Reminder:
* Labs begin next week.
* HW1 will be posted on Mon, Sept 10.
  Due date: Sept 17
* Quiz 1: Mon, Sept 24

Lines:

Slope of a line: A quantity that measures how fast the line is rising or falling moving from left to right.

Given two points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$, the slope of the line segment $PQ$ (secant line) is given by:

$$\text{slope of } PQ = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

One common notation for slope = $m$.

Sign of the slope:

* positive slope $\rightarrow$ Rising line
* negative slope $\rightarrow$ Falling line
* slope = 0 $\rightarrow$ Horizontal line
  $\Delta y = 0$
  $\Delta x$ is Any value
* slope undefined $\rightarrow$ Vertical line
  $\Delta x = 0$
  $\Delta y$ is Any value
Equation of the line:

To find the equation of a line we need two pieces of info:

1. slope of the line: $m$
2. A point on the line: $P = (x_1, y_1)$

Then the equation of the line is:

$$y - y_1 = m(x - x_1)$$

**Example 1:** Find the equation of a line with slope 3 that goes through the point $(2, 5)$.

**Solution:**

$$m = 3$$

$P = (2, 5)$

we have all we need

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 3(x - 2)$$

Now let's simplify the equation, by distributing 3 and have $y$ at one side and every other term at the other side of $=$.

$$y - 5 = 3x - 6$$

$$y = 3x - 6 + 5$$

$$y = 3x - 1$$

* The line equation in the form:

$$y = mx + b$$

is called the slope-intercept formula. This is because the value $b$ is actually the $y$-intercept of the line i.e. the line crosses the $y$-axis at $b$ at which $x = 0$.

**y-intercept**
Example 2: What is the equation of the line through \((-1,1)\) and \((1,-5)\)?

**Solution:**

We need to find the two pieces \(m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 1}{1 - (-1)} = -3\) \( \rightarrow \)

\[ P = (1,-5) \quad \Rightarrow \quad y - y_1 = m(x - x_1) \]

\[ y - (-5) = -3(x - 1) \]

\[ y + 5 = -3x + 3 \]

\[ y = -3x - 2 \]

*You can pick \((-1,1)\) and write the equation \(\Rightarrow\) the same answer*

You can pick any of the points as \((x_1, y_1)\) and \((x_2, y_2)\) but be consistent:

\((1,-5) = (x_2, y_2)\)

\((-1,1) = (x_1, y_1)\)

Example 3: What is the slope and \(y\)-intercept of the line \(2x - 3y = 6\)?

A. \(m = 2, \ y\text{-int} = 6\)

B. \(m = \frac{2}{3}, \ y\text{-int} = -2\)

C. \(m = \frac{3}{2}, \ y\text{-int} = -2\)

D. \(m = 3, \ y\text{-int} = 6\)

E. None of the above.

Rewrite the equation in the slope-intercept form:

\[
2x - 3y = 6 \quad \Rightarrow \quad -3y = -2x + 6 \quad \Rightarrow \quad y = \frac{-2}{-3}x + \frac{6}{-3}
\]

\[
\Rightarrow y = \frac{2}{3}x - 2
\]

Compare with \(y = mx + b\)

B
Example 4. Which of the following lines is parallel to 

\[ y - 2 = \frac{5}{2} (x+1) \] 

What’s \( m \)? Coefficient of \( x \) \( \Rightarrow m_1 = \frac{5}{2} \)

A. \( y + 4 = \frac{2}{5} (x-7) \) \( m_A = \frac{2}{5} \)

B. \( y + 1 = -\frac{5}{2} (x+1) \) \( m_B = -\frac{5}{2} \)

C. \( y = \frac{5}{2} (x+7) \) \( m_C = \frac{5}{2} \)

D. \( y - 6 = -\frac{2}{5} (x-4) \) \( m_D = -\frac{2}{5} \)

Which one is perpendicular? 

* Parallel lines have equal slopes 
  \( L_1 \parallel L_2 \Rightarrow m_1 = m_2 \)

* Perpendicular lines have slopes negative and reciprocal of each other. 
  \( L_1 \perp L_2 \Rightarrow m_1 = -\frac{1}{m_2} \)

\( \Rightarrow \) parallel to \( C \)

\( \Rightarrow \) perpendicular to \( D \)

Practice problem. Find the equation of the line that goes through the point \((1,2)\) and is perpendicular to the line \(2x - 3y = 10\).

Solution: 

\( P = (1,2) \)

The other piece is \( m \) \( \Rightarrow \) The line we look for is perp to \(2x - 3y = 10\) so they must satisfy \( m_1 = -\frac{1}{m_2} \) for their slope.

\[ 2x - 3y = 10 \quad \text{y} = \text{mx+b} \quad 2x - 10 = 3y \quad \Rightarrow \quad \frac{2}{3}x - \frac{10}{3} = y \]

Slope of perp. line

\[ m = -\frac{3}{2} \]

So \( m = -\frac{3}{2} \) and \( P = (1,2) \)

\[ y - y_1 = m(x-x_1) \]

\[ y - 2 = -\frac{3}{2} (x-1) \]

\( \Rightarrow \)

\[ y = -\frac{3}{2} (x-1) + 2 \]
**Functions**

What is a *function*? A function is a rule or relation that takes an input and assigns to it a unique output.

![Diagram of function concept]

Examples of functions:

1. **Set of Inputs**: Students in our course
   **Set of Outputs**: Final grade

2. \( f(x) = x^2 \)

   \[ 1 \rightarrow f(1) = 1^2 \rightarrow 1 = 1 \rightarrow f(1) = 1 \]
   \[ -2 \rightarrow f(-2) = (-2)^2 \rightarrow (-2)^2 = 4 \rightarrow f(-2) = 4 \]
   \[ \frac{1}{3} \rightarrow f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^2 \rightarrow \left(\frac{1}{3}\right)^2 = \frac{1}{9} \rightarrow f\left(\frac{1}{3}\right) = \frac{1}{9} \]

   ✓ The points help us sketch the graph of the function \( f(x) = x^2 \)
Question: Come up with a relation/rule that is NOT a function.

Notation for Domain is usually \( X \).

Domain: The set of all acceptable inputs that a function can take.

Range: The set of all produced outputs, usually denoted by \( Y \).

Example 5. How many of the following graphs are graphs of a function? Each input must go to only one output.

- **(I)** \( x \) NOT a function
- **(II)** Function
- **(III)** Function
- **(IV)** Function
- **(V)** Function
- **(VI)** Function

Answer: C. 4

How to test a graph for a function?

**Vertical Line Test:**

A given graph is the graph of a function, if any vertical line intersects the graph at most at one point.

* Discontinuities and jumps in the graph are OK, as long as we don't get more than y-value for any x-value.