1. Find all $x \in \mathbb{R}$ that solves the following equation.

$$x^4 + 3x^2 - 10 = 0.$$ 

2. Recall that the absolute value function is defined by

$$|x| = \begin{cases} 
  x, & x \geq 0 \\
  -x, & x < 0 
\end{cases}$$

(a) Plot $|x|$ 
(b) Plot the piecewise function

$$f(x) = \begin{cases} 
  |x|, & x \geq -2 \\
  2x + 6, & -4 \leq x < -2 \\
  -2, & x < -4 
\end{cases}$$

(c) Use the graph to find the domain and range of $f$. 
(d) Use the graph to find the zeros (roots) of $f$.

3. Consider the functions

$$f(x) = x^2 + 6x \quad \text{and} \quad g(x) = 3 - 2x$$

Find all real numbers $x$ such that $f \circ g(x) = g \circ f(x)$.

4. Using the relevant graphs/triangles/unit circle explain why

$$\sin \left( \frac{11\pi}{6} \right) = -\frac{1}{2}.$$
5. Consider the following functions

\[ g(x) = 3 \cos(2018x) \]

and

\[ f(x) = \begin{cases} 
  x^4 + 8x^2, & x > 3 \\
  5, & -3 \leq x \leq 3 \\
  2^x, & x < -3 
\end{cases} \]

Determine the range of the function \( f \circ g \). Ensure your answer is fully justified.

6. Below, the graph of functions \( f \) and \( g \) are given. Using the graph answer the following questions.

(a) Write the domain and range of \( f \). \quad \text{Dom: } [0, 6] \quad \text{Range: } [-1, 1]

(b) Write the domain and range of \( g \). \quad \text{Dom: } [0, 6] \quad \text{Range: } [-2, 2]

(c) Evaluate

i. \( f \circ g(0) = f(g(0)) = f(2) = 1 \)

ii. \( f \circ g(4) = f(g(4)) = f(1) = -1 \)

iii. \( g \circ g(4) = g(g(4)) = g(2) = 2 \)

iv. \( g \circ f(4) = g(f(4)) = g(1) = \text{undefined} \rightarrow -1 \text{ is NOT in the domain of } g \)

v. \( f \circ g \circ f(3) = f(g(f(3))) = f(g(1)) = f(2) = 1 \)

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[Graph of \( g(x) \) and \( f(x) \) with arrows and labeled points]
\[ x^4 + 3x^2 - 10 = 0. \]

Take \( x^2 = t \), then the equation becomes

\[ t^2 + 3t - 10 = 0 \Rightarrow (t + 5)(t - 2) = 0 \Rightarrow t + 5 = 0 \Rightarrow t = -5 \]
\[ t - 2 = 0 \Rightarrow t = 2 \]

\[ t = x^2 \Rightarrow x^2 = -5 \quad \therefore \text{No solution} \Rightarrow \text{squared number = negative number} \]
\[ \Rightarrow x^2 = 1 \Rightarrow \pm \sqrt{1} \Rightarrow x = \pm 1 \]

2) \( f(x) = |x| \)

Only pick the part of \(|x|\) for which \( x > -2 \)

\[ f(x) = \begin{cases} 
  |x|, & x \geq -2 \\
  2x + 6, & -4 \leq x < -2 \\
  -2, & x < -4 
\end{cases} \]

Pick the piece of line with these \( y \)-values.

(c) \( \text{Domain} : (-\infty, \infty) \text{ or } \mathbb{R} \)
\( \text{Range} : [-2, \infty) \text{ or } y \geq -2 \)

(d) \( \text{Roots are } x \text{-intercepts} : \text{line } y = 2x + 6 \text{ crosses } x \text{-axis when} \)

\[ 2x + 6 = 0 \Rightarrow 2x = -6 \Rightarrow x = -3 \]

Also the graph has a root at the origin

\[ \Rightarrow \text{roots} : \boxed{x = -3, \ x = 0} \]
\[ f(x) = x^2 + 6x \quad \text{and} \quad g(x) = 3 - 2x \]

\[ \text{fog}(x) = f(g(x)) = f(3-2x) = (3-2x)^2 + 6(3-2x) \]

\[ g\circ f(x) = g(f(x)) = g(x^2 + 6x) = 3 - 2(x^2 + 6x) \]

Equate and find \( x \):

\[ (3-2x)^2 + 6(3-2x) = 3 - 2(x^2 + 6x) \]

\[ 9 - 12x + 4x^2 + 18 - 12x = 3 - 2x^2 - 12x \]

Group

\[ 6x^2 - 12x + 24 = 0 \]

\[ \Rightarrow \quad 6(x^2 - 2x + 4) = 0 \]

\[ \Rightarrow \quad x^2 - 2x + 4 = 0 \]

\[ \Rightarrow \quad \text{Rearrangement does not work} \rightarrow \text{quadratic formula:} \quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ b^2 - 4ac = (-2)^2 - 4(1)(4) = -12 < 0 \]

\[ \Rightarrow \quad \text{NO solution} \]

\[ \Rightarrow \quad \text{For NO} \ x, \ \text{fog} = g\circ f \]

(4) With the unit circle:

\[ \frac{11\pi}{6} = \frac{12\pi - \pi}{6} = 2\pi - \frac{\pi}{6} \rightarrow \frac{\pi}{6} \quad \text{less than one complete cycle} \]

\[ \sin\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2} \]
\[ g(x) = 3 \cos(2018x) \]

\[ f(x) = \begin{cases} 
  x^3 + 8x^2, & x > 3 \\
  3, & -3 \leq x \leq 3 \\
  2^x, & x < -3 
\end{cases} \]

\[ f \circ g(x) = f(g(x)) = f(3 \cos(2018x)) \]

We know for any angle \( \theta \)

\[-1 \leq \cos \theta \leq 1 \quad \Rightarrow \quad -1 \leq \cos(2018x) \leq 1 \]

\[ \Rightarrow \quad -3 \leq 3 \cos(2018x) \leq 3 \]

\[ \Rightarrow \quad f(3 \cos(2018x)) = 3 \]

This is a value between -1 and 1.

So we input it in the 2nd line of \( f \).

\[ \Rightarrow \quad f \circ g(x) = 5 \quad \text{for any } x \quad \Rightarrow \quad \text{Range of } f \circ g = \{5\} \]