Math 104 section 108 Quiz #3 Column B   Date: Grade:

First Name: ________________ Last Name: ________________

Student-No: ________________ Section: ________________

Short answer questions — you must show your work

1. [4 marks] Each part is worth 2 marks.

   (a) Compute $y^{(3)}$ for $y = e^{x^3} + x^3$.

   Answer: $e^{x^3}\left(27x^6 + 54x^3 + 6\right) + 6$

   Solution:

   $$y' = e^{x^3} 3x^2 + 3x^2$$
   $$y'' = e^{x^3} 9x^4 + e^{x^3} 6x + 6x$$
   $$y^{(3)} = e^{x^2} 27x^6 + e^{x^2} 36x^3 + e^{x^3} 18x^3 + 6e^{x^3} + 6$$

   (b) Compute $\frac{dy}{dz}$ where $\cos(yz) = 3yz$.

   Answer: $y' = -\frac{3y + y\sin(yz)}{z\sin(yz) + 3z}$

   Solution: We have

   $$\frac{d}{dz}\left(\cos(yz)\right) = -\sin(yz)\frac{d(yz)}{dz} = -\sin(yz)\left(y + zy'\right)$$
   $$\frac{d}{dz}\left(3yz\right) = 3y + 3y'z.$$

   And so

   $$-\sin(yz)\left(y + zy'\right) = 3y + 3y'z.$$

   Solving for y' gives:

   $$y' = -\frac{3y + y\sin(yz)}{z\sin(yz) + 3z}$$

Long answer questions — you must show your work

2. [6 marks] Each part is worth 3 marks.

   (a) The number q of umbrellas that a store will sell per week and the price p (in dollars per item) are related by the demand equation $q^2 = 4,200 - 6p^2$. If the price of an umbrella is falling at the rate of \$ 1 per week, find how the sales will change if the current price is \$10.

   Answer: 1 umbrellas per week

   Solution: At the given time t_0 we have

1. $p(t_0) = \$10$
2. $p'(t_0) = -1 \ \frac{\$}{week}$
3. When $p(t_0) = 10$ we have

$$q^2 = 4,200 - 6 \cdot 10^2 = 3,600.$$

Since $q \geq 0$ it follows that $q = 60$ which means they sell 60 umbrellas a week.

We write $p(t)$ and $q(t)$ as functions of time. Using implicit differentiation we find

$$2qq' = -12pp'.$$

And hence

$$q'(t_0) = -\frac{6p(t_0)p'(t_0)}{q(t_0)} = -\frac{6 \cdot 10 \cdot (-1)}{60} = 1 \ \frac{units}{week}$$

That is, the sales increase by 1 umbrellas per week.

(b) Assume that $f(x)$ is one-to-one and its tangent at $x = 1$ is given by $l(x) = 4x + 1$. Compute the tangent of the inverse of $f(x)$ at $f(1)$.

Answer: $l(x) = \frac{1}{4}x - \frac{1}{4}$

Solution: First note that $f(1) = l(1) = 4 + 1 = 5$ and $f'(1) = l'(1) = 4$. Denote the inverse of $f(x)$ by $g(x)$ so $g(f(x)) = x$ for any x. By the chain rule we have $g'(f(x))f'(x) = 1$ for any x. In particular

$$g'(f(1))f'(1) = 1.$$

In other word

$$g'(5) = \frac{1}{f'(1)} = \frac{1}{4}$$

and so, the tangent of $g(x)$ at $x = 5$ is given by $l(x) = \frac{1}{4}(x - 5) + 1 = \frac{1}{4}x - \frac{1}{4}$.