Gravity, Market Potential, and Economic Development: Supplemental Material *

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1 Time-varying linkage coefficients

Figure 1 present the schedule of estimated coefficients for colonial linkages and common RTA membership across time. The preferential trading relationship between ex-colonies and their ex-hegemon has a striking downward trend. While the effect remains strongly positive in the early 2000s, the relative deterioration of historical preferences is extremely clear, and should have important consequences for the market potential of the ex-colonies, which are usually small markets located near to other small markets. We will return to that point in the next section. The evolution of the RTA coefficient seems to be strongly influenced by changes in the composition of the main agreements. The effect drops massively around 1973 and 1986 which are dates of significant entries into the European Community (UK, Ireland and Denmark in the first case, Spain and Portugal in the second). Entries of countries into an RTA tends to initially lower the statistical estimate of its effect quite naturally. The effect is also present in 1994, when Mexico adds to the already free trade area between the USA and Canada to form NAFTA.

2 Mapping the World's Market Potential

Moving away from cross-section, one can exploit the new dimension of our market potential estimates to evaluate whether this tight relationship shown in 2003 in the paper has had some persistence over time. Figure 2 confirms that this is the case. In 1970, a year where the United States were still the richest economy in the world the statistical association of GDP per capita with RMP was just as high as in 2003: 0.72 in logs. The correlation between log FMP and log income has risen from 0.38 in 1970 to 0.60 in 2003. The correlations in 1985 (0.66 for log RMP and 0.42 for log FMP) show little change relative to 1970.

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Figure 1: The effects of colonial linkages and regional agreements on trade

(a) ex-colonies (b) RTAs

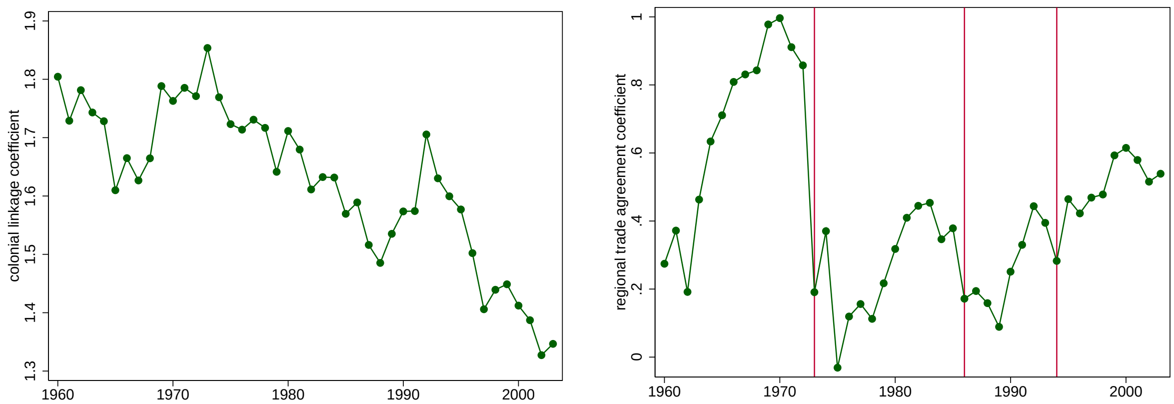
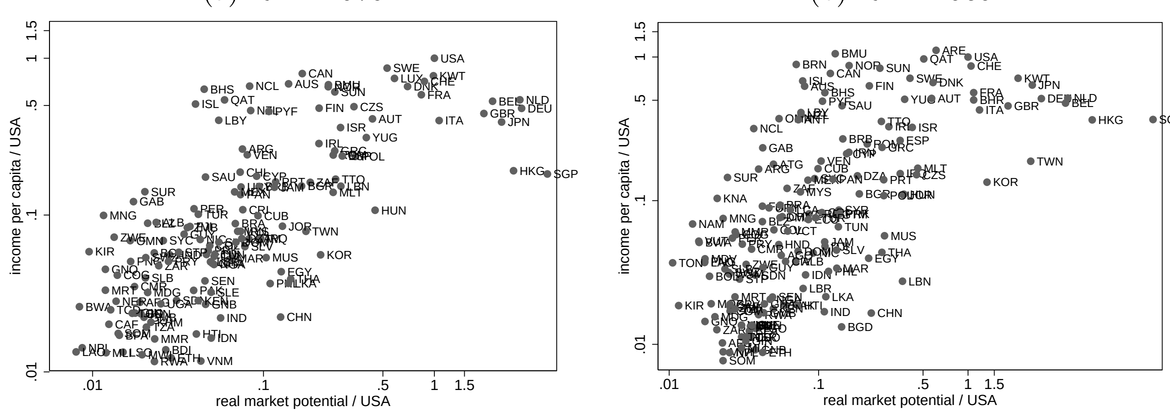


Figure 2: Market Potential and development over time

(a) RMP 1970 (b) RMP 1985



We continue the illustration with maps. The preceding graphs show an interesting correlation between RMP and income, but makes it hard to detect what is core in the concept of market potential, the spatial correlation of the forces behind economic development. Indeed, the theory of market potential tells us that being near large markets makes a country richer, and therefore itself a large market. This suggests that in equilibrium, "spatial clubs" of development will form. It will be very hard for a country surrounded by small and poor economies to reach a high level of income per capita, and inversely, the proximity of large and wealthy countries is a strong advantage in this economic geography world. The maps contained in figures 3 and 4 represent the levels of RMP and FMP in each country in the world, expressed again relative to the United States in 2003.

Those figures indeed show evidence of spatial correlation in RMP and even more in FMP. Moran's I statistics for spatial correlation support the visual evidence. Using inverse distances as weights, we obtain Moran statistics of 0.196 for log RMP and 0.398 for log FMP. By way of comparison, in our 2003 sample, log per capita income exhibits spatial correlation of 0.256.

Western Europe, North America and to a lesser extent East Asia are places were the spatial proximity of high GDP countries fuels each other's market potential and therefore income. The case of the United States and its immediate neighbors is illustrative of the problems raised by FMP. While the RMP figure in 2003 predicts the USA to have a much higher income per capita than Canada and Mexico, the reverse is true for FMP. One can also see in the FMP map the extent to which high demand zones exert a positive influence on their neighbors. The "pull-factor" of Western Europe is particularly visible in Eastern Europe and Northern Africa, while central America is clearly benefiting from being close to NAFTA countries in terms of FMP.

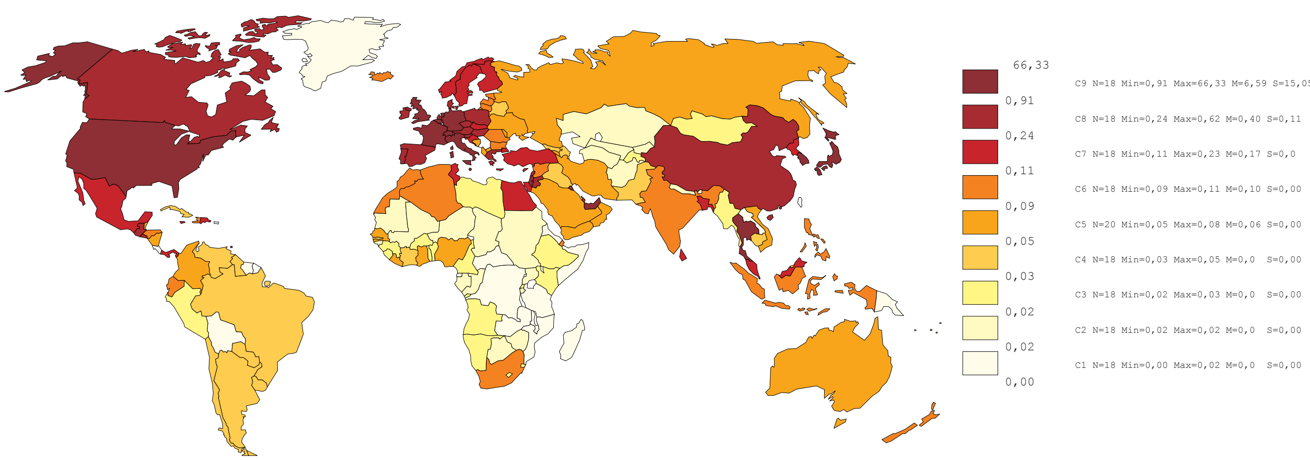


Figure 3: RMP 2003

Figures 5 and 6 are probably the most illustrative of the market potential forces at work over time. Those two figures present maps of the evolution of market potential over time for each country in the world. The precise figure represented is the change in terms of ranks (gained or lost) in the market potential hierarchy, relative to the United States. Both

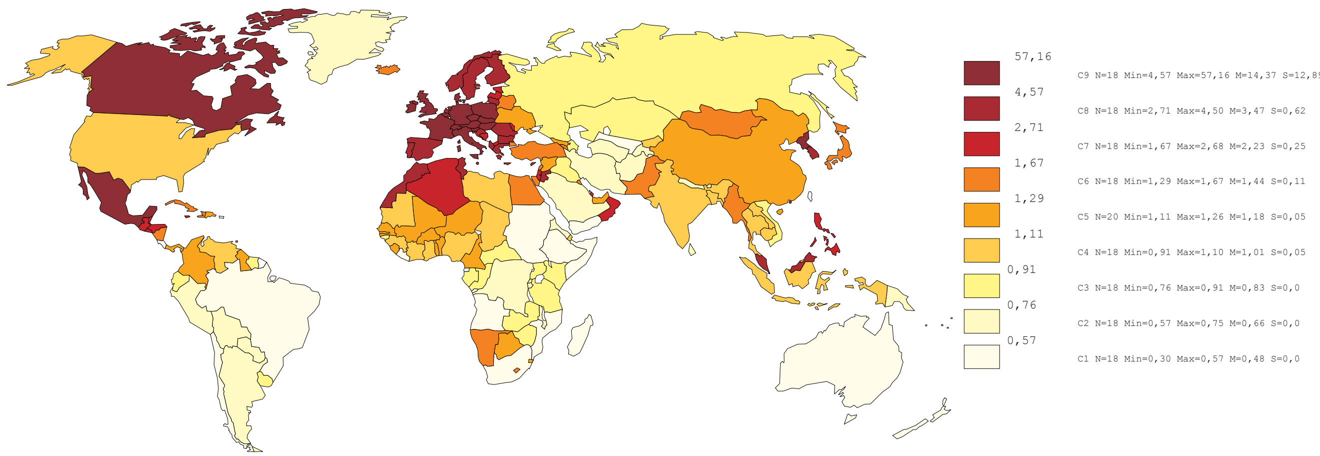


Figure 4: FMP 2003

figures, and in particular the Foreign Market Potential one makes very apparent the existence of market potential clubs of countries geographically proximate and having similar rates of high or low income growth that fuel each other market potential, and therefore income growth. East Asian countries are characterized by a very fast growing market potential during the period, while most if not all African countries are faced with neighbors receding in the worldwide hierarchy of market potential, which dampens their possibilities of economic expansion. In Latin and South America, there seem to be a clear gradient, where proximity to the Northern part of the continent helps the growth of market potential. Note also that Eastern Europe suffers from a low growth of the overall market potential during this period, despite a high growth of their FMP, driven by increased access to Western European markets. Particularly striking is the strong performance of three emerging countries over that period in terms of RMP: Mexico, Turkey and Malaysia. The performance of Turkey is particularly remarkable since figure 6 reveals that its FMP, that is the dynamism of its neighbors, actually decreased during that period. On the contrary, Mexico and Malaysia benefited largely from a very dynamic geographic environment.

3 Special cases of the general gravity formulation

In this appendix we show that the best-known theories underlying gravity can all fit neatly within the structure outlined above. The specifications we consider use iceberg trade costs, such that $\tau_{ij} - 1$ is the *ad valorem* tariff equivalent of all trade costs. A single factor of production, denoted L receives w as wages. Costs of production are given by w_i/z_i where z_i is productivity. With the exception of linear monopolisitic competition models, prices can be expressed as $p_i = mw_i/z_i$, where $m = 1$ in competitive models and $\sigma/(\sigma - 1)$ in CES monopolistic competition.

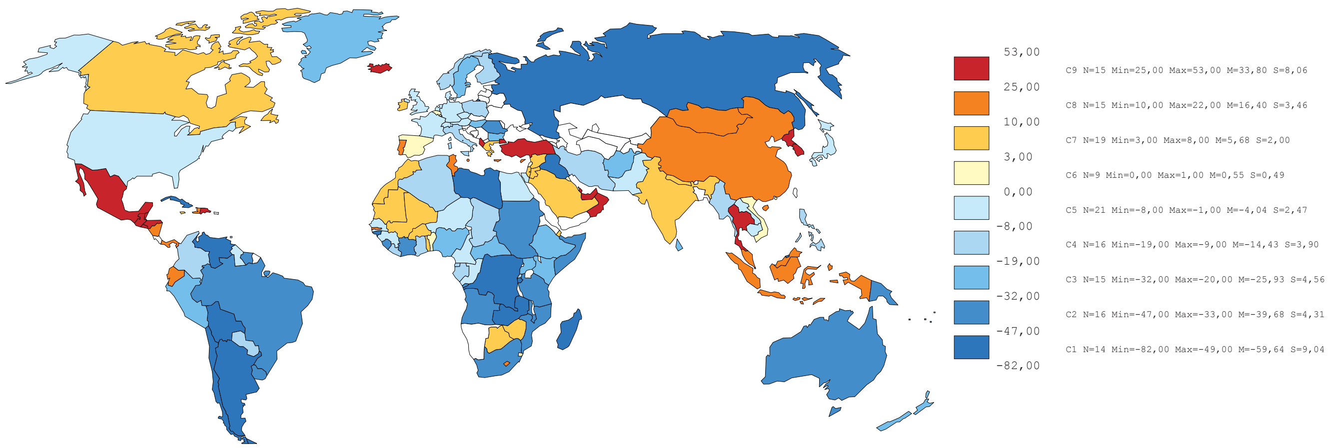


Figure 5: RMP rank evolution 1970–2003

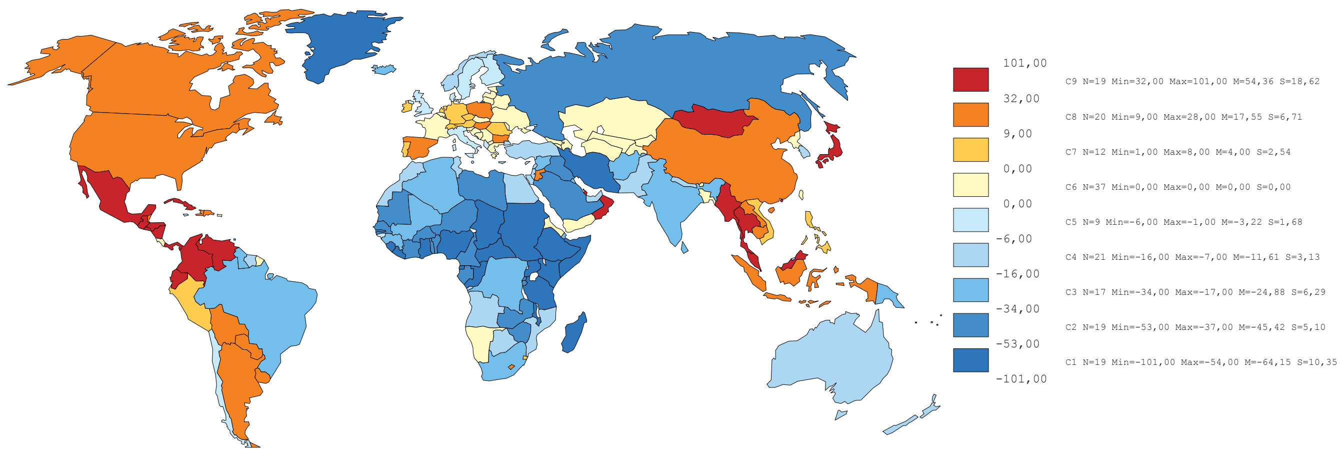


Figure 6: FMP rank evolution 1970–2003

3.1 CES national product differentiation

The earliest derivation of the gravity equation for trade must be Anderson (1979). As in Armington (1968), each country is the unique source of each product. Consumers in country j consume q_{ij} units of the product from country i. Utility exhibits a constant elasticity of substitution (CES), $\sigma > 1$, over all the national products:

$$U_j = \left(\sum_i (b_{ij} q_{ij})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

The exporter attribute and the dyadic integration term are given by

$$A_i = (p_i/b_i)^{1-\sigma} = (b_i z_i/m)^{\sigma-1} w_i^{1-\sigma}, \quad \phi_{ij} = \tau_{ij}^{1-\sigma} \quad \Phi_j = \sum_h A_h \phi_{hj}. \tag{2}$$

3.2 CES Monopolistic Competition

The earliest derivation of a gravity equation using monopolistic competition of the Dixit-Stiglitz form is Bergstrand (1985). Bergstrand used a more general set of preferences than has become standard. In particular, he allowed for a nested structure in which domestic varieties are closer substitutes for each other than are foreign varieties. Bergstrand also generalized the production side to allow for the possibility that output might not be transferable to the export sector on a one-for-one basis. Instead he allows for a "constant elasticity of transformation." While both of these assumptions seem plausible, they have the cost of making the model less tractable. So far as we know, the data do not strongly reject the simpler model in favour of Bergstrand's generalizations.

The gravity equation based on standard Dixit-Stiglitz-Krugman (DSK) assumptions was derived in the late 1990s by multiple authors. It assumes that each country has N_i firms that supply single varieties to the world. CES utility and productivity are symmetric: $b_i = 1$ and $z_i = 1$.

The exporter attribute and the dyadic integration term are given by

$$A_i = N_i p_i^{1-\sigma} = (N_i/m)^{\sigma-1} w_i^{1-\sigma}, \quad \phi_{ij} = \tau_{ij}^{1-\sigma} \quad \Phi_j = \sum_h A_h \phi_{hj}. \tag{3}$$

3.3 Heterogeneous consumers

The taste for variety present in the CES utility functions may be plausible in some contexts but it would be bizarre to apply it to products like passenger cars or laundry detergents. In those and many other cases, the natural way to think about consumer choice is that the large variety of products purchased results from consumers making different decisions. If they face the same prices, then the different selections result from a variety of tastes.[1] Anderson, de Palma, and Thisse (1992) show that two strong functional form assumptions are enough to yield a demand equation that is observationally equivalent to the CES. To our knowledge, no one has published a derivation of the gravity equation using their approach. This is

[1] Income differences would also produce different choices if utility were not homothetic.

probably because the derivation is too trivial. However, we provide it here for reference and because it achieves three small goals. First, it allows for easy synthesis of the national product differentiation and monopolistic competition models. Second, it introduces the idea of using parametric distributions for heterogeneity to obtain the gravity equation. Third, the approach can be used to investigate what happens to the gravity equation when preferences are not homothetic.

Consumers from country j, indexed with $j\ell$, have utility functions defined over the products made by each supplier s in each country i,

$$u_{j\ell is} = \ln[\epsilon_{j\ell is} q_{j\ell is}], \tag{4}$$

where $q_{j\ell is}$ represents the quantity of products consumed, $\epsilon_{j\ell is}$ is the perceived quality of the goods. The heterogeneity is assumed to be distributed Fréchet with a cumulative distribution function (CDF) of $\exp\{-(\epsilon/b_i)^{-\theta}\}$, where θ is an inverse measure of consumer heterogeneity and b_i is a location parameter that is specific to the origin country. In an analogous way to equation (1), an increase in b_i shifts up the utility derived from varieties produced in i, which can be interpreted as an increase in perceived quality.

Each of the L_j consumers chooses the product giving highest utility and then spends X_j/L_j on it. Hence, individual demand is $q_{j\ell is} = (X_j/L_j)/p_{ij}$ for the selected variety and zero on all other varieties. p_{ij} is the price consumers in country j face for product varieties from country i. The conditional indirect utility function is given by

$$v_{j\ell is} = \ln(X_j/L_j) - \ln p_{ij} + \ln \epsilon_{j\ell is}.$$

The Fréchet form for ϵ implies a Gumbel (which Anderson et al., 1992 call "double exponential") form for $\ln \epsilon$ and thereby implies multinomial logit forms for the probabilities of choosing one of the N_i varieties produced in country i for consumers in j:

$$\mathbb{P}_{ij} = \frac{(p_{ij}/b_i)^{-\theta}}{\sum_h (p_{hj}/b_h)^{-\theta}}. \tag{5}$$

The final step to obtain the gravity equation is to recognize that the aggregate value of bilateral demand multiplies the above probabilities by the number of consumers in j, their conditional individual demand in value, and the number of products available from i: $X_{ij} = N_i L_j \times p_{ij} q_{j\ell is} \mathbb{P}_{ij}$.

The exporter attribute and the dyadic integration term are given by

$$A_i = N_i(p_i/b_i)^{-\theta} = (N_i b_i z_i/m)^{\theta} w_i^{-\theta} \quad \phi_{ij} = \tau_{ij}^{-\theta} \quad \Phi_j = \sum_h A_h \phi_{hj}. \tag{6}$$

Note that the key difference in this model compared to the two former ones lies in the parameter $-\theta$ substituting for $1-\sigma$ when the demand system is CES. There is a very strong parallel though since an increase in σ means that products are becoming more homogenous, and an increase in θ means that consumers are becoming less heterogenous. Whether consumers are becoming more alike in their tastes, or whether products are becoming more substitutable yields similar aggregate predictions for trade flows, which is quite intuitive.

3.4 Heterogeneous Industries (Comparative Advantage)

Eaton and Kortum (2002) derive a gravity equation that departs from the the CES-based approaches in almost every respect and yet the results they obtain bear a striking resemblance. In contrast to the national product differentiation approach, each country produces a very large number of goods (modeled as a continuum) that are homogeneous across countries. In contrast to the DSK approach, every industry is perfectly competitive. Bernard, Jenson, Eaton, and Kortum (2003) reformulate the Eaton and Kortum model to allow for Bertrand competition in each sector. Remarkably, they do so in a way that does not change the form of the gravity equation.

Productivity, z is assumed to be distributed Fréchet with a cumulative distribution function (CDF) of $\exp\{-T_i\epsilon^{-\theta}\}$, where T_i is a technology parameter increasing the chances of country i being the lowest cost producer. θ is an inverse measure of heterogeneity in this distribution of productivity. Note that the θ parameter has a different signification than in the heterogenous consumers' section. It reflects variance in productivity of firms rather than variance in tastes. However, since this heterogeneity parameter plays the same key role in both models, we maintain the notation in order to emphasize the similarity in resulting terms.

The exporter attribute and the dyadic integration terms are given by

$$A_i = T_i w_i^{-\theta}, \quad \phi_{ij} = \tau_{ij}^{-\theta} \quad \Phi_j = \sum_h A_h \phi_{hj}. \tag{7}$$

3.5 Heterogeneous firms

Up to know we have allowed consumers to be heterogenous in their preferences and industries to differ in terms of production costs. The next step is to let each realization of cost be unique so that they can be used to identify individual firms. Then define $\pi_{ij}(c)$ as the share of expenditures of a representative consumer in country j on the goods supplied by the firm from country i with cost c. Suppose there is mass of firms in country i given by N_i. The CDF of costs is denoted $F(c)$. A key variable in heterogeneous firms models is the threshold cost, above which firms do not enter a market. We will denote that as $\hat{c}$ and recognize that it is a dyadic variable since the threshold must depend on trade costs between i and j. We can now use this notation to obtain an expression for the aggregate share of the market as the integral over all the individual firms' shares:

$$\Pi_{ij} = N_i \int_0^{\hat{c}_{ij}} \pi_{ij}(c) dF(c). \tag{8}$$

To obtain Π_{ij} we therefore need to specify $\hat{c}_{ij}$, $\pi_{ij}(c)$ and $F(c)$. The goal is to choose functional forms that yield a closed form for the integral. Two approaches have been shown to work so far: CES monopolistic competition (Chaney, 2008) and linear monopolistic competition (Melitz and Ottaviano, 2008).

Productivity, z, is distributed Pareto with shape parameter θ and minimum productivity given by $\underline{z} > 0$. Maximum possible costs are given by $\bar{c} = w/\underline{z}$. Pareto distribution for z implies a power distribution for costs with CDF of $F(c) = Kc^{\theta}$, where $K \equiv (w/\underline{z})^{-\theta}$.

3.5.1 CES monopolistic competition

Chaney (2008) and Helpman, Melitz, and Rubinstein (2008) embed heterogeneous firms in a Dixit-Stiglitz framework generalizing the Melitz (2003) paper to multiple countries. The market share and pricing equations are now specific to each firm indexed with their marginal cost c:

$$\pi_{ij}(c) = p_{ij}^{1-\sigma} P_j^{\sigma-1} \quad \text{where} \quad p_{ij}(c) = \frac{\sigma}{\sigma-1} c\tau_{ij}. \tag{9}$$

The aggregate market share of i firms in j is therefore obtained after solving for the integral:

$$\Pi_{ij} = \zeta_1 N_i \underline{z}_i^{\theta} w_i^{-\theta} \hat{c}_{ij}^{1-\sigma+\theta} \tau_{ij}^{1-\sigma} P_j^{\sigma-1}, \tag{10}$$

where ζ_1 is a constant.[2] In this model, the equilibrium threshold $\hat{c}_{ij}$ such that the corresponding firm is the last one to serve market j (zero profit condition with f_{ij} the fixed cost of serving j from i) is

$$\hat{c}_{ij} = \frac{\sigma-1}{\sigma}\left(\frac{f_{ij}}{X_j}\right)^{\frac{1}{1-\sigma}} \frac{P_j}{\tau_{ij}},$$

which brings an equilibrium aggregate market share

$$\Pi_{ij} = \zeta_2 N_i \underline{z}_i^{\theta} w_i^{-\theta} \tau_{ij}^{-\theta} f_{ij}^{1+\frac{\theta}{1-\sigma}} P_j^{\theta} X_j^{\frac{\theta}{\sigma-1}-1}, \tag{11}$$

where ζ_2 is a constant.[3]

The exporter attribute and the dyadic integration term are given by

$$A_i = N_i \underline{z}_i^{\theta} w_i^{-\theta}, \quad \phi_{ij} = \tau_{ij}^{-\theta} f_{ij}^{1+\frac{\theta}{1-\sigma}} \quad \Phi_j = \sum_h A_h \phi_{hj}. \tag{12}$$

3.5.2 Linear monopolistic competition

In Melitz and Ottaviano (2008), the bilateral exporter's cost threshold $\hat{c}_{ij}$ is simply a function of the domestic production threshold $\hat{c}_j$, such that $\hat{c}_j = \hat{c}_{ij}\tau_{ij}$. With the linear demand structure used

$$p_{ij}(c) = \frac{1}{2}(\hat{c}_j + \tau_{ij}c) \qquad \text{and} \qquad q_{ij}(c) = \frac{L_j}{2\gamma}(\hat{c}_j - \tau_{ij}c), \tag{13}$$

implying the following market share of firm c:

$$\pi_{ij}(c) = \frac{p_{ij}(c)q_{ij}(c)}{X_j} = \frac{\hat{c}_j^2 - (\tau_{ij}c)^2}{4\gamma(X_j/L_j)}. \tag{14}$$

Integrating over all firms, the collective share of the market is

$$\Pi_{ij} = \frac{N_i \underline{z}_i^{\theta} w_i^{-\theta} \hat{c}_j^{\theta+2} \tau_{ij}^{-\theta}}{2\gamma(\theta+2)w_j}. \tag{15}$$

Melitz and Ottaviano have a single factor of production so the wage does double duty. In the numerator, w_i enters as a determinant of the cost of production in the exporting country. In the denominator w_j is per-capita expenditure (X_j/L_j).[4]

[2] $\zeta_1 = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{\theta}{1-\sigma+\theta}$.

[3] $\zeta_2 = \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \frac{\theta}{1-\sigma+\theta}$.

[4] The free-entry assumption dissipates profits: $\delta = 0$.

The exporter attribute and the dyadic integration term are given by

$$A_i = N_i \underline{z}_i^{\theta} w_i^{-\theta}, \quad \phi_{ij} = \tau_{ij}^{-\theta} \quad \Phi_j = w_j / \hat{c}_j^{\theta+2}. \tag{16}$$

. However, the importer term in the gravity equation takes on a somewhat different form from the normal one:

$$X_j/\Phi_j = (w_j L_j)/(w_j/\hat{c}_j^{\theta+2}) = L_j \hat{c}_j^{\theta+2}.$$

It is increasing in the *population* of the importing country but not in the per-capita income. This is due to the non-homotheticity of preferences. In the linear-quadratic utility structure, a higher income individual lowers the share of income spent on the traded varieties and spends a higher share on good zero.

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