Math 104 section 108 Quiz #1 Column C Date: Grade:

First Name: ______________________ Last Name: ______________________

Student-No: ______________________ Section: ______________________

Very short answer questions

1. [2 marks] Each part is worth 1 mark. Please write your answers in the boxes.

   (a) What is the worth, after 9 months, of an investment of \$300 with a nominal interest rate of 11% compounded quarterly?

   Answer: $300 \cdot (1.0275)^3$

   Solution:

   $$FV = PV \cdot \left(1 + \frac{i}{n}\right)^{nt}, \quad PV = 300, \quad n = 4, \quad t = \frac{3}{4}, \quad i = 0.11$$

   $$FV = 300 \cdot \left(1 + \frac{0.13}{4}\right)^{4 \cdot \frac{3}{4}} = 300 \cdot (1.0275)^3$$

   (b) Compute $\lim_{x \to 2} \frac{x^3 - 3x^2 + 1}{x^2 + x + 1}$.

   Answer: $\frac{23}{5}$

   Solution: $\lim_{x \to 2} \frac{x^3 - 3x^2 + 1}{x^2 + x + 1} = \frac{2^3 - 3 \cdot 2^2 + 1}{2^2 + 2 + 1} = \frac{8 - 12 + 1}{7} = -\frac{3}{7}$

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

   (a) An investment of \$600 gained \$200 in the last two months, what is the nominal interest rate, assuming it is compunded monthly?

   Answer: $12 \cdot \left(\sqrt{\frac{4}{3}} - 1\right)$

   Solution: We use $FV = PV \cdot \left(1 + \frac{i}{n}\right)^{n \cdot t}$ with

   $$PV = 600, \quad FV = PV + 200 = 800, \quad n = 12, \quad t = \frac{2}{12} = \frac{1}{6}$$

   and so

   $$800 = 600 \cdot \left(1 + \frac{i}{12}\right)^{12 \cdot \frac{1}{6}} = 600 \cdot \left(1 + \frac{i}{12}\right)^2.$$

   Simplifying yields

   $$i = 12 \cdot \left(\sqrt{\frac{4}{3}} - 1\right)$$

(b) Compute the limit $\lim_{x \to 2} \frac{\frac{1}{x-4}+\frac{1}{2}}{x-2}$.

Answer: $-\frac{1}{4}$

Solution:

$$\lim_{x \to 2} \frac{\frac{1}{x-4}+\frac{1}{2}}{x-2} = \lim_{x \to 2} \frac{\frac{2+x-4}{2(x-4)}}{x-2} = \lim_{x \to 2} \frac{x-2}{2(x-4)(x-2)} = \lim_{x \to 2} \frac{1}{2(x-4)} = -\frac{1}{4}$$

Long answer question — you must show your work

3. 4 marks A global conglomerate manufactures chalk. They sell a box of chalk for \$1 and and their monthly revenue is \$3,000,000. Their research team figured that if they increase the price by a quarter a box they will sell 1,000,000 units less each month. Their fixed production cost is \$750,000 and each box costs an extra ¢75 to make.

(a) Find the linear demand equation. Use the notation p for price and q for the monthly demand.

Answer:
$q = -4,000,000 \cdot p + 7,000,000$

Solution: The revenue is given by $R = p \cdot q$ and the first piece of information says that $3,000,000 = 1 \cdot q$ and hence, for the price of \$1 the company sells 5,000,000 boxes. Write $q = A \cdot p + B$. We have

$$\begin{cases} 3,000,000 = A \cdot 1 + B \\ 2,000,000 = A \cdot 1.25 + B \end{cases}$$

Subtracting the two equations yields

$$1,000,000 = -\frac{1}{4} \cdot A$$

or otherwise $A = -4,000,000$. Plugging this back to the first equations gives

$$B = 3,000,000 - A \cdot 1 = 7,000,000$$

(b) Find the monthly profit as a function $P(q)$.

Answer: $\frac{(q-1,000,000)(q-5,000,000)}{-4,000,000}$

Solution: First write

$$p = \frac{q - 7,000,000}{-4,000,000}$$

so the revenue function is

$$R(q) = \frac{(q - 7,000,000)\, q}{-4,000,000}.$$

By the data, the cost function is

$$C(q) = F + V(q) = 750,000 + \frac{3}{4} q.$$

Hence the profit is given by

$$P(q) = R(q) - C(q) = \frac{(q - 7,000,000)\,q}{-4,000,000} - \left(750,000 + \frac{3}{4}q\right)$$
$$= \frac{(q - 1,000,000)\,(q - 3,000,000)}{-4,000,000}$$

(See solution of column A for a more detailed simplification)