Math 104 section 108 Quiz #3 Column A Date: Grade:

First Name: ____________________ Last Name: ____________________

Student-No: ____________________ Section: ____________________

Short answer questions — you must show your work

1. [4 marks] Each part is worth 2 marks.

   (a) Compute $y^{(3)}$ for $y = e^{x^2} + x^4$.

   Answer: $e^{x^2}\left(8x^3 + 12x\right) + 24x$

   Solution:

   $$y' = e^{x^2} 2x + 4x^3$$
   $$y'' = e^{x^2} 4x^2 + 2e^{x^2} + 12x^2$$
   $$y^{(3)} = e^{x^2} 8x^3 + e^{x^2} 8x + e^{x^2} 4x + 24x$$

   (b) Compute $\frac{dy}{dz}$ where $e^{yz} = 2yz$.

   Answer: $y' = \frac{2y - ye^{yz}}{ze^{yz} - 2z}$

   Solution: We have

   $$\frac{d}{dz}\left(e^{yz}\right) = e^{yz}\frac{d(yz)}{dz} = e^{yz}\left(y + zy'\right)$$
   $$\frac{d}{dz}\left(2yz\right) = 2y + 2y'z.$$

   And so

   $$e^{yz}\left(y + zy'\right) = 2y + 2y'z.$$

   Solving for y' gives:

   $$y' = \frac{2y - ye^{yz}}{ze^{yz} - 2z}$$

Long answer questions — you must show your work

2. [6 marks] Each part is worth 3 marks.

   (a) The number q of umbrellas that a store will sell per week and the price p (in dollars per item) are related by the demand equation $q^2 = 4,500 - 5p^2$. If the price of an umbrella is falling at the rate of \$ 1 per week, find how the sales will change if the current price is \$20.

   Answer: 2 umbrellas per week

   Solution: At the given time t_0 we have

   1. $p(t_0) = \$20$

2. $p'(t_0) = -1 \frac{\$}{week}$
3. When $p(t_0) = 20$ we have

$$q^2 = 4,500 - 5 \cdot 20^2 = 2,500.$$

Since $q \geq 0$ it follows that $q = 50$ which means they sell 50 umbrellas a week.

We write $p(t)$ and $q(t)$ as functions of time. Using implicit differentiation we find

$$2qq' = -10pp'.$$

And hence

$$q'(t_0) = -\frac{5p(t_0)p'(t_0)}{q(t_0)} = -\frac{5 \cdot 20 \cdot (-1)}{50} = 2 \frac{units}{week}$$

That is, the sales increase by 2 umbrellas per week.

(b) Assume that $f(x)$ is one-to-one and its tangent at $x = 1$ is given by $l(x) = 3x + 1$. Compute the tangent of the inverse of $f(x)$ at $f(1)$.

Answer: $l(x) = \frac{1}{3}x - \frac{1}{3}$

Solution: First note that $f(1) = l(1) = 3 + 1 = 4$ and $f'(1) = l'(1) = 3$. Denote the inverse of $f(x)$ by $g(x)$ so $g(f(x)) = x$ for any x. By the chain rule we have $g'(f(x))f'(x) = 1$ for any x. In particular

$$g'(f(1))f'(1) = 1.$$

In other word

$$g'(4) = \frac{1}{f'(1)} = \frac{1}{3}$$

and so, the tangent of $g(x)$ at $x = 4$ is given by $l(x) = \frac{1}{3}(x - 4) + 1 = \frac{1}{3}x - \frac{1}{3}$.