Environmental Cooperation and Trade - The Impact of Heterogeneity in Environmental Damages: An Endogenous Solution

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Abstract

This paper analyzes environmental damage heterogeneity through a three-country model of environmental cooperation with trade. In the context of international trade, governments face a tradeoff between higher environmental taxes to cooperatively reduce emissions and higher tariffs on exports when acting non-cooperatively. According to Diamantoudi et al. (2018), stable coalitions among homogeneous countries are larger and provide more significant welfare gains than the basic model without trade.

The objectives of this paper are to:(i) determine if environmental cooperation among countries with different environmental damage parameters provides environmental gains, overall welfare gains, or both, (ii) identify which cooperative scenarios will emerge in a stable environmental coalition to exploit these gains, and (iii) examine the effect of heterogeneity in environmental damages on the stability and success of these environmental coalitions.

In our model, each country has one firm producing an emission-intensive, homogeneous good, while generating an equal number of transboundary emissions. In stage one, each country decides its coalition membership. In stage two, each country chooses the emissions tax rate and the import tariff that maximizes the coalition's welfare. In stage three, each firm chooses its profit-maximizing production rate non-cooperatively. A coalition is considered stable if no firm has an incentive to enter or exit the coalition (D'Aspremont et al. 1983).

The numerical simulation shows that the grand coalition is stable at different levels of environmental damage heterogeneity. At sufficiently small market sizes, the grand coalition results in both environmental and overall welfare gains. However, at sufficiently large market sizes, the grand coalition only provides overall welfare gains without environmental gains. Therefore, this paper provides evidence that combining environmental and trade policies can result in a decrease in global emissions in sufficiently small markets, even in the presence of country heterogeneity.

Keywords: Coalition Formation, Environmental Agreements, Heterogeneity, International Trade, Environmental Taxes, JEL: Q56, F18, H23, C7

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1. Introduction

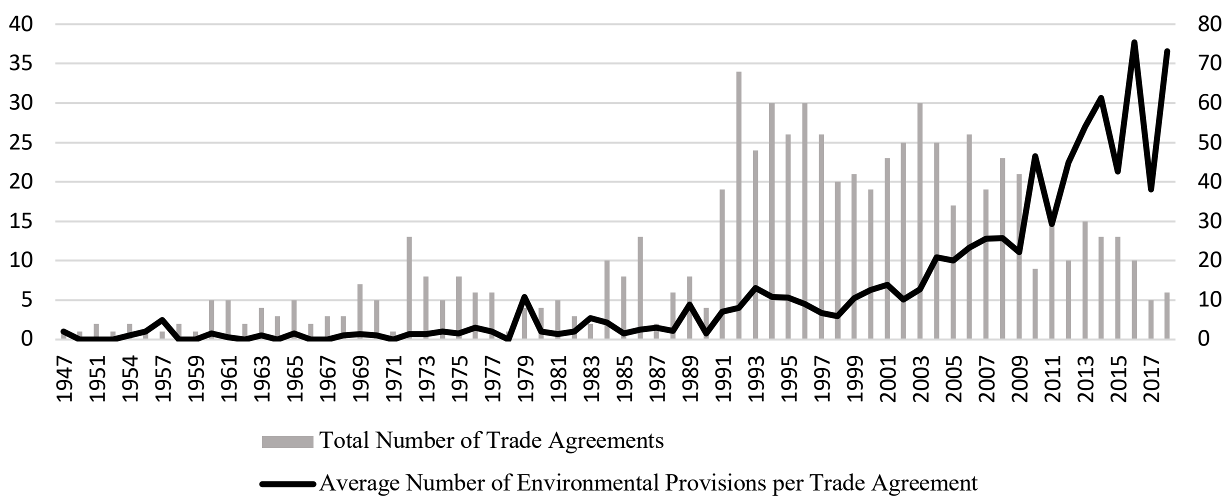
Transboundary pollution and greenhouse gas emissions are some of the most challenging and pressing environmental problems of the twenty-first century (UNEP, 2019). The United Nations World Meteorological Organization's latest report (WMO, 2021) warns that the last seven years (2015-2021) have been the warmest on record. Despite the initial fall in global emissions during the peak of the pandemic confinement measures, the increase in global average temperature in 2021 was 1.2 °C above pre-industrial levels (1850-1900), approaching the lower limit of 1.5 °C set by the Paris Agreement (WMO, 2021). To meet the 2015 Paris target, nations need to cut their global emissions in half by 2030 (UNEP, 2019), and global carbon emissions will have to reach net zero in the early 2050s (IPCC, 2022). However, strong incentives to free ride, and difficulties in enforcing International Environmental Agreements (IEAs), make international cooperation a challenging task (Diamantoudi et al., 2018a).

Developed and developing countries have long been divided on the responsibilities to reduce greenhouse gas emissions. The United States and Canada, for example, signed the original Kyoto Protocol in 1997 but later pulled out. The 2019 UN Climate Conference (COP25) in Madrid, aimed at finalizing the Paris Agreement rulebook, could not reach a consensus in many areas. China, the world's largest emitter of CO2, shows no plans to stop building coal plants at home or shutter old ones (Standaert, 2021). Similarly, Canada has promised to cut emissions by funding greener infrastructure while still subsidizing one of the largest sources of emissions in the country, the oil and gas industry (Carter and Dordi, 2021). Moreover, as governments prioritize post-pandemic economic recovery, many major emitters, such as Japan, Russia, and Saudi Arabia, are not on track to meet Paris pledges (CAT, 2020). While countries are predominantly focused on the economic and social disruptions of the global pandemic, the emissions reductions in the 2021 Glasgow Climate Pact (COP26) still fall short of the reductions needed to meet the Paris Climate Agreement targets (COP21). Therefore, Paris pledges to reduce greenhouse gas emissions remain vague promises rather than credible plans and actions, and the fight against climate change is just being delayed.

Addressing environmental damage heterogeneity, this paper examines environmental cooperation among heterogeneous trading partners to analyze the feasibility of partial and global international environmental agreements. In the context of international trade, governments face the tradeoff of higher taxes to cooperatively reduce emissions or higher tariffs on exports when acting non cooperatively. However, countries do not necessarily have the same vulnerability to aggregate emissions exposure and suffer different environmental damage consequences when faced with transboundary pollution. While property damages from wildfires made the headlines in Canada in 2021, in East Africa, farmers battling the drought were forced to remove their children from school. Indeed, developing countries like Bangladesh, Bhutan, Kenya, and Gambia have been more vulnerable to environmental damages than other countries, experiencing severe economic and non-economic losses (Mckibben, 2021).

The relationship between trade and the environment has often been seen as one of divergence between economic development and environmental degradation. The opportunities to bring trade and environmental policies closer together were often overlooked. Trade can play a vital role in reducing countries' free-riding incentives and increasing their willingness to cooperate, all while providing adequate support for stable climate coalitions (Diamantoudi et al. 2018c). Bridging the divide between trade and the environment, preferential trade agreements today enclose increasing amounts of environmental provisions. As indicated in Figure1, these have become a regular feature of almost 85% of the trade agreements signed between 1947 and 2018 (Morin et al. 2018). These environmental provisions are becoming gradually more diverse and extensive, covering an increasingly wide range of environmental protection, with some directly addressing the reduction of greenhouse gases (GHG) emissions and the cooperation on climate change. Some of these clauses are even more specific and restrictive than those found in multilateral environmental agreements (Morin and Jinnah, 2018). Assessing the environmental impact of these environmental provisions, scholars have found that they increased green exports from developing countries (Brandi et al., 2020) and reduced the greenhouse gases (GHG) emissions resulting from trade flows (Baghdadi et al. 2013, Zhou et al. 2017, Martínez-Zarzoso and Oueslati 2018, Tharakan and Zakaria 2020). These recent studies suggest that the coordination of trade and environmental policies can be a valuable strategy in diffusing environmental policies across borders and strengthening international environmental cooperation beyond what is currently implemented.

Figure 1: Average Number of Environmental Provisions in Trade Agreements 1947- 2018



Note. Data from https://www.chaire-epi.ulaval.ca/sites/chaire-epi.ulaval.ca/files/trend_2_public_version.xlsx

Environmental damage heterogeneity would imply that environmental and trade policies can have different welfare implications for different countries and create different incentives concerning environmental cooperation. Thus, the main objectives are to i) Determine whether environmental cooperation among countries with different environmental damage parameters provides environmental gains, overall welfare gains, or both, ii) Identify which cooperative scenarios will emerge in a stable coalition among countries, and iii) Capture the effect of heterogeneity in environmental damages on the stability and success of these environmental coalitions.

This research contributes to and connects two branches of the theoretical literature: the one on Environmental Cooperation and Trade and the other on International Environmental Agreements.

Concerning the literature on Environmental Cooperation and Trade, scholars (Conrad 1993, Barrett 1994, Kennedy 1994, Tanguay 2001) have long examined strategic environmental policy in a symmetric trade framework without addressing heterogeneity among countries. Duval and Hamilton (2002) analyzed environmental tax policy under cooperative and non-cooperative equilibria allowing for differences in consumer market size, production costs, number of firms, and pollution diffusion in a two-country trade union. They did not tackle, however, the welfare implications of each scenario. Cheikbossian (2010) focused on market size heterogeneity under free trade in a two-country global market model. He did not address the sustainability of environmental cooperation among the two heterogeneous entities. Gautier (2017) addressed differences in abatement and production costs on environmental policy reforms in the context of trade and Cournot oligopoly. He focused on the effectiveness of tax policy reforms and their welfare implications ignoring the incentives behind countries' coordination. Baksi and Chaudhuri (2017) investigated environmental damage heterogeneity in a two-country repeated game model. They used trigger strategies and border tax adjustments to assess the robustness of environmental cooperation. They found that environmental cooperation among heterogeneous countries provided significant overall welfare gains, which are bound to increase with trade liberalization. This paper is comparable to the one developed by Baksi and Chaudhuri (2017), who focused on environmental damage heterogeneity in a 2-country repeated game. In contrast to Baksi and Chaudhuri (2017), the current research considers a three-stage static coalition formation game with three heterogeneous countries. The sustainability of environmental coordination is based on internal and external stability criteria (d'Aspremont et al., 1983) rather than exogenous trigger strategies and trade linkages.

Examining the impact of environmental damage heterogeneity on the formation and stability of environmental coalitions in trade, this paper also contributes to the literature on International Environmental Agreements (IEAs) among heterogeneous countries. Hoel (1992) and Barrett (1997) were among the first to model heterogeneity in international environmental agreement games. Using internal and external stability criteria, they found that the number of signatories was still small even when countries were modelled as heterogeneous. Later, Barrett (2001), Finus and Rundshagen (2003), Pavlova and de Zeeuw (2013), Hagen and Eisenack (2015), and Diamantoudi et al. (2018b) examined the stability of coalition formation with heterogeneous countries but not in the context of international trade, transfer payments or trade linkages. It was found that, in pure IEAs, where a coalition only generates positive externalities to non-members, heterogeneity does not increase the size of stable coalitions and can reduce the likelihood of cooperation. Heterogeneity, however, when associated with direct transfer payments, can improve the prospect of cooperation and support the stability of larger coalitions (Botteon and Carraro 2001, McGinty 2007, Biancardi and Villani 2010, Diamantoudi et al. 2018c, Bakalova and Eyckmans 2019, Finus and McGinty 2019). In this case, the coalition generates a positive externality due to lower emissions and a negative externality to non-signatories due to the forgone transfer. By not signing the IEA, non-members essentially lose the transfer payment, which constitutes a form of a negative externality generated by the coalition. Heterogeneity, when associated with trade linkages, such as trade sanctions, can reduce free-riding incentives and increase the size of stable coalitions (Cirone and Urpelainen 2013, Nordhaus 2015, Hagen and Schneider 2021). Scholars who have examined IEAs with R&D linkages (Biancardi and Villani 2018, Eichner and Kollenbach 2021) have found that R&D linkages, like trade linkages, improved environmental coalitions' stability and increased cooperation relative to pure IEAs models.

Few scholars, Cavagnac and Cheikbossian (2017), have examined the stability of international environmental agreements in the context of international trade with heterogeneous countries. Using Coalition-Proof Nash Equilibria (Bernheim et al., 1987) in a free trade setting, they found that market size heterogeneity fostered partial rather than global agreements. This paper is comparable to the one developed by Cavagnac and Cheikbossian in 2017. However, using internal and external stability criteria (d'Aspremont et al., 1983) rather than Coalition-Proof Nash Equilibria to evaluate the stability of environmental coalitions, we focus on environmental damage rather than market size heterogeneity in a segmented market setting with positive import tariffs rather than free trade.

The present model is built as follows. There are three countries, each with a different environmental damage parameter. Each country has only one firm producing a homogenous polluting product. The production process generates transboundary air pollution such as carbon dioxide (CO_2). Consumers in each country are affected by aggregate global emissions, and every unit produced generates one unit of global emissions. The firm's choice variable is production (emissions). Abatement is not modelled as a separate choice variable, but forgone profit is the firm's abatement cost. Firms can reduce emissions by producing less output at the expense of reducing profits and thus face a tradeoff between emissions and profits. Firms compete à la Cournot in a segmented market where each firm faces its demand domestically rather than shared global market demand.

International trade occurs in domestic markets. Therefore, each country can use import tariffs as a trade policy tool to protect local production. There are no transfer payments between countries; fiscal revenues remain in the state of origin. Each government uses a per-unit production (emissions) tax rate as an environmental policy instrument. Thus, governments face the tradeoff of imposing higher taxes to cooperatively reduce emissions or paying higher tariffs on exports when acting non cooperatively.

The simple coalition formation game is composed of three stages. Stage one is the coalition formation game where each country chooses its coalition membership. A coalition is stable if no firm has an incentive to either enter or exit the coalition (D'Aspremont et al., 1983). In the second stage, each country chooses the emissions tax rate and the import tariff rate that maximize the coalition's welfare. In the third stage, each firm chooses noncooperatively the production rate that maximizes the firm's profit. The static coalition formation game is solved by backward induction, starting from the third stage, and moving backward to the first stage.

It is shown that the grand coalition is stable at various levels of environmental damage heterogeneity, and countries can achieve environmental gains and overall welfare gains under this cooperative equilibrium.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 examines the heterogeneous endogenous solution, and section 4 concludes the paper.

2. The Model Defined

The model considers an open economy with three heterogeneous countries, $N = \{i, j, k\}$. Each country has only one profit-maximizing firm, producing a homogenous polluting product. The total production of the firm located in country i is given by

$$X_i = \left(x_{ii} + x_{ij} + x_{ik}\right), \tag{1}$$

where x_{ii} is produced and sold in country i, and x_{ij} is produced in country i and exported to country j $\forall\, i \neq j, k$. For the market structure to be maintained throughout the game, it is assumed that $X_i, x_{ii} \in \mathbb{R}_n^{++}$ $\forall\, i, j, k$ and $x_{ij} \in \mathbb{R}_n^{+}$ $\forall\, i, j, k$. The production process generates transboundary air pollution such as carbon dioxide (CO_2). Every unit produced generates one unit of global emissions. The firm's choice variables are local production and exports, which also represent emissions. Firms can reduce emissions by producing less output at the expense of reducing profits and thus face a tradeoff between emissions and profits. Hence, abatement is neither an option nor a choice variable.

Total consumption in country i is given by:

$$Q_i = \left(x_{ii} + x_{ji} + x_{ki}\right), \tag{2}$$

where x_{ii} is locally produced and x_{ji} is imported from country j $\forall\, i \neq j, k$.

Firms compete à la Cournot in a segmented market where each firm faces its own demand domestically. The market demand in country i is given by:

$$Q_i = (\alpha - P_i), \tag{3}$$

where Q_i is the total consumption of the polluting good in country i, P_i is the price of the good in market i, and α is the marginal utility derived from its consumption. For simplification, it is assumed that the marginal cost of production is equal to zero, and each firm can export to the other two foreign markets at no transaction costs.

Pollution generates environmental damage in each country; the social cost of pollution is linear in global emissions:

$$D_i(X) = \beta_i(X_i + X_j + X_k), \tag{4}$$

where β_i is the marginal environmental damage in country i caused by aggregate production, that is, by global emissions. The linear environmental damage function makes the analysis more readable and tractable. For the market to be active and the model's solution to be interior, it is assumed that the marginal environmental damage parameter cannot be higher than the marginal utility of good X given by α, and thus $\beta_i \in (0, \alpha)$. Consumers in each country are affected by the aggregate level of emissions. As such, variance in environmental damages does not manifest through different emissions exposure levels, but how the same number of emissions translates into costs, given the underlying determinants of heterogeneity, such as income, health stock, defensive investment, or baseline exposure (Hsiang et al., 2018). In this model, therefore, different environmental damages are a result of different impacts of the same emissions levels. In other words, all three countries face the same global emissions, but they are impacted differently.

The government in country i imposes a non-negative tariff $\tau_{i,j}$ per unit of imports from country j and $\tau_{i,k}$ per unit of imports from country k where $i \neq j, k$. As a result, $\tau_{j,i}$ and $\tau_{k,i}$ is the effective marginal cost of the firm operating in country i on exports to country j and k, respectively.

In addition to import tariffs as a trade policy tool, each government uses a per-unit of production tax rate t_i that is imposed on the local firm as an environmental policy instrument. Since every unit produced precisely generates a unit of pollution emissions, then a tax per unit of production t_i is equivalent to a tax per unit of emissions. Thus, the government in country i collects tariff revenues on imports from foreign markets given by,

$$TR_i = \left(\tau_{i,j}x_{ji} + \tau_{i,k}x_{ki}\right), \tag{5}$$

and emissions tax revenues expressed as,

$$ER_i = t_i\left(x_{ii} + x_{ij} + x_{ik}\right) = t_i X_i. \tag{6}$$

It is assumed that there are no transfer payments between countries and fiscal revenues remain in the state of origin.

Let $\mathcal{S}$ be a coalition where $\mathcal{S} \subset N = \{i, j, k\}$. $\mathcal{S}$ represents a group of countries cooperating on environmental and trade policies. Coalition members will determine their emissions tax rates $t_\mathcal{S}$ jointly. Each coalition $\mathcal{S}$ is associated with two import tariff rates: $\tau_\mathcal{S}$ represents the common tariff rate that members of $\mathcal{S}$ would charge to each other, and $\tau_{\mathcal{S},k}$ where $k \notin \mathcal{S}$, represents the tariff rate that members of $\mathcal{S}$ would charge to each of its non-members. Therefore, it is explicitly assumed that coalition members will charge the same tariff rate to each other $\tau_\mathcal{S} = \tau_{i,j} = \tau_{j,i}$, if $i, j \epsilon\, \mathcal{S}$ and the same tariff rate to non-members $\tau_{\mathcal{S},k} = \tau_{i,k} = \tau_{j,k}$, if $i, j \epsilon\, \mathcal{S}$ and $k \notin \mathcal{S}$ even if coalition members are heterogeneous and not associated with each other.

In a three-country model, there are three types of coalition structures: i) the grand coalition, ii) the singletons, and iii) a pair and a singleton. The simple coalition formation game is composed of three stages. Stage one is the coalition formation game; each country chooses its coalition membership $\mathcal{S}$. In the second stage, each country chooses the emissions tax rate $t_\mathcal{S}$, and the import tariff rates τ_S and $\tau_{S,k}$, that maximizes the coalition's welfare $W_S = \sum_{i \in S} W_i$, given the coalition structure C. In the third stage, each firm chooses noncooperatively its profit-maximizing production rate X_i, given the coalition structure C, import tariffs, and the emissions tax rate. The coalition formation game is solved by backward induction, starting from the third stage, and moving backward to the first stage.

2.1 Stage Three − The Firm's Optimization Problem

In stage three, each firm chooses noncooperatively the output rate by maximizing its profit function, taking as given the policies set by all three governments and the output decisions of the other two foreign firms. Firms compete à la Cournot in domestic markets, and each firm has three choice variables: production for the local market x_{ii}, and exports to the other two foreign markets x_{ij} and x_{ik}. The total profit function of the firm located in country i consists of total revenues from the domestic market i and the foreign markets j and k, minus the emissions tax imposed on production and the tariff costs incurred on exports. Thus, the firm's optimization problem[1] is:

$$\max_{x_{ij}\ j \in N} \pi_i = \max_{x_{ij}\ j \in N} \sum_{j \in N} \left(P_j(x_{ij}) x_{ij} - t_i x_{ij}\right) - \sum_{j \in N/\{i\}} \tau_{ji} x_{ij} \qquad \forall i \in N \tag{7}$$

The first order conditions with respect to local production and exports are as follows:

$$\frac{\partial \pi_i}{\partial x_{ii}} = 0 => {x_{ii}}^* = \frac{1}{2}\left(\alpha - x_{ji} - x_{ki} - t_i\right) \tag{8}$$

$$\frac{\partial \pi_i}{\partial x_{ij}} = 0 => {x_{ij}}^* = \frac{1}{2}(\alpha - x_{jj} - x_{kj} - t_i - \tau_{j,i}) \tag{9}$$

Using the first order conditions (8) and (9), the equilibrium quantities produced by the firm operating in country i are as follows $\forall i, j, k \in N$,

$${x_{ii}}^* = \frac{1}{4}(\alpha - 3t_i + \left(t_j + t_k\right) + \tau_{i,j} + \tau_{i,k}) \tag{10}$$

$${x_{ij}}^* = \frac{1}{4}\left(\alpha - 3t_i + \left(t_j + t_k\right) + \tau_{j,k} - 3\tau_{j,i}\right) \tag{11}$$

The second order conditions (SOCs) are satisfied, as we have:

$$\frac{\partial^2 \pi_i}{\partial {x_{ii}}^2} < 0, \frac{\partial^2 \pi_i}{\partial {x_{ij}}^2} < 0 \text{ and } \frac{\partial^2 \pi_i}{\partial^2 x_{ii}} \frac{\partial^2 \pi_i}{\partial {x_{ij}}^2} - \left(\frac{\partial^2 \pi_i}{\partial x_{ii} \partial x_{ij}}\right) > 0$$

The Cournot Equilibrium would imply that domestic production and exports are decreasing in the local emission tax rate t_i and increasing in the tax rates imposed on foreign firms t_j and t_k. As expected, domestic production increases in the local tariff rates $\tau_{i,j}$ and $\tau_{i,k}$ and exports are decreasing in foreign tariffs $\tau_{j,i}$ and $\tau_{k,i}$. Note that tariff rates are assumed to be sufficiently low

[1] The firm's optimization problem is detailed in appendix A.

and below the prohibitive rates, so that $x_{ij} \in \mathbb{R}_n^+ \; \forall \, i, j, k$. The third stage of the game is common to all coalition structures. The welfare function of any country is based on the optimal output quantities obtained in this stage.

2.2 Stage Two − The Government's Optimization Problem

In a 3-country global economy, there are three types of coalition structures:

- A coalition structure C_{NC}, composed of three singletons, containing one country each, where $C_{NC} = \{\{i\}, \{j\}, \{k\}\}$.
- A coalition structure C_G, composed of one coalition containing all three countries, the grand coalition $C_G = \{\{i, j, k\}\}$.
- A coalition structure C_P, composed of two coalitions, a pair and a singleton. There are 3 such coalition structures. For example, $C_P^k = \{\{i, j\}, \{k\}\}$ is composed of the pair formed by countries i and j, and of country k which remains a singleton.

The emissions tax rate $t_{\mathcal{S}}$, and the tariff rates $\tau_{\mathcal{S}}$ and $\tau_{\mathcal{S},k}$, are determined by maximizing the coalition's welfare $W_{\mathcal{S}}$, given the firms' optimal output quantities derived in stage three:

$$\max_{t_{\mathcal{S}},\, \tau_{\mathcal{S}},\, \tau_{\mathcal{S},k}} W_{\mathcal{S}} = \max_{t_{\mathcal{S}}, \tau_{\mathcal{S}}, \tau_{\mathcal{S},k}} \sum_{i \subset \mathcal{S}} W_i \left(t_{\mathcal{S}}\right) \text{ where } t_{\mathcal{S}} = t_i{}^{2} \qquad \forall i \in \mathcal{S} \tag{12}$$

Under the singleton structure C_{NC}, each government sets independently a non-cooperative emissions tax rate t_i^{NC} and two non-negative import tariff rates, $\tau_{i,j}$ and $\tau_{i,k}$. We end up, therefore, with three emissions tax rates t_i^{NC}, t_j^{NC}, t_k^{NC} and six tariff rates $\tau_{i,j}, \tau_{i,k}, \tau_{j,i}, \tau_{j,k}, \tau_{k,i}$, and $\tau_{k,j}$.

Under the grand coalition structure C_G, countries collectively decide to tax the production of the polluting good at rates that maximize the joint welfare of all three countries. Countries in the grand coalition set a common uniform tax rate t_G and have one non-negative common tariff rate, τ_G.

Under the partial coalition structure C_P, two countries i and j form a coalition $\mathcal{S}$, and the third country k remains a singleton. Countries in the partial coalition $\mathcal{S}$ set a common uniform tax rate $t_{\mathcal{S}} = t_{ij}$, and have non-negative common tariff rates, $\tau_{\mathcal{S}} = \tau_{ij}$ to be levied on each other, and $\tau_{\mathcal{S},k} = \tau_{ij,k}$ where $i, j \in \mathcal{S}$ and $k \notin \mathcal{S}$, to be imposed on the singleton k.

Recall, that firms regardless to which coalition their countries belong, they are competing à la Cournot, and still act independently of each other in the third stage of the oligopoly game.

The welfare function of country i, denoted by W_i, consists of the domestic consumer surplus CS_i, the domestic firm's profits π_i, the government's tariff revenues TR_i and emissions tax revenues ER_i, minus the environmental damages D_i caused by global emissions. Thus, country i 's total welfare function can be written as,

[2] t_i and $t_{\{i\}}$ will be used in this paper interchangeably.

$$W_i = (CS_i + \pi_i + ER_i + TR_i - D_i) \tag{13}$$

Expanding the terms of country i 's total welfare function, then equation (13) can be expressed as:

$$W_i = \begin{bmatrix} \frac{1}{2}(Q_i)^2 - \beta_i\big(X_i + X_j + X_k\big) + (\tau_{i,j}x_{ji} + \tau_{i,k}x_{ki}) \\ +(\alpha - Q_i)x_{ii} + \big(\alpha - Q_j - \tau_{j,i}\big)x_{ij} + \big(\alpha - Q_k - \tau_{k,i}\big)x_{ik} \end{bmatrix} \tag{14}$$

The current model assumes that any cooperative equilibrium under the grand coalition and the partial coalition structure, would imply a uniform emissions tax rate, that is a single emissions tax rate adopted by all countries within the coalition $\mathcal{S}$, and common import tariff rates $\tau_{\mathcal{S}}$ and $\tau_{\mathcal{S},k}$. Indeed, economists and academics have frequently advocated uniform emissions tax solutions as an efficient policy instrument to tackle global environmental problems (Hoel 1992, Finus and Rundshagen 1998, Nordhaus 2006, Weitzman 2014). Advocates of uniform solutions often argue that these solutions are straightforward, typically involving less negotiation time and thus fewer transaction costs than differentiated solutions. It is also argued that uniform emissions taxes appear equitable since every country faces the same tax rate and are generally viewed as "fair" by the public (Finus and Rundshagen 1998, McEvoy and McGinty 2018). Moreover, uniform emissions tax rates are easily verifiable in an agreement. "Uniform solutions constitute some kind of focal point in the sense of Schelling (1960) on which bargaining partners feel relatively easy to agree." (Finus and Rundshagen 1998, page 149).

2.3 Stage One − Coalition Formation

In the first stage, the formation and stability of each coalition structure is analyzed. A coalition structure is stable if no country has an incentive to either enter or exit a coalition within the structure. This definition of stability is based on the original definition of cartel stability developed by D'Aspremont al. (1983).

Let C be the coalition structure to which a coalition $\mathcal{S}$ belongs; $W_{i\in\mathcal{S}}^{C}$ denotes the welfare of country i, where i belongs to $\mathcal{S}$. As such, $W_i^{C_{NC}}$, $W_i^{C_G}$, $W_i^{C_P^k}$, and $W_i^{C_P^i}$, represent respectively, the welfare function of country i when i is a singleton, a member of the grand coalition, a pair member of a partial coalition formed by countries i and j, and an outsider to a partial coalition formed by countries j and k.

Definition: A coalition $\mathcal{S} \subset N$, where $\mathcal{S} \in C$ is stable if it is both internally and externally stable.

- $\mathcal{S}$ is internally stable $\Leftrightarrow \forall i \in \mathcal{S}, W_i^C \geq W_i^{C^f}$ where $C^f = C/\mathcal{S} \quad \cup \quad \{\mathcal{S}/\{i\}, \{i\}\}$ (15)
- $\mathcal{S}$ is externally stable $\Leftrightarrow \forall i\ , W_{\{i\}}^{C} \geq W_{i\subset\mathcal{S}}^{C^c}$ where $C^c = \{C/\{i\} \quad \cup \quad \{\mathcal{S} \cup \{i\}\}\}$ (16)

In particular, C^f is a finer coalition structure than C; that is, as country i leaves the coalition $\mathcal{S}$ to become a singleton, C^f contains the remaining members of $\mathcal{S}$ and a singleton $\{i\}$.

In contrast, C^c is a coarser coalition structure than C; since country i, initially behaving as a singleton $\{i\}$, now joins the other member(s) in the coalition $\mathcal{S}$.

Notably, the singleton coalition structure C_{NC} is internally stable by default as it is the finest coalition structure, and no country has the possibility to leave a coalition formed by itself. Similarly, the grand coalition C_G is externally stable by default as all countries are members of the coalition and there no outsiders left to join the coalition. The partial coalition C_P is externally stable if no outsider has an incentive to join and is internally stable if no member has an incentive to exit the coalition and become a singleton.

Within our context, therefore, in the partial coalition structures, we need to investigate whether both internal and external stability conditions are satisfied. Whereas in the singleton structure, we only need to check for external stability, and in the case of the grand coalition, we only need to check for internal stability.

3. The Heterogeneous Tie-In Endogenous Case

The heterogeneous case assumes that countries have different environmental damage parameters, where $\beta_i > \beta_j > \beta_k$. Considering a Tie-In scenario, members of a coalition $\mathcal{S}$ coordinate all their actions with other members. They impose a uniform emissions tax rate $t_{\mathcal{S}}$ and uniform non-negative import tariff rates, $\tau_{\mathcal{S}}$ to be levied on each other, and $\tau_{\mathcal{S},k}$ to be imposed on non-members.

3.1 The Singleton Structure C_{NC} − Noncooperative Equilibrium

Under the singleton structure C_{NC}, each government sets independently a noncooperative emissions tax rate ${t_i}^{NC}$ $\forall i \in N = \{i, j, k\}$, and tariff rates on imports from other countries. Let $\tau_{i,j}$ be the tariff rate imposed by country i on imports from country j, and $\tau_{i,k}$ be the tariff rate imposed by country i on imports from country k, $\forall i, j, k$ and $i \neq j, k$. We end up, therefore, with three emissions tax rates ${t_i}^{NC}$, ${t_j}^{NC}$, ${t_k}^{NC}$ and six import tariff rates, $\tau_{i,j}, \tau_{i,k}, \tau_{j,i}, \tau_{j,k}, \tau_{k,i}$, and $\tau_{k,j}$.

The equilibrium quantities produced by the firm operating in country i, given by equations (10) and (11), can thus be rewritten as follows $\forall i, j, k \in N$ and $i \neq j, k$:

$$ {x_{ii}}^*(C_{NC}) = \frac{1}{4}(\alpha - 3{t_i}^{NC} + {t_j}^{NC} + {t_k}^{NC} + \tau_{i,j} + \tau_{i,k}) \tag{17} $$

$$ {x_{ij}}^*(C_{NC}) = \frac{1}{4}\left(\alpha - 3{t_i}^{NC} + {t_j}^{NC} + {t_k}^{NC} + \tau_{j,k} - 3\tau_{j,i}\right) \tag{18} $$

To guarantee an interior solution and for the market structure to be maintained throughout the game, it is assumed that ${x_{ii}}^* \in \mathbb{R}_n^{++}$ and ${x_{ij}}^* \in \mathbb{R}_n^{+}, \forall i, j, k \in N$.

Accordingly, country i's maximization problem[3] (12) can be written as,

[3] Country i's optimization problem under the singleton structure is detailed in appendix B.

$$\max_{t_i^{NC},\tau_{i,j},\tau_{i,k}} W_i^{C_{NC}} = \max_{t_i^{NC},\tau_{i,j},\tau_{i,k}} \begin{bmatrix} \frac{1}{2}\left(Q_i\left(t_i^{NC}\right)\right)^2 - \beta_i\left(X_i\left(t_i^{NC}\right) + X_j\left(t_i^{NC}\right) + X_k\left(t_i^{NC}\right)\right) \\ +\tau_{i,j}x_{ji}^*\left(t_i^{NC}\right) + \tau_{i,k}x_{ki}^*\left(t_i^{NC}\right) + \left(\alpha - Q_i\left(t_i^{NC}\right)\right)x_{ii}^*\left(t_i^{NC}\right) \\ +\left(\alpha - Q_j\left(t_i^{NC}\right) - \tau_{j,i}\right)x_{ij}^*\left(t_i^{NC}\right) + \left(\alpha - Q_k\left(t_i^{NC}\right) - \tau_{k,i}\right)x_{ik}^*\left(t_i^{NC}\right) \end{bmatrix} \quad (19)$$

The first order conditions of the above welfare maximization problem (19) with respect the emissions tax rate, $t_{i,j}(C_{NC})$, and bilateral tariff rates, $\tau_{i,j}(C_{NC})$ and $\tau_{i,k}(C_{NC})$, are given by the following equations, $\forall i \epsilon N = \{i,j,k\}$ and $i \neq j,k$:

$$\frac{\delta W_i^{C_{NC}}}{\delta t_i^{NC}} = 0 \Rightarrow \left(-9\alpha + 12\beta_i - 17t_i^{NC} - 5\left(t_j^{NC} + t_k^{NC}\right) + 3(\tau_{i,j} + \tau_{i,k}) + 6(\tau_{j,i} + \tau_{k,i}) - 2(\tau_{j,k} + \tau_{k,j})\right) = 0 \quad (20)$$

$$\frac{\delta W_i^{C_{NC}}}{\delta \tau_{i,j}} = 0 \Rightarrow \left(4\beta_i + 3\alpha + 3t_i - 9t_j + 7t_k - 21\tau_{i,j} + 11\tau_{i,k}\right) = 0 \quad (21)$$

$$\frac{\delta W_i^{C_{NC}}}{\delta \tau_{i,k}} = 0 \Rightarrow \left(4\beta_i + 3\alpha + 3t_i - 9t_k + 7t_j - 21\tau_{i,k} + 11\tau_{i,j}\right) = 0 \quad (22)$$

The first order condition of the welfare maximization problem (19) with respect to the emissions tax rate $t_{i,j}(C_{NC})$ yields the following negative best response function, $\forall i \epsilon N = \{i,j,k\}$:

$$t_i^{NC}\left(t_j^{NC}, t_k^{NC}\right) = \frac{1}{17}\left(-9\alpha + 12\beta_i - 5\left(t_j^{NC} + t_k^{NC}\right) + 3(\tau_{i,j} + \tau_{i,k}) + 6(\tau_{j,i} + \tau_{k,i}) - 2(\tau_{j,k} + \tau_{k,j})\right) \quad (23)$$

The singleton behaves non cooperatively; hence, it has a negative best response function implying strong free riding behavior.

The first order conditions (20), (21), and (22), yield the following equilibrium emissions tax rate and non-negative import tariff rates $\forall i \epsilon N = \{i,j,k\}$:

$$t_i^{NC*}(C_{NC}) = \frac{1}{688}\left(-129\alpha + 499\beta_i - 13\left(\beta_j + \beta_k\right)\right) \quad (24)$$

$$\tau_{i,j}{}^*(C_{NC}) = \frac{1}{1376}\left(387\alpha + 19\left(45\beta_i - 19\beta_j\right) + 151\beta_k\right) \quad (25)$$

$$\tau_{i,k}{}^*(C_{NC}) = \frac{1}{1376}\left(387\alpha + 19(45\beta_i - 19\beta_k) + 151\beta_j\right) \quad (26)$$

Under this noncooperative equilibrium, each country's emission tax rate is positively related to its environmental damage parameter and inversely related to the other two countries' environmental damage parameters, implying solid free riding incentives.

$\forall i,j,k$ and $i \neq j,k$, country i's local production $x_{ii}(C_{NC})$ and exports $x_{ij}(C_{NC})$ are respectively:

$$x_{ii}(C_{NC}) = \frac{1}{688}\left(301\alpha - 167\beta_i + 105\left(\beta_j + \beta_k\right)\right) \quad (27)$$

$$x_{ij}(C_{NC}) = \frac{1}{1376}(215\alpha - 453\beta_i - 165\beta_j + 59\beta_k) \quad (28)$$

Country i's total quantity produced $X_i(C_{NC})$ and total quantity consumed $Q_i(C_{NC})$ are given by:

$$X_i(C_{NC}) = \frac{1}{172}\Big(129\alpha - 155\beta_i + 13\big(\beta_j + \beta_k\big)\Big) \tag{29}$$

$$Q_i(C_{NC}) = \frac{1}{172}\Big(129\alpha - 83\beta_i - 23\big(\beta_j + \beta_k\big)\Big) \tag{30}$$

The world market clears. Global production equals global consumption, as shown by the expression:

$$\sum_i X_i(C_{NC}) = \sum_i Q_i\,(C_{NC}) = \frac{129}{172}\big(3\alpha - \sum \beta_i\big) \tag{31}$$

Given the assumption that every unit of production generates exactly one unit of emissions, then equation (31) represents global emissions as well.

Country i's welfare, $W_i^{C_{NC}}$, $\forall i, j, k$ and $i \neq j, k,$ is given by:

$$W_i^{C_{NC}} = \frac{\mathbf{1}}{\mathbf{688}}\frac{\mathbf{1}}{\mathbf{(86)}}\left[\begin{array}{c} \alpha\Big(27735\alpha - 129602\beta_i - 7310\big(\beta_j + \beta_k\big)\Big) \\ +\beta_i\Big(20087\beta_i + 41618\big(\beta_j + \beta_k\big)\Big) + 8171\big({\beta_j}^2 + {\beta_k}^2\big) - 3178\beta_j\beta_k \end{array}\right] \tag{32}$$

3.2 The Grand Coalition Structure C_G − Fully Cooperative Equilibrium

Under the grand coalition, countries collectively decide to tax the production of the polluting good at a uniform tax rate, $t_G(C_G)$, that maximizes the joint welfare of all countries, such that,

$$t_i(C_G) = t_j(C_G) = t_k(C_G) = t_G(C_G).$$

Members of the grand coalition not only coordinate their environmental policies with other members, but their trade policies as well. They impose a uniform emissions tax rate and a common non-negative import tariff rate, such that,

$$\tau_{i,j}(C_G) = \tau_{i,k}(C_G) = \tau_{j,i}(C_G) = \tau_{j,k}(C_G) = \tau_{k,i}(C_G) = \tau_{k,j}(C_G) = \tau_G(C_G).$$

Hence, the equilibrium quantities produced by the firm operating in country i, given by equations (10) and (11), can be reduced to $\forall i, j, k \in N$ and $i \neq j, k$:

$$x_{ii}^*(C_G) = \frac{1}{4}(\alpha - t_G + 2\tau_G) \tag{33}$$

$$x_{ij}^*(C_G) = x_{ik}^*(C_G) = \frac{1}{4}(\alpha - t_G - 2\tau_G) \tag{34}$$

All the above optima are indeed interior solutions given the restrictions imposed on the model's parameters.

Given country i's maximization problem (12), then the grand coalition's maximization problem[4] can be written as,

$$\max_{t_G,\tau_G} W^{C_G} = \max_{t_G,\tau_G} \sum_i \begin{bmatrix} \frac{1}{2}\left(Q_i(t_G)\right)^2 - \beta_i\left(X_i(t_G) + X_j(t_G) + X_k(t_G)\right) \\ +\tau_G\left(x_{ji}^*(t_G) + x_{ki}^*(t_G)\right) + \left(\alpha - Q_i(t_G)\right)x_{ii}^*(t_G) \\ +\left(\alpha - Q_j(t_G) - \tau_G\right)x_{ij}^*(t_G) + \left(\alpha - Q_k(t_G) - \tau_G\right)x_{ik}^*(t_G) \end{bmatrix} \tag{35}$$

The first order conditions of the above welfare maximization problem (35) with respect to t_G and τ_G are respectively,

$$\frac{\delta W^{C_G}}{\delta t_G} = 0 => 3t_G = \left(-\alpha + 4\sum_i \beta_i - 2\tau_G\right) \tag{36}$$

$$\frac{\delta W^{C_G}}{\delta \tau_G} = 0 => 2\tau_G = \left(-\alpha + 4\sum_i \beta_i - 3t_G\right) \tag{37}$$

The first order conditions (36) and (37) yield the following cooperative set of uniform solutions where any emissions tax rate and non-negative import tariff rate (t_G, τ_G) satisfying the following two conditions (38) and (39) is an equilibrium solution:

$$3t_G^*(C_G) + 2\tau_G^*(C_G) = \left(4\sum_i \beta_i - \alpha\right) \tag{38}$$

$$t_G^*(C_G) \le \frac{1}{3}\left(4\sum_i \beta_i - \alpha\right) \tag{39}$$

For example, if we were to assume that members of the grand coalition would operate under free trade, then $({t_G}^*, {\tau_G}^*) = \left(\frac{1}{3}(4\sum_i \beta_i - \alpha), 0\right)$ would be an equilibrium uniform solution. With positive import tariffs, $({t_G}^*, {\tau_G}^*) = \left(\frac{1}{4}(4\sum_i \beta_i - \alpha), \frac{1}{8}(4\sum_i \beta_i - \alpha)\right)$ is another equilibrium solution. Note that $\frac{\partial t_G^*(C_G)}{\partial \tau_G} = -\frac{2}{3} < 0$, which would imply that trade liberalization in the form of lower import tariffs would entail higher production levels and thus higher environmental damage, and would, therefore, require higher emissions taxes.

The fully cooperative agreement denotes that the uniform emissions tax rate is positively related to all three environmental damage parameters and negatively related to tariffs τ_G. This inverse relationship between the emissions tax rate and tariffs is unique to grand coalition's equilibrium, where changes in tariffs are offset by changes in emissions taxes. Hence, trade liberalization in the form of lower tariffs, will increase the emissions tax rate under this cooperative scenario, fostering a "race to the top" in terms of environmental regulations and standards.

$\forall i \in N$, and $i \neq j, k$, local production and exports of each coalition member are, respectively:

[4] Country i's optimization problem under the grand coalition structure is detailed in appendix C.

$$x_{ii}(C_G) = \frac{1}{3}(\alpha - \sum_i \beta_i + 2\tau_G) \tag{40}$$

$$x_{ij}(C_G) = x_{ik}(C_G) = \frac{1}{3}(\alpha - \sum_i \beta_i - \tau_G) \tag{41}$$

To guarantee that any member's local production $x_{ii}(C_G)$ is strictly positive, and exports $x_{ij}(C_G)$ and $x_{ik}(C_G)$ are positive, then $\frac{1}{2}(\sum_i \beta_i - \alpha) < \tau_G{}^*(C_G) \leq (\alpha - \sum_i \beta_i)$.

Assuming that $\tau_G{}^*(C_G) = 0$, then the equilibrium tax rate, $t_G^*(C_G)$, is given by:

$$t_G^*(C_G) = \frac{1}{3}(4\sum_i \beta_i - \alpha) \tag{42}$$

$\forall i \in N$, and i$\neq j, k$, local production (40) and exports (41) of each coalition member are given by:

$$x_{ii}(C_G) = \frac{1}{3}(\alpha - \sum_i \beta_i) \tag{43}$$

$$x_{ij}(C_G) = x_{ik}(C_G) = \frac{1}{3}(\alpha - \sum_i \beta_i) \tag{44}$$

The total quantity produced in country i is equal to the total quantity consumed in that market,

$$X_i(C_G) = Q_i(C_G) = (\alpha - \sum_i \beta_i) \tag{45}$$

The global production level, which is equal to the global consumption level, is equal to,

$$\sum_i X_i\,(C_G) = \sum_i Q_i\,(C_G) = 3(\alpha - \sum_i \beta_i) \tag{46}$$

As such, the world market clears since global production equals to global consumption. Given the assumption that every unit of production generates a unit of emissions, then equation (46) represents the global level of emissions generated by the grand coalition.

Country i's welfare as a member of the grand coalition, $W_i^{C_G}$, is given by,

$$W_i^{C_G} = \frac{1}{2}(\alpha - \sum_i \beta_i)\left(\alpha - 5\beta_i + (\beta_j + \beta_k)\right) \tag{47}$$

The grand coalition's collective welfare W^{C_G} is expressed as,

$$W^{C_G} = \sum_i W_i^{C_{NC}} = \frac{3}{2}(\alpha - \sum_i \beta_i)^2 \tag{48}$$

Note that a member individual welfare, $W_i^{C_G}$, and the grand coalition aggregate welfare, W^{C_G}, are both independent of the import tariff rate, $\tau_G(C_G)$. Under the grand coalition, changes in tariffs are offset by changes in the emissions tax rates, as indicated by equations (36) and (37); Accordingly, the individual production rate $X_i(C_G)$, the welfare of any member country $W_i^{C_G}$, global production $\sum_i X_i\,(C_G)$, and collective welfare W^{C_G}, do not change if we were to assume a different equilibrium solution $(t_G{}^*, \tau_G{}^*)$.

3.3 The Partial Coalition Structure (C_P) − Partial Cooperative Equilibrium

Under the partial coalition structure C_P, two countries, i and j for example, form a coalition $\mathcal{S}$, and the third country, k in this case, remains a singleton. Pair members cooperatively decide to tax the production of the polluting good at a uniform tax rate, $t_{ij}(C_P^k)$, that maximizes the joint welfare of the two members, where $W_{ij}^{C_P^k} = W_i^{C_P^k} + W_j^{C_P^k}$. As such, $\forall i, j, k \in N$,

$$t_i(C_P^k) = t_j(C_P^k) = t_{ij}(C_P^k).$$

In a Tie-in scenario, environmental coordination spans over global pollution and trade flows. It is assumed, therefore, that pair members within a partial coalition structure have zero tariffs among themselves and levy the same positive tariff rate on imports from the outsider, that is, $\forall i, j, k \in N$,

$$\tau_{i,j}\left(C_P^k\right) = \tau_{j,i}\left(C_P^k\right) = \tau_{ij}\left(C_P^k\right) = 0$$

$$\tau_{i,k}\left(C_P^k\right) = \tau_{j,k}\left(C_P^k\right) = \tau_{ij,k}\left(C_P^k\right).$$

The use of preferential tariffs as a carrot-and-stick mechanism to promote environmental policy and other non-trade policy objectives, such as human rights, labor standards, the production of narcotic drugs, and security, has been a common practice in the European Union (EU). For example, in 2010, when Sri Lanka violated a few of the UN human rights conventions, the European Union denied its trading partner the preferential market access at lower tariffs. Also, in 2010, the EU denied Venezuela the preferential access to the European market, when it failed to ratify the UN convention against corruption (Borchert et al. 2021). In line with these negative conditionality practices, it is assumed that $\tau_{ij}\left(C_P^k\right) = 0$, and pair members would restrict the preferential tariffs access to the outsider by imposing a positive tariff rate $\tau_{ij,k}\left(C_P^k\right)$, such that $\tau_{ij,k}\left(C_P^k\right) > \tau_{ij}\left(C_P^k\right)$.

Note that the two firms located in the countries forming the coalition $\mathcal{S}$ still act independently of each other and compete à la Cournot in the third stage of the oligopoly game.

Let $\tau_{i,jk}\left(C_P^i\right)$ be the positive tariff rate that a singleton will charge to the pair of countries in the same coalition structure. The singleton within the partial coalition structure treats the pair as one entity and thus charges the same tariff rate to each member of the pair, that is, $\forall i, j, k \in N$,

$$\tau_{k,i}\left(C_P^k\right) = \tau_{k,j}\left(C_P^k\right) = \tau_{k,ij}\left(C_P^k\right).$$

The outsider to the pair, country k in this case, behaves noncooperatively as a singleton, maximizing its individual welfare function, given the pair's emissions tax $t_{ij}\left(C_P^k\right)$ and tariff rate $\tau_{ij,k}\left(C_P^k\right)$.

There are three possible arrangements under the partial coalition structure, namely $\{\{i,j\},\{k\}\}$, $\{\{i,k\},\{j\}\}$, and $\{\{j,k\},\{i\}\}$. We end up, therefore, with three pair members emissions tax rates $t_{ij}(C_P^k)$, $t_{ik}(C_P^j)$, $t_{jk}(C_P^i)$ and the corresponding outsider's emissions tax rate $t_k^P(C_P^k)$, $t_j^P(C_P^j)$, and $t_i^P(C_P^i)$. With respect to import tariffs, there are three pair members tariff rates, imposed by the pair on the outsider, $\tau_{ij,k}(C_P^k), \tau_{ik,j}(C_P^j)$, and $\tau_{jk,i}(C_P^i)$, and three import tariff rates levied by the outsider on pair members, $\tau_{k,ij}(C_P^k), \tau_{j,ik}(C_P^j)$, and $\tau_{i,jk}(C_P^i)$.

3.3.1. Pair Members

Given the outsider's emissions tax rate, $t_k^P(C_P^k)$, and tariff rate, $\tau_{k,ij}(C_P^k)$, the equilibrium quantities produced by the firm operating in a pair member country, given by equations (10) and (11), can thus be reduced as follows $\forall i, j, k \in N$ and $i \neq j, k$:

$$x_{ii}{}^*(C_P^k) = \frac{1}{4}(\alpha - 2t_{ij} + t_k^P + \tau_{ij,k}) \tag{49}$$

$$x_{ij}{}^*(C_P^k) = \frac{1}{4}(\alpha - 2t_{ij} + t_k^P + \tau_{ij,k}) \tag{50}$$

$$x_{ik}{}^*(C_P^k) = \frac{1}{4}(\alpha - 2t_{ij} + t_k^P - 2\tau_{k,ij}) \tag{51}$$

Note that local production, $x_{ii}{}^*(C_P^k)$, is strictly positive, and exports, $x_{ij}{}^*(C_P^k)$ and $x_{ik}{}^*(C_P^k)$, are positive, given the imposed restrictions on the parameters of the model.

The pair members maximization problem[5] (12) can be written as,

$$\max_{t_{ij},\tau_{ij,k}} W_{ij}^{C_P^k} = \max_{t_{ij},\tau_{ij,k}} \begin{bmatrix} \frac{1}{2}\left(Q_i(t_{ij})\right)^2 + \frac{1}{2}\left(Q_j(t_{ij})\right)^2 - \left(\beta_i + \beta_j\right)\left(X_i(t_{ij}) + X_j(t_{ij}) + X_k(t_{ij})\right) \\ \left(\alpha - Q_i(t_{ij})\right)\left(x_{ii}^*(t_{ij}) + x_{ji}^*(t_{ij})\right) + \left(\alpha - Q_j(t_{ij})\right)\left(x_{ij}^*(t_{ij}) + x_{jj}^*(t_{ij})\right) \\ +\left(\alpha - Q_k(t_{ij}) - \tau_{k,ij}\right)\left(x_{ik}^*(t_{ij}) + x_{jk}^*(t_{ij})\right) + \left(\tau_{ij,k}(x_{ki}^*(t_{ij}) + x_{kj}^*(t_{ij})\right) \end{bmatrix} \tag{52}$$

The first order conditions of the above welfare maximization problem (52) with respect to t_{ij} and $\tau_{ij,k}$ are respectively,

$$\frac{\delta W_{ij}^{C_P^k}}{\delta t_{ij}} = 0 \Rightarrow 10t_{ij} = \left(-3\alpha + 6(\beta_i + \beta_j) + t_k^P + 5\tau_{ij,k}\right) \tag{53}$$

$$\frac{\delta W_{ij}^{C_P^k}}{\delta \tau_{ij,k}} = 0 \Rightarrow 19\tau_{ij,k} = \left(4(\beta_i + \beta_j) + 5\alpha + 10t_{ij} - 7t_k^P\right) \tag{54}$$

The first order condition (53) yields the following upward sloping best response function,

$$t_{ij}(t_k^P, \tau_{ij,k}) = \frac{1}{10}\left(-3\alpha + 6(\beta_i + \beta_j) + t_k^P + 5\tau_{ij,k}\right) \tag{55}$$

[5] Country i's optimization problem as a pair member under the partial coalition structure is detailed in appendix D.

Interestingly, a pair member exhibits a positive upward sloping best response function implying a cooperative response towards the outsider, while the latter is behaving noncooperatively as a singleton. A higher emission tax rate levied on the firm operating in the noncooperative country, country k in this case, increases the cost and reduces the competitiveness of that firm. Hence, it prompts pair members to increase the emissions tax rate levied in their own countries, fostering an environmental "race to the top" despite the singleton noncooperative behavior.

Unlike the grand coalition structure, the best response function (55) shows a positive relationship between the pair's emissions tax rate and the tariff rate imposed by the pair on the outsider $\tau_{ij,k}(C_P^k)$. Lower tariffs between the pair and the outsider will reduce the emissions tax rate under this partially cooperative scenario, fostering looser environmental standards and regulations.

Solving simultaneously the first order conditions (53) and (54) of the pair's maximization problem (52) with those derived from the outsider's maximization problem, (68) and (69), yields the following pair members equilibrium tax rate, $t_{ij}^*(C_P^k)$, and import tariff rate, $\tau_{ij,k}{}^*(C_P^k)$:

$$t_{ij}^*(C_P^k) = \frac{1}{834}\left(-176\alpha + 809(\beta_i + \beta_j) - 72\beta_k\right) \tag{56}$$

$$\tau_{ij,k}{}^*(C_P^k) = \frac{1}{139}\left(29\alpha + 106(\beta_i + \beta_j) - 45\beta_k\right) \tag{57}$$

Domestic production in each pair member market within the partial coalition structure is,

$$x_{ii}(C_P^k) = x_{jj}(C_P^k) = \frac{1}{834}\left(308\alpha - 269(\beta_i + \beta_j) + 126\beta_k\right) \tag{58}$$

And exports among pair members are given by,

$$x_{ij}(C_P^k) = x_{ji}(C_P^k) = \frac{1}{834}\left(308\alpha - 269(\beta_i + \beta_j) + 126\beta_k\right) \tag{59}$$

While exports from any of the two pair members to the outsider are given by,

$$x_{ik}(C_P^k) = x_{jk}(C_P^k) = \frac{3}{834}\left(47\alpha - 111(\beta_i + \beta_j) - 25\beta_k\right) \tag{60}$$

The total quantity produced and consumed in any pair member country are respectively,

$$X_i(C_P^k) = X_j(C_P^k) = \frac{1}{834}\left(757\alpha - 871(\beta_i + \beta_j) + 177\beta_k\right) \tag{61}$$

$$Q_i(C_P^k) = Q_j(C_P^k) = \frac{9}{139}\left(13\alpha - 10(\beta_i + \beta_j) - \beta_k\right) \tag{62}$$

Given the assumption that every unit of production generates one unit of emissions, then the total level of emissions generated by the pair of countries within the partial coalition structure amounts to:

$$X_i(C_P^k) + X_j(C_P^k) = \frac{1}{417}\left(757\alpha - 871(\beta_i + \beta_j) + 177\beta_k\right) \tag{63}$$

The welfare of any pair member within the partial coalition structure, $\forall i, j, k \in N$ and $i \neq j, k$, is given by the following expression:

$$W_i^{C_P^k} = \frac{1}{12}\frac{1}{(139)^2}\begin{bmatrix} \alpha\left(112581\alpha - 558007\beta_i + 5221\beta_j - 21982\beta_k\right) \\ +(\beta_i + \beta_j)(278248\beta_i - 101500\beta_j - 14879\beta_k) \\ + 4\beta_k(8938\beta_k + 34611\beta_i) \end{bmatrix} \tag{64}$$

3.3.2. The Partial Coalition's Outsider

Given the pair's emissions tax rate, $t_{ij}(C_P^k)$, and tariffs, $\tau_{ij}(C_P^k)$ and $\tau_{ij,k}(C_P^k)$, the equilibrium quantities produced by the firm operating in country k, the outsider to a pair, given by equations (10) and (11), can thus be reduced as follows $\forall i, j, k \in N$ and $i \neq j, k$:

$$x_{kk}{}^*(C_P^k) = \frac{1}{4}(\alpha - 3t_k^P + 2t_{ij} + 2\tau_{k,ij}) \tag{65}$$

$$x_{ki}{}^*(C_P^k) = x_{kj}{}^*(C_P^k) = \frac{1}{4}(\alpha - 3t_k^P + 2t_{ij} - 3\tau_{ij,k}) \tag{66}$$

The imposed restrictions on the model's parameters guarantee that the outsider's local production $x_{kk}{}^*(C_P^k)$ is strictly positive, and exports to pair members $x_{ki}{}^*(C_P^k)$ and $x_{kj}{}^*(C_P^k)$ are positive.

The outsider to the pair will continue to behave non-cooperatively, and thus its maximization problem[6] (12) can be written as,

$$\max_{t_k^P, \tau_{k,ij}} W_k^{C_P^k} = \max_{t_k^P, \tau_{k,ij}} \begin{bmatrix} \frac{1}{2}\left(Q_k(t_k^P)\right)^2 - \beta_k\left(X_i(t_k^P) + X_j(t_k^P) + X_k(t_k^P)\right) \\ +\tau_{k,ij}\left((x_{ik}^*(t_k^P) + x_{jk}^*(t_k^P)\right) + \left(\alpha - Q_k(t_k^P)\right)x_{kk}^*(t_k^P) \\ +(\alpha - Q_i(t_k^P) - \tau_{ij,k})x_{ki}^*(t_k^P) + (\alpha - Q_j(t_k^P) - \tau_{ij,k})x_{kj}^*(t_k^P) \end{bmatrix} \tag{67}$$

The first order conditions of the above welfare maximization problem (67) with respect to t_k^P and $\tau_{k,ij}$ are, respectively:

$$\frac{\delta W_k^{C_P^k}}{\delta t_k^P} = 0 \Rightarrow 17t_k^P = (12\beta_k - 9\alpha - 10t_{ij} + 12\tau_{ij,k} + 6\tau_{k,ij}) \tag{68}$$

$$\frac{\delta W_k^{C_P^k}}{\delta \tau_{k,ij}} = 0 \Rightarrow 10\tau_{k,ij} = (4\beta_k + 3\alpha + 3t_k^P - 2t_{ij}) \tag{69}$$

The first order condition (68) yields the following downward sloping best response function,

$$t_k^P(t_{ij}) = \frac{1}{17}(12\beta_k - 9\alpha - 10t_{ij} + 12\tau_{ij,k} + 6\tau_{k,ij}) \tag{70}$$

[6] Country k's optimization problem as an outsider under the partial coalition structure is detailed in appendix D.

Unlike pair members, country k, the outsider to the pair, has a downward sloping best response function, implying a noncooperative behavior and solid free-riding incentives.

Solving the system of equations (53), (54), (68), and (69), the outsider's equilibrium emissions tax and tariff rates, ${t_k^P}^*(C_P^k)$ and ${\tau_{k,ij}}^*(C_P^k)$, are given, respectively, by the following two expressions:

$$ {t_k^P}^*(C_P^k) = \frac{1}{417}\left(315\beta_k - 47(\beta_i + \beta_j) - 64\alpha\right) \tag{71} $$

$$ {\tau_{k,ij}}^*(C_P^k) = \frac{1}{834}\left(247\alpha + 537\beta_k - 190(\beta_i + \beta_j)\right) \tag{72} $$

Clearly, the outsider, behaving non-cooperatively as a singleton, imposes an emissions tax rate that is directly related to the country's own environmental damage parameter β_k, and indirectly related to the pair members' environmental damage parameters β_i and β_j.

The outsider's local production and exports to pair members are respectively,

$$ x_{kk}(C_P^k) = \frac{10}{417}\left(17\alpha + 19(\beta_i + \beta_j) - 12\beta_k\right) \tag{73} $$

$$ x_{ki}(C_P^k) = x_{kj}(C_P^k) = \frac{1}{417}\left(43\alpha - (\beta_i + \beta_j) - 153\beta_k\right) \tag{74} $$

The total quantity produced and consumed by the outsider are respectively,

$$ X_k(C_P^k) = \frac{1}{417}\left(256\alpha + 188(\beta_i + \beta_j) - 426\beta_k\right) \tag{75} $$

$$ Q_k(C_P^k) = \frac{1}{417}\left(311\alpha - 143(\beta_i + \beta_j) - 195\beta_k\right) \tag{76} $$

The world market clears, as global production equals global consumption, such that

$$ \sum_i X_i(C_P^k) = \sum_i Q_i(C_P^k) = \frac{1}{417}\left(1013\alpha - 683(\beta_i + \beta_j) - 249\beta_k\right) \tag{77} $$

Given the assumption that every unit of production generates a unit of emissions, then the global level of emissions generated by the partial coalition structure is also given by equation (77).

The outsider to the pair welfare is expressed as follows:

$$ W_k^{C_P^k} = \frac{1}{2}\frac{1}{(417)^2}\begin{bmatrix} \alpha\left(163976\alpha - 117259(\beta_i + \beta_j) - 827364\beta_k\right) \\ 138251(\beta_i + \beta_j)^2 + 59472{\beta_k}^2 + 529329(\beta_i + \beta_j)\beta_k \end{bmatrix} \tag{78} $$

3.4 Simulation Results − Stable Coalitions

Having examined all possible coalition structures and their equilibria, the aim is to identify which cooperative scenarios will emerge in a stable environmental coalition among countries and to capture the effect of environmental damage heterogeneity on the stability of these environmental coalitions.

The model restricts local production to be strictly positive and exports to be positive. Consequently, each firm's total production is strictly positive, and the market structure is maintained throughout the game. The Cournot Equilibrium quantities (10) and (11) derived in stage three are applicable to all possible coalition structures defined in the second stage. Also, the imposed restrictions ensure that the market is active, by assuming that any marginal environmental damage parameter cannot be higher than the marginal utility of good X given by α, that is, $\alpha > \beta_i > \beta_j > \beta_k$. Finally, the constrained parameters guarantee positive import tariff rates and positive trade flows. The most restrictive condition on the model's parameters is expressed as $\alpha \geq \frac{1}{47}\left(111\left(\beta_i + \beta_j\right) + 25\beta_k\right)$. The parameters chosen in the numerical simulation comply with this condition.

Given the stability conditions (15) and (16), the parametrical simulation of the model shows that the grand coalition is stable, at different degrees of environmental damage heterogeneity and alternative market sizes. The parametrical simulation results are summarized in the following propositions:

Proposition 1: *Based on the simulation results, the grand coalition is stable at different levels of environmental damage heterogeneity.*

Let $2(\beta_i - \beta_k) \div (\beta_i + \beta_k)$ be a measure of environmental damage heterogeneity, where β_i refers to the country that suffers the most from environmental damage, and β_k refers to the country that suffers the least. The parametrical simulation reveals that the grand coalition is stable over a large spectrum of environmental damage heterogeneity, where $0 \leq 2(\beta_i - \beta_k) \div (\beta_i + \beta_k) \leq 1$.

We have examined the homogenous benchmark case, where $\beta_i = \beta_j = \beta_k$, and considered the scenario where $\beta_j = \beta_k < \beta_i$, and the alternative where $\beta_i = \beta_j > \beta_k$. In each of these cases, the grand coalition is stable at different levels of the environmental damage heterogeneity.

We have also studied dynamic behavior under the partial coalition structure as opposed to the static simultaneous game. The dynamic game assumes that pair members would move first and coordinate their environmental tax and trade policies, and the outsider, behaving as a singleton, would respond subsequently. The simulation of the dynamic game indicates that the pair members do not benefit from a first move advantage. Like the static game, the simulation results show that the grand coalition is stable at alternative levels of environmental damage heterogeneity.

To curb the negative externality associated with the production of good X, a coalition can levy a positive emissions tax rate to reduce production and thus emissions. It can also enforce a subsidy to increase production since the polluting good is underproduced due to the Cournot competition among the three firms. While the restrictions imposed on the model's parameters would entail that any country behaving noncooperatively as a singleton, would levy a negative tax rate on the production of the polluting good, the pair members under a partial coalition structure and members of the grand coalition may either impose a tax or a subsidy on local firms. Table 1 summarizes the conditions under which members of the grand coalition would levy a positive uniform emissions tax rate on all firms and those under which they would enforce a subsidy, in comparison to the tax rates imposed by the singleton and the partial coalition structures.

As indicated in Table 1, the grand coalition would impose a uniform positive emissions tax rate on all firms when the market size is sufficiently small, $\frac{1}{47}\left(111\left(\beta_i+\beta_j\right)+25\beta_k\right)<\alpha\leq 4\sum_i \beta_i$. On the other hand, at a sufficiently large market size, when $\alpha>4\sum_i \beta_i$, members of the grand coalition would cooperatively agree to subsidize the local firms.

Table 1: The grand coalition emissions tax rate in comparison to the singleton and the partial coalition structures.

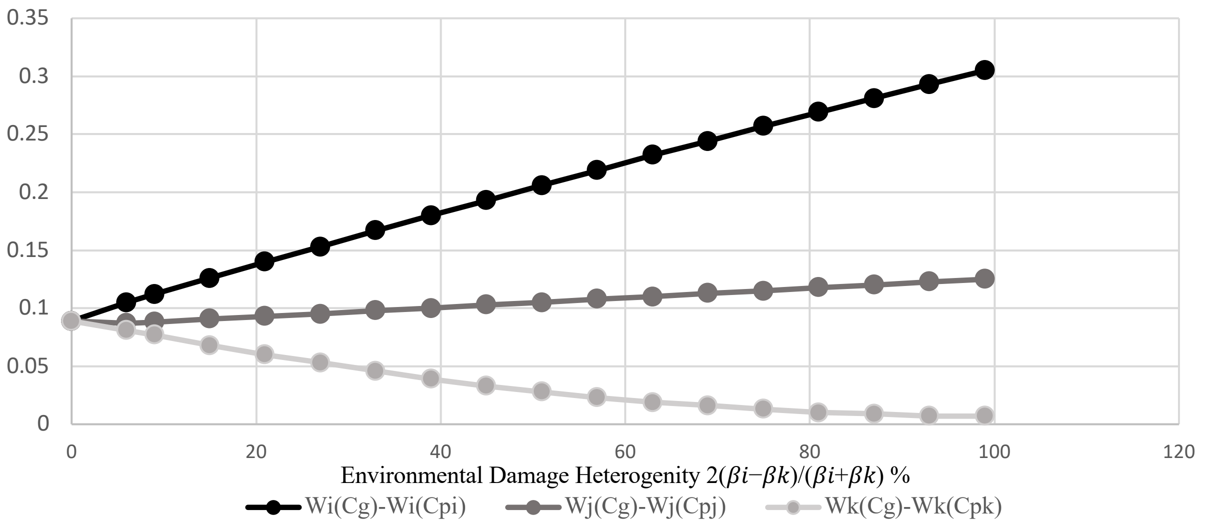
	Grand Coalition C_G	Pair Members in a Partial Coalition C_p^k	Singleton C_{NC}	Outsider in a Partial Coalition C_p^k
Parameters Range	$t_G(C_G)$	$t_{ij}(C_p^k)$	$t_i(C_{NC})$	$t_k(C_p^k)$
$\frac{1}{47}\left(111(\beta_i+\beta_j)+25\beta_k\right)\leq\alpha\leq\frac{1}{176}\left(809(\beta_i+\beta_j)-72\beta_k\right)$	$t_G(C_G)\geq 0$ Tax	$t_{ij}(C_p^k)\geq 0$ Tax	$t_i(C_{NC})<0$ Subsidy	$t_k(C_p^k)<0$ Subsidy
$\frac{1}{176}\left(809(\beta_i+\beta_j)-72\beta_k\right)<\alpha\leq 4\sum_i\beta_i$	$t_G(C_G)\geq 0$ Tax	$t_{ij}(C_p^k)<0$ Subsidy	$t_i(C_{NC})<0$ Subsidy	$t_k(C_p^k)<0$ Subsidy
$4\sum_i\beta_i<\alpha$	$t_G(C_G)<0$ Subsidy	$t_{ij}(C_p^k)<0$ Subsidy	$t_i(C_{NC})<0$ Subsidy	$t_k(C_p^k)<0$ Subsidy

Let $W_i^{C_G}-W_i^{C_P^i}$ be the overall welfare gains achieved by country i for being a member of the grand coalition as opposed to being an outsider to a pair within a partial coalition structure. Given the stability conditions (15) and (16), the grand coalition is externally stable by default, and internally stable $\Leftrightarrow W_i^{C_G}-W_i^{C_P^i}\geq 0, \forall\, i\in N=\{i,j,k\}$. These overall welfare gains are given by:

$$W_i^{C_G}-W_i^{C_P^i}=\frac{1}{2}\frac{1}{(417)^2}\left[\begin{array}{c}\alpha\left(9913\alpha-215970\beta_i+117259\left(\beta_j+\beta_k\right)\right)+809973{\beta_i}^2\\+\left(\beta_j+\beta_k\right)\left(166227\beta_i-312140\left(\beta_j+\beta_k\right)\right)\end{array}\right] \tag{79}$$

Given the model's parameters restrictions, and assuming that $\alpha=\frac{1}{46}(111(\beta i+\beta j)+25\beta k)$, Figure 2 shows that the overall welfare gains achieved by country $i, j,$ and k, for being a member of the grand coalition as opposed to being an outsider to a pair in a partial coalition structure. Because of their heterogeneity, and as indicated in Figure 2, all three countries do not benefit equally from joining the grand coalition. While country i's welfare gains increase with a higher degree of environmental damage heterogeneity, country k's welfare gains are inversely related to degree of environmental damage heterogeneity. As such, country k, in contrast to countries i and j, has less incentives to join the fully cooperative agreement when the level of environmental damage heterogeneity increases. At higher levels of heterogeneity, the stability of the grand coalition can be reinforced by a larger market size, as captured by α.

Figure 2: Welfare Gains from the Grand Coalition at Different levels of Environmental Damage Heterogenity



Proposition 2: *The Grand Coalition provides environmental gains measured in terms of lower aggregate emissions only at sufficiently low market sizes.*

The collective environmental gains measured in terms of lower aggregate emissions, provided by the grand coalition as compared to the singleton and the partial coalition structures are given, respectively, by the following two expressions,

$$X(C_{NC}) - X(C_G) = \sum_i X_i(C_{NC}) - \sum_i X_i\,(C_G) = \frac{129}{172}\left(3\sum_i \beta_i - \alpha\right) \tag{80}$$

$$X\left(C_P^k\right) - X(C_G) = \sum_i X_i\left(C_P^k\right) - \sum_i X_i\,(C_G) = \frac{2}{417}\left(-119\alpha + 284\left(\beta_i + \beta_j\right) + 501\beta_k\right) \tag{81}$$

It is clear from Equation (80) that global production and thus global emissions are lower under the grand coalition as opposed to the singleton structure, only when the market size, as captured by α, is sufficiently low, that is when $\frac{1}{47}(111(\beta_i + \beta_j) + 25\beta_k) \leq \alpha < 3\sum_i \beta_i$.

It should be noted that at this market size range, the grand coalition is imposing a positive emissions tax rate on all local firms, whereas, under the noncooperative equilibrium, the governments, behaving as singletons, are subsidizing the local firms, and thus encouraging the production of the polluting good.

Comparing the grand coalition's aggregate production to that of the partial coalition C_P^k, equation (81) indicates that the grand coalition provides environmental gains measured in terms of lower aggregate emissions, only when the market size, as captured by α, is sufficiently low, that is when $\frac{1}{47}(111(\beta_i + \beta_j) + 25\beta_k) \leq \alpha < \frac{1}{119}\left(284\left(\beta_i + \beta_j\right) + 501\beta_k\right)$.

At this market size range, both the grand coalition and the pair members, that is countries i and j, are imposing a positive emissions tax rate on the production of the polluting good; however, the outsider to the pair, behaving non-cooperatively is subsidizing its local firm. As such, the global production that results from the partial coalition structure would exceed what is globally produced by the grand coalition.

Remarkably, these environmental gains are independent of the degree of environmental damage heterogeneity and are positively related to all three environmental damage parameters. In the homogeneous benchmark case, for example, where $\beta_i = \beta_j = \beta_k$, equations (80) and (81) can be reduced as follows:

$$\sum_i \hat{X}_i\,(C_{NC}) - \sum_i \hat{X}_i\,(C_G) = \tfrac{129}{172}\left(9\hat{\beta} - \hat{\alpha}\right) \tag{82}$$

$$\sum_i \hat{X}_i\,(C_P^k) - \sum_i \hat{X}_i\,(C_G) = \tfrac{2}{417}\left(1069\hat{\beta} - 119\hat{\alpha}\right) \tag{83}$$

Like the heterogeneous case, equations (82) and (83) indicate that the grand coalition provides environmental gains measured in terms of lower aggregate emissions, when compared to the singleton and partial coalition structures, only when market sizes are sufficiently low, that is, when $\hat{\alpha} < 9\hat{\beta}$ and $\hat{\alpha} < \frac{1069}{119}\hat{\beta}$, respectively.

Proposition 3: *Although collective welfare is always highest under the grand coalition, the simulation shows that a member of the grand coalition is not always better off individually compared to the singleton structure.*

Let $W_i^{C_G} - W_i^{C_{NC}}$ be the overall welfare gain achieved by country i for being a member of the grand coalition as opposed to behaving as a singleton, then $\forall i \in N = \{i, j, k\}$,

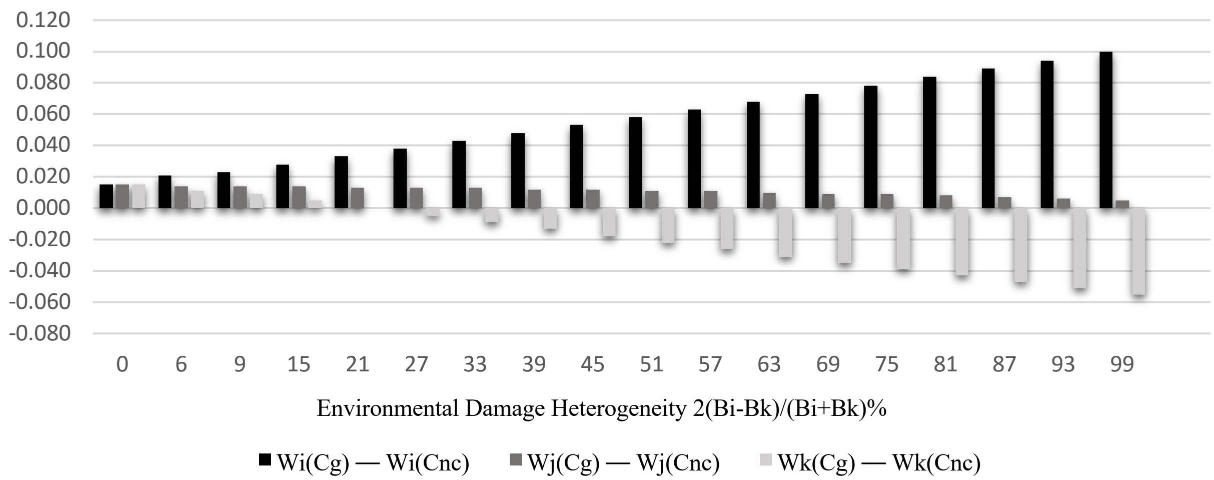
$$W_i^{C_G} - W_i^{C_{NC}} = \frac{1}{688^2}\begin{bmatrix} 344\alpha\left(43\alpha - 2\left(557\beta_i - 85(\beta_j+\beta_k)\right)\right) + 1022664{\beta_i}^2 \\ +156235\beta_j\beta_k + 8(\beta_j + \beta_k)\left(76718\beta_i - 37755(\beta_j + \beta_k)\right) \end{bmatrix} \tag{84}$$

The overall welfare gains achieved by country i, as a member of the grand coalition as opposed to behaving as a singleton, depend on the degree of environmental damage heterogeneity, as captured by the two terms $\left(557\beta_i - 85(\beta_j+\beta_k)\right)$ and $\left(76718\beta_i - 37755(\beta_j + \beta_k)\right)$. Although the impact of environmental damage heterogeneity on these overall welfare gains is not straightforward, the simulation results show that members of the grand coalition do not benefit equally from the cooperative equilibrium as opposed to the singleton structure. Figures 3 and 4 illustrate the overall welfare gains obtained by all 3 members of the grand coalition as compared to the singleton structure, at two alternative market sizes.

In Figure 3, the market size, as captured by $\alpha = 2.3\sum_i \beta_i$, is sufficiently small so that the grand coalition imposes a positive emissions tax rate and generates environmental gains, measured in terms of lower emissions, that is, $\sum_i X_i(C_{NC}) - \sum_i X_i\,(C_G) > 0$. Given the assumption that $\beta_i >$

$\beta_j > \beta_k$, country i achieves more significant welfare gains with a higher degree of environmental damage heterogeneity. By contrast, countries j and k realize lower welfare gains with a higher level of environmental damage heterogeneity. Despite the environmental gains generated by the grand coalition, country k, for example, experiences overall welfare losses at any degree of environmental damage heterogeneity that exceeds 21%. In fact, for country k, the increase in the firm's profits generated by the grand coalition, are more than offset by the decrease in net consumer surplus and the loss of tariff revenues associated with the fully cooperative agreement.

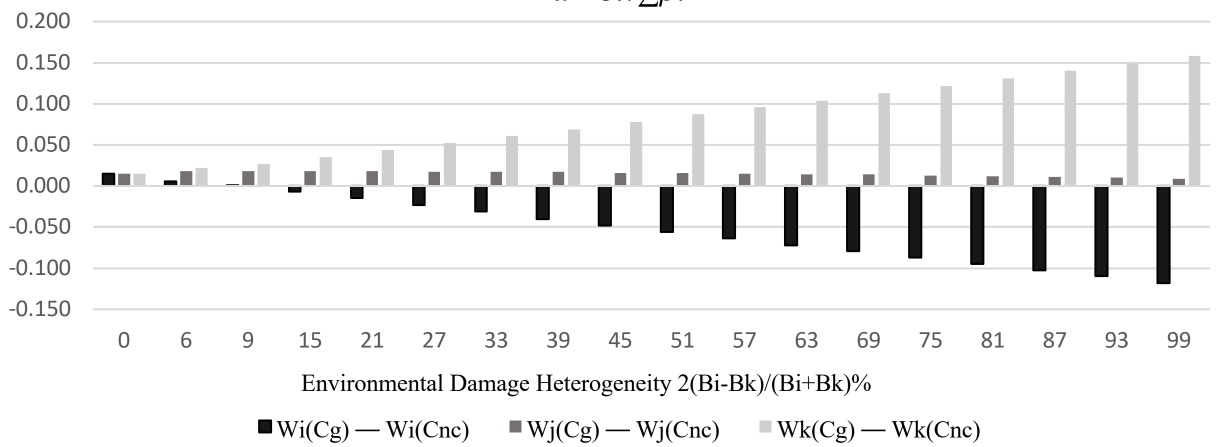
Figure 3: Individual welfare gains of countries i, j, and k, as members of the grand coalition as compared to the singleton structure
$\alpha = 2.3\sum\beta i$



Under these conditions, the biggest winner would be country i. Having the highest environmental damage parameter, it is better off individually at sufficiently low market sizes, where $\frac{1}{47}\left(111(\beta_i + \beta_j) + 25\beta_k\right) \leq \alpha \leq 3\sum_i \beta_i$. The curtailed production of the polluting good boosts the firm's profit and the net consumer surplus, leading to both environmental and individual welfare gains.

In Figure 4, the market size, as captured by $\alpha = 3.7\sum_i \beta_i$, is sufficiently small so that the grand coalition imposes a positive emissions tax rate, but it does not generate environmental gains. Despite the positive emissions tax rate, the grand coalition's aggregate production exceeds what would be produced globally under the singleton structure, that is $\sum_i X_i(C_{NC}) - \sum_i X_i\,(C_G) < 0$. In this case, country k, having the lowest environmental damage parameter, would achieve more significant welfare gains with a greater degree of environmental damage heterogeneity. By contrast, countries i and j would realize lower welfare gains with a higher degree of environmental damage heterogeneity. Country i, for example, experiences overall welfare losses at any degree of environmental damage heterogeneity that exceeds 10%. In this case, country i enjoys a significant increase in the firm's profit and net consumer surplus, but the total loss in tariff revenues outweighs these gains.

Figure 4: Individual welfare gains of countries i, j, and k, as members of the grand coalition as compared to the singleton structure
α = 3.7$\sum\beta i$



Observing figures 3 and 4, it is easy to see that in the homogeneous benchmark case, that is when $\beta_i = \beta_j = \beta_k$, all countries benefit equally from the grand coalition, and each country is better off as a member of the grand coalition as opposed to behaving as a singleton, $\forall i, j, k \in N$.

Let $W^{C_G} - W^{C_{NC}}$ be the collective welfare gains provided by the grand coalition structure as opposed to the singleton structure, then,

$$W^{C_G} - W^{C_{NC}} = \frac{3}{688^2}\begin{bmatrix} 14792\alpha(\alpha - 6\sum_i \beta_i) \\ 139528({\beta_i}^2 + {\beta_j}^2 + {\beta_k}^2) + 259881(\beta_i\beta_j + \beta_i\beta_k + \beta_j\beta_k) \end{bmatrix} \tag{85}$$

Unlike individual welfare gains, collective welfare gains are independent from the degree of environmental damage heterogeneity. The parametrical simulation shows that they are strictly positive at alternative market sizes and levels of environmental damage.

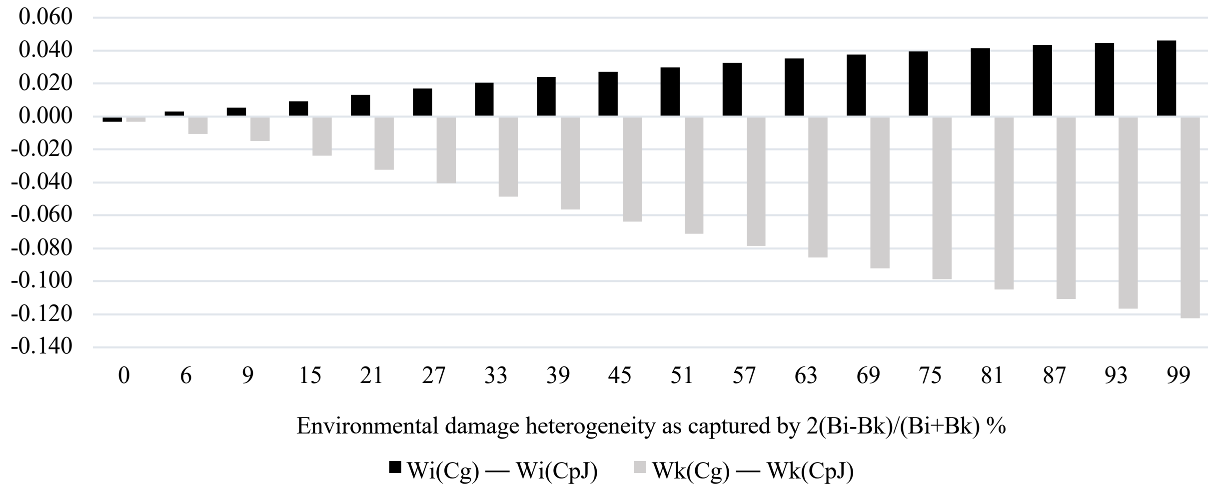
Proposition 4: *Although collective welfare is always highest under the grand coalition, the simulation shows that a member of the grand coalition is often worse off than a pair member in a partial coalition structure.*

Let $W_i^{C_G} - W_i^{C_P^k}$ be the overall welfare gains achieved by country i for being a member of the grand coalition as opposed to being a pair member within the partial coalition structure C_p^k, where the outsider is country k. Then, these welfare gains are expressed as follows:

$$W_i^{C_G} - W_i^{C_P^k} = \begin{bmatrix} \alpha(3345\alpha - 137549\beta_i - 5221\beta_j + 21982\beta_k) \\ 301382{\beta_i}^2 - 14426{\beta_j}^2 - 151678{\beta_k}^2 \\ 286956\beta_i\beta_j + 340139\beta_i\beta_k - 216973\beta_j\beta_k \end{bmatrix} \tag{86}$$

The simulation shows that a member of the grand coalition is often worse off than a pair member in a partial coalition agreement. Clearly, members of the grand coalition benefit differently from the fully cooperative equilibrium in comparison to a pair member under the partial coalition structure because they do not suffer equally the consequences of environmental damage. Thus, we compare the individual welfare gains of countries i and k, as members of the grand coalition, in comparison to what they would have achieved under the partial coalition arrangement C_p^j at alternative market sizes.

Figure 5: Individual welfare gains of countries i and k as members of the grand coalition in comparaison to the partial coalition agreement Cpj
α = 1/45 (111(βi+βj)+25βk)

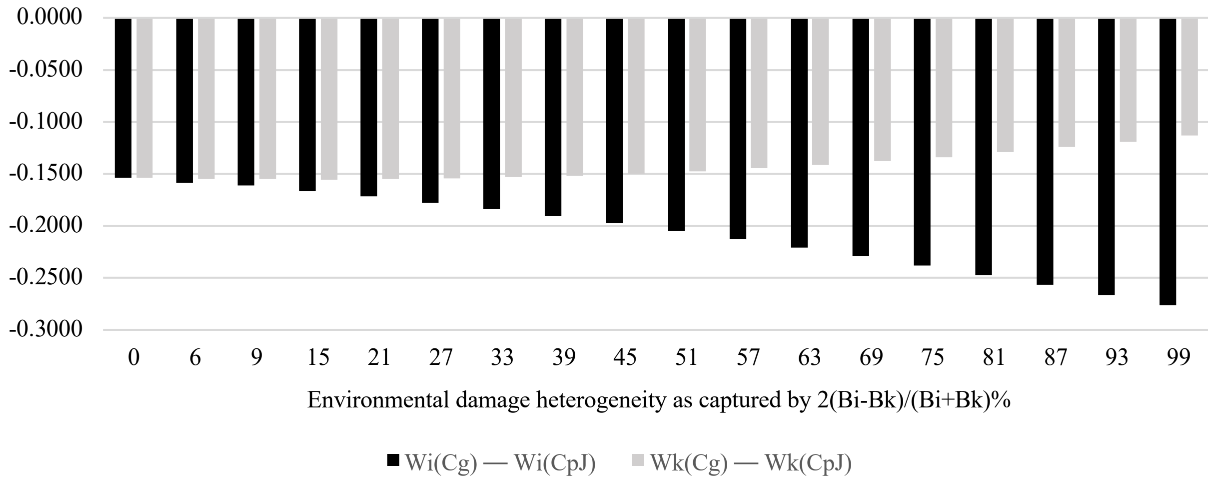


In Figure 5, the market size, as captured by $\alpha = \frac{1}{45}\left(111(\beta_i + \beta_j) + 25\beta_k\right)$ is sufficiently small so that the grand coalition imposes a positive emissions tax rate and generates environmental gains, measured in terms of lower emissions, that is $\sum_i X_i(C_G) - \sum_i X_i\left(C_p^j\right) > 0$. Under these conditions, country i derives minor positive welfare gains and these are bound to increase with the level of environmental damage heterogeneity. By contrast, country k experiences welfare losses which are increasing with a higher degree of environmental damage heterogeneity.

Having the highest environmental damage parameter, country i benefits from sufficiently low market sizes, where $\frac{1}{47}\left(111(\beta_i + \beta_j) + 25\beta_k\right) \leq \alpha \leq \frac{1}{119}\left(284(\beta_i + \beta_j) + 501\beta_k\right)$. The reduced production of the polluting good significantly increases the local firm's profit and net consumer surplus, leading to both environmental and individual welfare gains. By contrast, country k individual welfare losses are mainly attributed to the loss in tariff revenues.

Alternatively, in Figure 6, the market size, as captured by $\alpha = \frac{1}{25}\left(111(\beta_i + \beta_j) + 25\beta_k\right)$ is sufficiently small so that the grand coalition imposes a positive emissions tax rate, yet it is large enough so that the grand coalition does not generate environmental gains, that is $\sum_i X_i(C_G) - \sum_i X_i\left(C_p^j\right) < 0$. Under these conditions, both countries are worse off under the grand coalition and experience individual welfare losses rather than overall welfare gains. However, country i's welfare losses are bound to increase with the level of environmental damage heterogeneity, while country k experiences welfare losses which are decreasing in the level of environmental damage heterogeneity.

Figure 6: Individual welfare gains of countries i and k as members of the grand coalition in comparaison to the partial coalition agreement Cpj
α = 1/25 (111(βi+βj)+25βk)



The numerical simulation also indicates that even when the three countries have the same environmental damage parameter, that is, $\beta_i = \beta_j = \beta_k$, any member of the grand coalition is worse off in comparison to a pair member under the partial coalition structure at alternative market sizes.

Let $W^{C_G} - W^{C_P^k}$ be the collective welfare gains provided by the grand coalition structure as opposed to the partial coalition structure C_p^k; these gains are given by the following expression:

$$W^{C_G} - W^{C_P^k} = \frac{1}{2}\frac{1}{(417)^2}\begin{bmatrix} 4\left(4987\alpha^2 - 24224\alpha(\beta_i+\beta_j) - 37506\alpha\beta_k\right) \\ +354939{\beta_k}^2 + 118294(\beta_i+\beta_j)^2 + 350976(\beta_i+\beta_j)\beta_k \end{bmatrix} \tag{87}$$

Based on the simulation results, the collective welfare gains provided by the grand coalition as opposed to the partial coalition structure C_p^k are positive at different market sizes and at alternative degrees of environmental damage heterogeneity. These collective welfare gains significantly increase at higher market sizes. However, they are driven by the fact that at sufficiently large market sizes, the grand coalition reduces its emissions tax rate or subsidies the industry. These collective welfare gains coincide with elevated production levels and thus emissions.

Hence, as we would have expected, countries are always collectively better off when they cooperate their environmental policies as members of the grand coalition, in comparison to the singleton and the partial coalition structures. Under certain conditions, some members may be better off individually under the fully cooperative equilibrium, all while benefiting from environmental and overall welfare gains.

4. Conclusion

The current model's solution with endogenous tariffs shows that the fully cooperative equilibrium is feasible and stable when environmental and trade policies are being negotiated simultaneously. Given the model's parameters restrictions to maintain the market structure throughout the game and guarantee an interior solution with positive trade flows, the grand coalition is stable at different degrees of environmental damage heterogeneity.

International environmental cooperation has been a real challenge. In the absence of effective enforcement methods, current international environmental agreements rely on the good faith of signatories. The current research provides a new way of framing the relationship between trade and environmental damage, often seen as one divergence rather than synergy. It is shown that at sufficiently low market sizes, the grand coalition provides environmental gains and welfare gains when trade and environmental policies are pursued in tandem. At sufficiently larger market sizes, the grand coalition does not provide environmental gains but only collective welfare gains, mainly higher profits for the monopoly firms.

The simplified framework of the current model, however, introduces some limitations. To focus on the impact of environmental damage heterogeneity, it was assumed that all three countries had the same market size, incurred identical marginal production costs, and each firm could export to the other two foreign markets at no transaction costs. It was also assumed that environmental damage is a linear function of aggregate production. These simplifications pave the way for many research questions that could be addressed in the future.

In sum, countries do not suffer equally the consequences of environmental damage, and despite the growing recognition of the climate crisis, ambitious climate policies are constantly being undermined by governments inaction or inept responses. The talks about climate action have certainly become louder, but little has been done to effectively reduce emissions. As the climate crisis worsens, it becomes absolutely urgent to strengthen our environmental policies beyond what is currently implemented. The current paper mirrors the real state of global climate action, where global environmental pledges are constantly being diluted by subsidies to polluting industries, particularly in the world’s largest economies. It also shows that the coordination of environmental and trade policies is a valuable strategy to reduce global emissions in sufficiently small markets despite countries' heterogeneity.

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6. Appendices

6.1 Appendix A

The Firm's Optimization Problem – Stage Three

The firm's optimization problem (7) is :

$$\max_{x_{ii},x_{ij},x_{ik}} \pi_i = \max_{x_{ii},x_{ij},x_{ik}} \begin{pmatrix} (\alpha - (x_{ii} + x_{ji} + x_{ki}) - t_i)x_{ii} \\ +(\alpha - (x_{jj} + x_{ij} + x_{kj}) - t_i - \tau_{j,i})\, x_{ij} + (\alpha - (x_{kk} + x_{ik} + x_{jk}) - t_i - \tau_{k,i})\, x_{ik} \end{pmatrix}$$

The first order conditions (8) and (9) with respect to local production and exports are as follows:

$$\frac{\partial \pi_i}{\partial x_{ii}} = 0 => (\alpha - 2x_{ii}{}^* - x_{ji} - x_{ki} - t_i) = 0$$

$$\frac{\partial \pi_i}{\partial x_{ij}} = 0 => (\alpha - 2x_{ij}{}^* - x_{jj} - x_{kj} - t_i - \tau_{j,i}) = 0$$

$$\frac{\partial \pi_i}{\partial x_{ik}} = 0 => (\alpha - 2x_{ik}{}^* - x_{kk} - x_{jk} - t_i - \tau_{k,i}) = 0$$

By symmetry, we obtain the following equations:

$$\frac{\partial \pi_j}{\partial x_{ji}} = 0 => (\alpha - 2x_{ji}{}^* - x_{ii} - x_{ki} - t_j - \tau_{i,j}) = 0$$

$$\frac{\partial \pi_k}{\partial x_{ki}} = 0 => (\alpha - 2x_{ki}{}^* - x_{ii} - x_{ji} - t_k - \tau_{i,k}) = 0$$

Solving the following system of equations:

$$\begin{Bmatrix} 2x_{ii}{}^* = \alpha - (x_{ji} + x_{ki}) - t_i \\ 2x_{ji}{}^* = \alpha - x_{ii} - x_{ki} - t_j - \tau_{i,j} \\ 2x_{ki}{}^* = \alpha - x_{ii} - x_{ji} - t_k - \tau_{i,k} \end{Bmatrix}$$

We obtain the equilibrium quantities (10) and (11) produced by the firm operating in country i, $\forall i, j, k \in N$:

$$x_{ii}{}^* = \frac{1}{4}(\alpha - 3t_i + (t_j + t_k) + \tau_{i,j} + \tau_{i,k})$$

$$x_{ij}{}^* = \frac{1}{4}(\alpha - 3t_i + (t_j + t_k) + \tau_{j,k} - 3\tau_{j,i})$$

$$x_{ik}{}^* = \frac{1}{4}(\alpha - 3t_i + (t_j + t_k) + \tau_{k,j} - 3\tau_{k,i})$$

6.2 Appendix B

The Optimization Problem – The Singleton Structure C_{NC} – Stage Two

Let $W_i^{C_{NC}}$ be the welfare equation of country i under the singleton structure C_{NC}, then country i's maximization problem (19) can be written as follows $\forall i \in N = \{i, j, k\}$:

$$\max_{t_i^{NC}} W_i^{C_{NC}} = \frac{1}{32} \max_{t_i^{NC}} \begin{bmatrix} (15\alpha^2 - 72\alpha\beta_i) \\ +t_i^{NC}\left(-18\alpha + 24\beta_i - 17t_i^{NC} - 10t_j^{NC} - 10t_k^{NC}\right) \\ +\left(t_j^{NC} + t_k^{NC}\right)\left(6\alpha + 24\beta_i + 7t_j^{NC} + 7t_k^{NC}\right) \\ +\tau_{i,j}\left(8\beta_i + 6\alpha + 6t_i^{NC} - 18t_j^{NC} + 14t_k^{NC} + 11\tau_{i,k} - 21\tau_{i,j}\right) \\ +\tau_{i,k}\left(8\beta_i + 6\alpha + 6t_i^{NC} + 14t_j^{NC} - 18t_k^{NC} + 11\tau_{i,j} - 21\tau_{i,k}\right) \\ +\tau_{j,i}\left(8\beta_i - 12\alpha + 12t_i^{NC} - 12t_j^{NC} - 12t_k^{NC} + 18\tau_{j,i} - 6\tau_{j,k}\right) \\ +\tau_{j,k}\left(8\beta_i + 4\alpha - 4t_i^{NC} + 4t_j^{NC} + 4t_k^{NC} + 2\tau_{j,k} - 6\tau_{j,i}\right) \\ +\tau_{k,i}\left(8\beta_i - 12\alpha + 12t_i^{NC} - 12t_j^{NC} - 12t_k^{NC} + 18\tau_{k,i} - 6\tau_{k,j}\right) \\ +\tau_{k,j}\left(8\beta_i + 4\alpha - 4t_i^{NC} + 4t_j^{NC} + 4t_k^{NC} + 2\tau_{k,j} - 6\tau_{k,i}\right) \end{bmatrix}$$

The first order condition (20) with respect to t_i^{NC} is:

$$\frac{\delta W_i^{C_{NC}}}{\delta t_i^{NC}} = 0 \Rightarrow \left(-9\alpha + 12\beta_i - 17t_i^{NC} - 5\left(t_j^{NC} + t_k^{NC}\right) + 3(\tau_{i,j} + \tau_{i,k}) + 6(\tau_{j,i} + \tau_{k,i}) - 2(\tau_{j,k} + \tau_{k,j})\right) = 0$$

The first order condition (20) yields the following negative best response function (23):

$$t_i^{NC}\left(t_j^{NC}, t_k^{NC}\right) = \frac{1}{17}\left(-9\alpha + 12\beta_i - 5\left(t_j^{NC} + t_k^{NC}\right) + 3\left(\tau_{i,j} + \tau_{i,k}\right) + 6\left(\tau_{j,i} + \tau_{k,i}\right) - 2\left(\tau_{j,k} + \tau_{k,j}\right)\right)$$

The first order conditions (21) and (22) with respect to $\tau_{i,j}$ and $\tau_{i,k}$ are, respectively:

$$\frac{\delta W_i^{C_{NC}}}{\delta \tau_{i,j}} = 0 \Rightarrow \left(4\beta_i + 3\alpha + 3t_i - 9t_j + 7t_k - 21\tau_{i,j} + 11\tau_{i,k}\right) = 0$$

$$\frac{\delta W_i^{C_{NC}}}{\delta \tau_{i,k}} = 0 \Rightarrow \left(4\beta_i + 3\alpha + 3t_i - 9t_k + 7t_j - 21\tau_{i,k} + 11\tau_{i,j}\right) = 0$$

Given the first order conditions (20), (21), and (22), and by symmetry, we derive the following system of equations:

$$\begin{bmatrix} 50t_i^{NC} = -12\alpha + 36\beta_i + 4\left(\beta_j + \beta_k\right) - 7\left(t_j^{NC} + t_k^{NC}\right) \\ 50t_j^{NC} = -12\alpha + 36\beta_j + 4\left(\beta_i + \beta_k\right) - 7\left(t_i^{NC} + t_k^{NC}\right) \\ 50t_k^{NC} = -12\alpha + 36\beta_k + 4\left(\beta_i + \beta_j\right) - 7\left(t_i^{NC} + t_j^{NC}\right) \end{bmatrix}$$

Solving the above system of equations , the singleton equilibrium tax rate and non-negative import tariff rates are given by, $\forall i \in N = \{i, j, k\}$ and $i \neq j, k$:

$$t_i^{NC*}(C_{NC}) = \frac{1}{688}\left(-129\alpha + 499\beta_i - 13(\beta_j + \beta_k)\right) \tag{24}$$

$$\tau_{i,j}^{*}(C_{NC}) = \frac{1}{1376}\left(387\alpha + 19(45\beta_i - 19\beta_j) + 151\beta_k\right) \tag{25}$$

$$\tau_{i,k}^{*}(C_{NC}) = \frac{1}{1376}(387\alpha + 19(45\beta_i - 19\beta_k) + 151\beta_j) \tag{26}$$

6.3 Appendix C

The Optimization Problem – The Grand Coalition Structure C_G – Stage Two

Let $W_i^{C_G}$ be the individual welfare equation of country i as a member of the grand coalition C_G, then $W_i^{C_G}, \forall i \in N = \{i, j, k\}$, is expressed as follows:

$$W_i^{C_G} = \frac{1}{32}\left[(15\alpha^2 - 72\alpha\beta_i) + t_G(-6\alpha + 72\beta_i - 9t_G) + \tau_G(48\beta_i - 4\alpha - 12t_G - 4\tau_G)\right]$$

Let $W^{C_G} = \sum_i W_i^{C_G}$, be the collective welfare equation of all 3 members in the grand coalition C_G, then the maximization problem (35) can be written as follows:

$$\underset{t_G}{\text{Max}}\, W^{C_G} = \max_{t_G} \frac{3}{32}\begin{bmatrix} (15\alpha^2 - 24\alpha \sum_i \beta_i) \\ + t_G(-6\alpha + 24\sum_i \beta_i - 9t_G) + \tau_G(16\sum_i \beta_i - 4\alpha - 12t_G - 4\tau_G) \end{bmatrix} \tag{35}$$

The first order conditions of the above welfare maximization problem (35) with respect to t_G and τ_G are respectively,

$$\frac{\delta W^{C_G}}{\delta t_G} = 0 => 3t_G = (-\alpha + 4\sum_i \beta_i - 2\tau_G) \tag{36}$$

$$\frac{\delta W^{C_G}}{\delta \tau_G} = 0 => 2\tau_G = (-\alpha + 4\sum_i \beta_i - 3t_G) \tag{37}$$

The first order conditions (36) and (37) yield the following cooperative set of uniform solutions, where any uniform emissions tax rate and non-negative import tariff rate (t_G, τ_G) satisfying the following two conditions is an equilibrium solution:

$$3t_G^*(C_G) + 2\tau_G^*(C_G) = (4\sum_i \beta_i - \alpha) \tag{38}$$

$$t_G^*(C_G) \leq \frac{1}{3}(4\sum_i \beta_i - \alpha) \tag{39}$$

For example, if we were to assume that members of the grand coalition would operate under free trade, then $(t_G{}^*, \tau_G{}^*) = \left(\frac{1}{3}(4\sum_i \beta_i - \alpha), 0\right)$ would be an equilibrium uniform solution.

With positive import tariffs, for example, $(t_G{}^*, \tau_G{}^*) = \left(\frac{1}{4}(4\sum_i \beta_i - \alpha), \frac{1}{8}(4\sum_i \beta_i - \alpha)\right)$ and $(t_G{}^*, \tau_G{}^*) = \left(\frac{1}{5}(4\sum_i \beta_i - \alpha), \frac{1}{5}(4\sum_i \beta_i - \alpha)\right)$ are another two possible equilibrium solutions.

Note that $\frac{\partial t_G^*(C_G)}{\partial \tau_G} = -\frac{2}{3} < 0$, which would imply that trade liberalization in the form of lower import tariffs, $\tau_G^*(C_G)$, would entail higher production levels and thus higher environmental damage, and would, therefore, require higher emissions taxes $t_G^*(C_G)$.

6.4 Appendix D

The Optimization Problem – The Partial Coalition Structure C_P – Stage Two

Let $W_i^{C_P^k}$ be the individual welfare equation of country i as a pair member under the partial coalition structure C_P^k, where country k is the outsider to the pair, then $W_i^{C_P^k}$ is expressed as follows:

$$W_i^{C_P^k} = \frac{1}{32}\begin{bmatrix} (15\alpha^2 - 72\alpha\beta_i) \\ +t_{ij}\left(-12\alpha + 48\beta_i - 20t_{ij} - 3{t_k}^P\right) + {t_k}^P\left(6\alpha + 24\beta_i + 7t_{ij} + 7{t_k}^P\right) \\ +\tau_{ij}\left(16\beta_i - 6\alpha - 12t_{ij} + 2{t_k}^P + 5\tau_{ij,k} - 3\tau_{ij}\right) \\ +\tau_{ij,k}\left(16\beta_i + 10\alpha + 20t_{ij} - 14{t_k}^P + 5\tau_{ij} - 19\tau_{ij,k}\right) \\ +\tau_{k,ij}\left(16\beta_i - 8\alpha - 8{t_k}^P + 8\tau_{k,ij}\right) \end{bmatrix}$$

Let $W_{ij}^{C_P^k} = W_i^{C_P^k} + W_j^{C_P^k}$ be the joint welfare equation for the pair members under the partial coalition structure (C_P^k), and given the assumption that $\tau_{ij}\left(C_P^k\right) = 0$, then the pair members' optimization problem (12) can be written as follows:

$$\max_{t_{ij},\, \tau_{ij,k}} W_{ij}^{C_P^k} = \frac{1}{16}\max_{t_{ij},\, \tau_{ij,k}}\begin{bmatrix} \left(15\alpha^2 - 36\alpha\beta_i - 36\alpha\beta_j\right) \\ +t_{ij}\left(-12\alpha + 24\left(\beta_i + \beta_j\right) - 20t_{ij} + 2t_k^P\right) \\ +t_k^P\left(6\alpha + 12\left(\beta_i + \beta_j\right) + 2t_{ji} + 7t_k^P\right) \\ +\tau_{ij,k}\left(8\left(\beta_i + \beta_j\right) + 10\alpha + 20t_{ij} - 14t_k^P - 19\tau_{ij,k}\right) \\ +8\tau_{k,ij}\left(\left(\beta_i + \beta_j\right) - \alpha - t_k^P + \tau_{k,ij}\right) \end{bmatrix} \tag{52}$$

The first order conditions of the above welfare maximization problem (52) with respect to $t_{ij}\left(C_P^k\right)$ and $\tau_{ij,k}\left(C_P^k\right)$ are, respectively,

$$\frac{\delta W_{ij}^{C_P^k}}{\delta t_{ij}} = 0 \Rightarrow 10t_{ij} = \left(-3\alpha + 6(\beta_i + \beta_j) + t_k^P + 5\tau_{ij,k}\right) \tag{53}$$

$$\frac{\delta W_{ij}^{C_P^k}}{\delta \tau_{ij,k}} = 0 \Rightarrow 19\tau_{ij,k} = \left(4(\beta_i + \beta_j) + 5\alpha + 10t_{ij} - 7t_k^P\right) \tag{54}$$

The first order condition (53) yields the following upward sloping best response function,

$$t_{ij}\left(t_k^P, \tau_{ij,k}\right) = \frac{1}{10}\left(-3\alpha + 6(\beta_i + \beta_j) + t_k^P + 5\tau_{ij,k}\right) \tag{55}$$

Given the pair members tax $t_{ij}\left(C_P^k\right)$ and positive tariff $\tau_{ij,k}\left(C_P^k\right)$ rates, the outsider to the pair behaves as a singleton, and its welfare maximization problem (12) can be expressed as:

$$\underset{t_k^P}{\text{Max}}\, W_k^{C_P^k} = \frac{1}{32}\underset{t_k^P}{\text{Max}}\begin{bmatrix} (15\alpha^2 - 72\alpha\beta_k) \\ +t_k^P\left(-18\alpha + 24\beta_k - 10t_{ij} - 17t_k^P\right) \\ +\left(t_{ij}\right)\left(12\alpha + 48\beta_k - 10t_k^P + 28t_{ij}\right) \\ +\tau_{k,ij}\left(16\beta_k + 12\alpha + 12t_k^P - 8t_{ij} - 20\tau_{k,ij}\right) \\ +\tau_{ij,k}\left(16\beta_k - 24\alpha + 24t_k^P - 48t_{ij} + 36\tau_{ij,k}\right) \end{bmatrix} \tag{67}$$

The first order conditions of the above welfare maximization problem (67) with respect to t_k^P and $\tau_{k,ij}$ are, respectively:

$$\frac{\delta W_k^{C_P^k}}{\delta t_k^P} = 0 => 17t_k^P = \left(12\beta_k - 9\alpha - 10t_{ij} + 12\tau_{ij,k} + 6\tau_{k,ij}\right) \tag{68}$$

$$\frac{\delta W_k^{C_P^k}}{\delta \tau_{k,ij}} = 0 => 10\tau_{k,ij} = \left(4\beta_k + 3\alpha + 3t_k^P - 2t_{ij}\right) \tag{69}$$

The first order condition (68) of the outsider's maximization problem (67) yields the following downward sloping best response function,

$$t_k^P\left(t_{ij}\right) = \frac{1}{17}\left(12\beta_k - 9\alpha - 10t_{ij} + 12\tau_{ij,k} + 6\tau_{k,ij}\right) \tag{70}$$

Solving simultaneously the system of equations provided by the first order conditions (53), (54), (68), and (69), we obtain the following pair members equilibrium tax rate $t_{ij}^*(C_P^k)$ and import tariff rate ${\tau_{ij,k}}^*(C_P^k)$:

$$t_{ij}^*(C_P^k) = \frac{1}{834}\left(-176\alpha + 809\left(\beta_i + \beta_j\right) - 72\beta_k\right) \tag{56}$$

$${\tau_{ij,k}}^*(C_P^k) = \frac{1}{139}\left(29\alpha + 106\left(\beta_i + \beta_j\right) - 45\beta_k\right) \tag{57}$$

And the outsider's equilibrium solution ${t_k^P}^*\left(C_P^k\right)$ and ${\tau_{k,ij}}^*\left(C_P^k\right)$:

$${t_k^P}^*\left(C_P^k\right) = \frac{1}{417}\left(315\beta_k - 47\left(\beta_i + \beta_j\right) - 64\alpha\right) \tag{71}$$

$${\tau_{k,ij}}^*\left(C_P^k\right) = \frac{1}{834}\left(247\alpha + 537\beta_k - 190\left(\beta_i + \beta_j\right)\right) \tag{72}$$