Math 104 section 108 Quiz #1 Column B Date: Grade:

First Name: ____________________ Last Name: ____________________

Student-No: ____________________ Section: ____________________

Very short answer questions

1. [2 marks] Each part is worth 1 mark. Please write your answers in the boxes.

   (a) What is the worth, after 9 months, of an investment of \$200 with a nominal interest rate of 15% compounded quarterly?

   Answer: $200 \cdot (1.0375)^3$

   Solution:

$$FV = PV \cdot \left(1 + \frac{i}{n}\right)^{nt}, \quad PV = 200, \quad n = 4, \quad t = \frac{3}{4}, \quad i = 0.15$$

$$FV = 200 \cdot \left(1 + \frac{0.15}{4}\right)^{4 \cdot \frac{3}{4}} = 200 \cdot (1.0375)^3$$

   (b) Compute $\lim_{x \to 3} \frac{3x^3 - 2x^2 + 1}{2x^2 + 1}$.

   Answer: $\frac{23}{5}$

   Solution: $\lim_{x \to 3} \frac{3x^3 - 2x^2 + 1}{2x^2 + 1} = \frac{3 \cdot 3^3 - 2 \cdot 3^2 + 1}{2 \cdot 3^2 + 1} = \frac{81 - 18 + 1}{18 + 1} = \frac{64}{19}$

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

   (a) An investment of \$400 gained \$80 in the last two months, what is the nominal interest rate, assuming it is compunded monthly?

   Answer: $12 \cdot \left(\sqrt{\frac{6}{5}} - 1\right)$

   Solution: We use $FV = PV \cdot \left(1 + \frac{i}{n}\right)^{n \cdot t}$ with

$$PV = 400, \quad FV = PV + 80 = 480, \quad n = 12, \quad t = \frac{2}{12} = \frac{1}{6}$$

   and so

$$480 = 400 \cdot \left(1 + \frac{i}{12}\right)^{12 \cdot \frac{1}{6}} = 400 \cdot \left(1 + \frac{i}{12}\right)^2.$$

   Simplifying yields

$$i = 12 \cdot \left(\sqrt{\frac{6}{5}} - 1\right)$$

(b) Compute the limit $\lim_{x\to 1} \frac{\frac{1}{2x-8}+\frac{1}{6}}{x-1}$.

Answer: $-\frac{1}{18}$

Solution:

$$\lim_{x\to 1} \frac{\frac{1}{2x-8}+\frac{1}{6}}{x-1} \lim_{x\to 1} \frac{\frac{1\cdot 6+1\cdot(2x-8)}{6\cdot(2x-8)}}{x-1} \lim_{x\to 1} \frac{2(x-1)}{6(2x-8)(x-1)} = \lim_{x\to 1} \frac{2}{6(2x-8)} = \frac{2}{6\cdot(-6)}$$

Long answer question — you must show your work

3. [4 marks] A global conglomerate manufactures chalk. They sell a box of chalk for \$1 and and their monthly revenue is \$4,000,000. Their research team figured that if they increase the price by a quarter a box they will sell 1,000,000 units less each month. Their fixed production cost is \$750,000 and each box costs an extra ¢37.5 ($\frac{3}{8}$ of a dollar) to make.

(a) Find the linear demand equation. Use the notation p for price and q for the monthly demand.

Answer:
$q = -4,000,000 \cdot p + 9,000,000$

Solution: The revenue is given by $R = p \cdot q$ and the first piece of information says that $5,000,000 = 1 \cdot q$ and hence, for the price of \$1 the company sells 5,000,000 boxes. Write $q = A \cdot p + B$. We have

$$\begin{cases} 4,000,000 = A \cdot 1 + B \\ 3,000,000 = A \cdot 1.25 + B \end{cases}$$

Subtracting the two equations yields

$$1,000,000 = -\frac{1}{4} \cdot A$$

or otherwise $A = -4,000,000$. Plugging this back to the first equations gives

$$B = 4,000,000 - A \cdot 1 = 8,000,000$$

(b) Find the monthly profit as a function $P(q)$.

Answer: $\frac{(q-2{,}000{,}000)(q-6{,}000{,}000)}{-4{,}000{,}000}$

Solution: First write

$$p = \frac{q - 8,000,000}{-4,000,000}$$

so the revenue function is

$$R(q) = \frac{(q - 8,000,000)\, q}{-4,000,000}.$$

By the data, the cost function is

$$C(q) = F + V(q) = 750,000 + \frac{3}{8}q.$$

Hence the profit is given by

$$\begin{aligned} P(q) = R(q) - C(q) &= \frac{(q - 8,000,000)\, q}{-4,000,000} - \left(750,000 + \frac{3}{8}q\right) \\ &= \frac{(q - 2,000,000)\,(q - 6,000,000)}{-4,000,000} \end{aligned}$$

(See solution of column A for a more detailed simplification)