Please hand in your homework

Reminder:

Quiz 1: Next Monday, Sept 24
  topics: trigonometry and exponential functions.

Clicker Q: Which of the following is true?

(I) \( \cos\left(\frac{15\pi}{2}\right) = 0 \)
(II) \( \sin\left(\frac{15\pi}{2}\right) = 1 \)
(III) \( \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \)
(IV) \( \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \)

A. I and II
B. II and IV
C. II, III and IV
D. I, III and IV
E. All of them

First we find the angles in the unit circle, then we find the endpoint corresponding to the terminal side of the angle and we read its x and y coordinates: 

- \( x = \cos \theta \), \( y = \sin \theta \)

\[ \theta = \frac{15\pi}{2} = \frac{14\pi + \pi}{2} = \frac{14\pi}{2} + \frac{\pi}{2} = 7\pi + \frac{\pi}{2} \]

7 half-circles

\((0, -1) \Rightarrow \cos\left(\frac{15\pi}{2}\right) = 0 \), \( \sin\left(\frac{15\pi}{2}\right) = -1 \Rightarrow \text{II is wrong} \)

\[ \theta = -\frac{\pi}{3} \rightarrow \frac{\pi}{3} \text{ in } \theta \text{ direction } \rightarrow 4^{th} \text{ quadrant } \rightarrow \sin \theta \]

\[ \text{table: } \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \rightarrow \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \rightarrow \text{III} \checkmark \]

\[ \theta = \frac{4\pi}{3} = \frac{3\pi + \pi}{3} = \pi + \frac{\pi}{3} \rightarrow 3^{rd} \text{ quadrant } \rightarrow \sin \theta \]

\[ \text{table: } \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \rightarrow \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \rightarrow \text{IV} \checkmark \]
Example 1. Find all real $x$ values in $[0, 2\pi]$ such that

$$2\cos x \sin x + \sqrt{3} \cos x = 0$$

What is the angle $x$ that solves this equation:

$$\cos x (2\sin x + \sqrt{3}) = 0$$

or

$$2\sin x + \sqrt{3} = 0$$

$\Rightarrow$ $2\sin x = -\sqrt{3}$

$\Rightarrow$ $\sin x = -\frac{\sqrt{3}}{2}$

*Angles whose $\sin$ is $-\frac{\sqrt{3}}{2}$?

Reference: $\frac{\pi}{3}$

3rd or 4th quadrant

$\Rightarrow$ $\sin x = -\frac{\sqrt{3}}{2}$

$\Rightarrow$ $x = \frac{\pi}{2}, x = \frac{3\pi}{2}$

So the $x$ values that solve the equation are:

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{4\pi}{3}, \frac{5\pi}{3}$
Example 2. Find the domain of \( y = \tan x \)

We know \( \tan x = \frac{\sin x}{\cos x} \), so we need to exclude the \( x \)-values that make the denominator \(-1\) from the domain, so let's solve

\[ \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \]

There are only in one cycle, if we rotate on the unit circle and keep adding half-cycles to \( \frac{\pi}{2} \) we'll get all the values in other cycle; we can do this in the negative direction as well so:

\[ \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{\pi}{2} + \pi, \frac{3\pi}{2} + \pi, \frac{5\pi}{2} + \pi, \ldots \]

\[ \Rightarrow x = \frac{\pi}{2} + n\pi \quad \text{for} \quad n = 0, \pm 1, \pm 2, \ldots \]

\[ \text{Domain: } \mathbb{R} - \left\{ \frac{\pi}{2} + n\pi, n = 0, \pm 1, \pm 2, \ldots \right\} \]

Example 3. Find all \( x \) in \( \mathbb{R} \) satisfying

\[ \cos x \sin x + 1 = 1 \]

\[ \cos x \cdot \sin x = 1 - 1 = 0 \]

\[ \downarrow \quad \text{Ex. 2} \quad \downarrow \]

\[ \cos x = 0 \quad \text{or} \quad \sin x = 0 \]

\[ \Rightarrow x = \frac{\pi}{2} + n\pi \]

\[ n = 0, \pm 1, \pm 2, \ldots \]

\[ \text{Ex. 2} \]

So

\[ x = n\pi \quad \text{or} \quad x = \frac{\pi}{2} + n\pi \]

For \( n = 0, \pm 1, \pm 2, \ldots \)

Note: We can combine the two sets of angles into one:

\[ x = \frac{n\pi}{2} \]

\[ n = 0, \pm 1, \pm 2, \ldots \]