

First Name: _____ Last Name: _____

Student-No: _____ Section: _____

Short answer questions — you must show your work

- 1.
- 4 marks
- Each part is worth 2 marks.

- (a) The cost to make
- q
- TVs in our factory is given by
- $C(q) = 700,000 + 400\sqrt{q}$
- . If we make 10,000 TVs, what are the marginal unit cost, marginal cost and average cost of making one more TV?

Solution:

- Marginal unit cost:

$$MUC(10,000) = C(10,001) - C(10,000) = 400 \left(\sqrt{10,001} - \sqrt{10,000} \right).$$

- Average cost $AC(10,000) = \frac{C(10,000)}{10,000} = \frac{700,000 + 400\sqrt{10,000}}{10,000} = 74$

- Marginal cost

$$MC(10,000) = C'(10,000) = \frac{200}{\sqrt{10,000}} = \frac{200}{100} = 2,$$

$$\text{since } C'(q) = 400 \cdot \frac{1}{2\sqrt{q}} = \frac{200}{\sqrt{q}}.$$

- (b) The distance, in meters, of a cream-pie from a clown's face is given
- $d(t) = -t^2 + t + 6$
- (time is measured in seconds). What is the velocity in which it hits the clown's face?

Answer: $-5 \frac{m}{sec}$

Solution: We start by solving $d(t_0) = 0$. There are two solutions, -2 and 3 . Since -2 doesn't make any sense we find that $t_0 = 3$. The velocity at time 2 is $d'(2) = -2 \cdot 3 + 1 = -5 \frac{m}{sec}$

Long answer questions — you must show your work

- 2.
- 6 marks
- Each part is worth 3 marks.

- (a) The demand equation for a certain product is
- $q^2 + p^{3/2} + 2p = 32$
- , where
- q
- is the number of units per hour the manufacturer can sell at a price of
- p
- dollars per unit. If the price is raised slightly from \$4 dollars, will the revenue increase or decrease (use the elasticity of demand to do this)?

Answer: Increase

Solution: We first find $\frac{dq}{dp}$. Differentiating the demand equation with respect to p yields

$$2q \frac{dq}{dp} + \frac{3}{2} \sqrt{p} + 2 = 0$$

So

$$\frac{dq}{dp} = -\frac{3\sqrt{p} + 4}{4q}$$

The elasticity is given by

$$\epsilon = \frac{p}{q} \frac{dq}{dp} = -\frac{p(3\sqrt{p} + 4)}{4q^2}.$$

On the other hand, when $p = \$4$ we have $q^2 + 8 + 8 = 32$, i.e. $q = \sqrt{16} = 4$. Plugging this into the elasticity function yields

$$\epsilon = -\frac{4(3\sqrt{4} + 4)}{4 \cdot 4^2} = -\frac{40}{64} > -1$$

Hence the good is price in-elastic and hence the revenue will increase if the price is increased.

- (b) Find the absolute maximum and minimum of $f(x) = x^{2/3} + 2x$ on $[-1, 1]$ (value and point).

Answer: Max 2, Min 0

Solution: We derive the function

$$f'(x) = \frac{2}{3x^{1/3}} + 2,$$

this is true for any $x \neq 0$. Solving the equation $f'(c) = 0$ for c yields one solution $c = -\left(\frac{1}{3}\right)^3$ which is in the interval. We have two critical points 0 and $-\left(\frac{1}{3}\right)^3$. We plug the edges and critical points into the function

$$f(-1) = -1, \quad f(0) = 0, \quad f\left(-\left(\frac{1}{3}\right)^3\right) = \frac{4}{27}, \quad f(1) = 3$$

The absolute maximum is 3 at 1 and the absolute minimum is -1 at -1 .