Math 104 section 108 Quiz #3 Column C   Date: Grade:

First Name: ________________ Last Name: ________________

Student-No: ________________ Section: ________________

Short answer questions — you must show your work

1. 4 marks Each part is worth 2 marks.

(a) Compute $y^{(3)}$ for $y = e^{x^4} + x^2$.

Answer:
$e^{x^4}\left(64x^9 + 96x^5 + 144x^5 + 24x\right)$

Solution:

$$\begin{aligned} y' &= e^{x^4}4x^3 + 2x \\ y'' &= e^{x^4}16x^6 + e^{x^4}12x^2 + 2 \\ y^{(3)} &= e^{x^4}64x^9 + e^{x^4}96x^5 + e^{x^4}48x^5 + e^{x^4}24x \end{aligned}$$

(b) Compute $\frac{dy}{dz}$ where $\sin(yz) = yz$.

Answer: $y' = \frac{y - y\cos(yz)}{z\cos(yz) - z}$

Solution: We have

$$\begin{aligned} \frac{d}{dz}\left(\sin(yz)\right) &= \cos(yz)\frac{d(yz)}{dz} = \cos(yz)\left(y + zy'\right) \\ \frac{d}{dz}\left(yz\right) &= y + y'z. \end{aligned}$$

And so

$$\cos(yz)\left(y + zy'\right) = y + y'z.$$

Solving for y' gives:

$$y' = \frac{y - y\cos(yz)}{z\cos(yz) - z}$$

Long answer questions — you must show your work

2. 6 marks Each part is worth 3 marks.

(a) The number q of umbrellas that a store will sell per week and the price p (in dollars per item) are related by the demand equation $q^2 = 7,700 - 7p^2$. If the price of an umbrella is falling at the rate of \$ 1 per week, find how the sales will change if the current price is \$20.

Answer: 2 umbrellas per week

Solution: At the given time t_0 we have

1. $p(t_0) = \$20$
2. $p'(t_0) = -1 \frac{\$}{week}$
3. When $p(t_0) = 20$ we have

$$q^2 = 7,700 - 7 \cdot 20^2 = 4,900.$$

Since $q \geq 0$ it follows that $q = 70$ which means they sell 70 umbrellas a week.

We write $p(t)$ and $q(t)$ as functions of time. Using implicit differentiation we find

$$2qq' = -14pp'.$$

And hence

$$q'(t_0) = -\frac{7p(t_0)p'(t_0)}{q(t_0)} = -\frac{7 \cdot 20 \cdot (-1)}{70} = 2 \frac{units}{week}$$

That is, the sales increase by 2 umbrellas per week.

(b) Assume that $f(x)$ is one-to-one and its tangent at $x = 1$ is given by $l(x) = 2x + 1$. Compute the tangent of the inverse of $f(x)$ at $f(1)$.

Answer: $l(x) = \frac{1}{2}x - \frac{1}{2}$

Solution: First note that $f(1) = l(1) = 2 + 1 = 3$ and $f'(1) = l'(1) = 2$. Denote the inverse of $f(x)$ by $g(x)$ so $g(f(x)) = x$ for any x. By the chain rule we have $g'(f(x))f'(x) = 1$ for any x. In particular

$$g'(f(1))f'(1) = 1.$$

In other word

$$g'(3) = \frac{1}{f'(1)} = \frac{1}{2}$$

and so, the tangent of $g(x)$ at $x = 3$ is given by $l(x) = \frac{1}{2}(x - 3) + 1 = \frac{1}{2}x - \frac{1}{2}$.