

Find all the vertical and horizontal asymptotes of the function $f(x) = \dfrac{x+2}{xe^x - 2x}$. *Show your work completely and clearly. No credit will be given for the answer without the correct accompanying work.*

Solution:

Horizontal asymptotes:

$$\lim_{x\to\infty} \frac{x+2}{xe^x - 2x} = \lim_{x\to\infty} \frac{x\left(1+\frac{2}{x}\right)}{xe^x\left(1-\dfrac{2}{e^x}\right)} = \lim_{x\to\infty} \frac{\left(1+\frac{2}{x}\right)}{e^x\left(1-\dfrac{2}{e^x}\right)} = \lim_{x\to\infty} \frac{1}{e^x} = 0$$

So $y = 0$ is a horizontal asymptote.

$$\lim_{x\to-\infty}\frac{x+2}{xe^x-2x}=\lim_{x\to-\infty}\frac{x\left(1+\frac{2}{x}\right)}{x(e^x-2)}=\lim_{x\to-\infty}\frac{\left(1+\frac{2}{x}\right)}{e^x-2}=\frac{1}{-2}$$

So $y=-1/2$ is a horizontal asymptote. *Vertical asymptotes:*

$$xe^x-2x=0\Rightarrow x(e^x-2)=0\Rightarrow\begin{cases}x=0\\ \text{OR}\\ e^x-2=0\Rightarrow x=\ln(2)\end{cases}$$

$$\lim_{x\to0}f(x)\stackrel{?}{=}\frac{2}{0}$$

$$\lim_{x\to\ln(2)}f(x)\stackrel{?}{=}\frac{\ln(2)+2}{0}$$

So $x=0$ and $x=\ln(2)$ are vertical asymptotes.