Recall our first class: Our main tool in differential calculus is derivative and derivative is based upon tangent line to the graph of a function \( y = f(x) \).

To see what \( \text{limit} \) means, let's start with graph of a function:

**Goal**: info about the slope of the tangent line.

We need to have info about \( f(x) \) "close" to the tangency point \( x = a \) and for this we need \( \text{limits} \).

**Question**: When \( x \) "gets close" to 1

\( f(x) \) "gets close" to 2

* Track function values on the y-axis when x gets close to 1.

* Translation of "As \( x \to 1 \) then \( f(x) \to 2 \)" in maths language is:

\[ \lim_{x \to 1} f(x) = 2 \]
We can compute some values of \( f(x) \) for \( x \)'s close to 1 and see what's going on.

Let's make a table of values with some \( x \) close to 1 and find \( f(x) = x + 1 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = x + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9, 0.99</td>
<td>1.9, 1.99</td>
</tr>
<tr>
<td>1.1, 1.01</td>
<td>2.1, 2.01</td>
</tr>
</tbody>
</table>

\[ \lim_{x \to 1} f(x) = 2. \]

**Question 2.**

When \( x \) "gets close" to \(-2\), \( f(x) \) "gets close" to \(-1\).

Again let's track \( y \)-values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = x + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.1, -2.01</td>
<td>-1.1, -1.01</td>
</tr>
<tr>
<td>-1.9, -1.99</td>
<td>-0.9, -0.99</td>
</tr>
</tbody>
</table>

But \( f(-2) = 1 \).

* From graph and from the table of values, it can be seen that:
  As we approach \(-2\) on the \( x \)-axis, the values of function on the \( y \)-axis approaches to \(-1\) i.e. \( \lim_{x \to -2} f(x) = -1 \).

Note that the exact value of the function and \( x = -2 \) is \( f(-2) = 1 \).

So **in finding limit the exact value at the given \( x \)-value does NOT matter. We are checking the values close to that \( x \) NOT exactly at \( x \).**
Limit

The limit is an operation that we perform on a function
\[ y = f(x) \].

\[ \lim_{x \to a} f(x) = L \]

* a constant number on the \( y \)-axis.

\( x \) \rightarrow \( a \)

* input variable

output variable

\[ f(x) \]

* a constant number on the \( x \)-axis.

Meaning: As the \( x \)-values are getting closer and closer
to the value "\( a \)" , the \( y \)-values of the function
are getting closer and closer to the value \( L \) on the \( y \)-axis.

* Again, note that limit is all about being close to some value NOT exactly at that value.

Note: Different letters can be used for variable on the \( x \)-axis
and functions and the given numbers.

* \[ \lim_{t \to a} g(t) = L \]

\( t \) \rightarrow \( a \)

* \[ \lim_{r \to c} h(r) = M \]

\( r \) \rightarrow \( c \)

some number on the \( x \)-axis

Don't get confused with the notation.

Understand the concept and translate that to any mathematical notations.
Example: Let \( f(x) \) be a function given by the following graph.

Evaluate the following limits:

(1) \( \lim_{{x \to 1}} f(x) = 2 \)  
   * In this case: \( \lim_{{x \to 1}} f(x) = 2 = f(1) \)
   
   A. 1  
   B. 2  
   C. 2.5  
   D. 3  
   E. Does NOT exist.

(2) \( \lim_{{x \to 2}} f(x) = 2 \)  
   * In this case: \( \lim_{{x \to 2}} f(x) = 2 \) but \( f(2) = 1 \)
   
   A. 1  
   B. 2  
   C. 2.5  
   D. 3  
   E. Does NOT exist.

(3) \( \lim_{{x \to 3}} f(x) = 1 \)  
   * In this case: \( \lim_{{x \to 3}} f(x) = 1 \) but \( f(3) \) is undefined.
   
   A. 1  
   B. 2  
   C. 2.5  
   D. 3  
   E. Does NOT exist (DNE)

(4) \( \lim_{{x \to 5^+}} f(x) = 2 \)  
   \( \lim_{{x \to 5^-}} f(x) = 3 \)  
   * NO unique limit
   
   A. 1  
   B. 2  
   C. 2.5  
   D. 3  
   E. DNE
Practice: Use the following graph for the function \( y = g(x) \) to find the limits.

\[
\begin{align*}
\lim_{x \to 0} f(x) &= \quad \text{Compare with } f(0) \\
\lim_{x \to 1} f(x) &= \quad \text{Compare with } f(1) \\
\lim_{x \to 2} f(x) &= \quad \text{with } f(2) \\
\lim_{x \to 3} f(x) &= \quad \text{with } f(3) \\
\lim_{x \to 4} f(x) &= \quad \text{with } f(4)
\end{align*}
\]