MATH 190, Lab 4: Oct 2 and 4, 2018

Work through the following problems while the TAs circulate. When you have completed the problems (to the satisfaction of the facilitators) you can spend the rest of the lab working on the limit practice problems given in the class worksheet (Lecture 11).

Warm-up.

Discuss in your groups why the following statement is false, come up with an example that contradicts this statement. Your example can be a graph or an algebraic expression for a function.

If a function $f$ is not defined at $x = a$ then $\lim_{x \to a} f(x)$ does not exist.

Problems

1. Evaluate the following limits.

(a) $\lim_{x \to 1} \frac{x^2 + 3x + 2}{x^2 - x - 2}$

(b) $\lim_{t \to 2} \frac{\ln(t - 1)}{t - 3}$

(c) $\lim_{x \to 1} \frac{x - 1}{|x - 1|}$

(d) $\lim_{h \to 0} f(h)$ where $f(h) = \begin{cases} \frac{\sqrt{3h + 1} - 1}{h} & h \neq 0 \\ 1 & h = 0 \end{cases}$

(e) $\lim_{x \to 2} f(x)$ where $f(x) = \begin{cases} \frac{x^3 - 2x^2}{x^2 - 4} & x < 2 \\ -x^3 + 3 & x \geq 2 \end{cases}$

2. Match each limit in Question 1 with one of the following statements describing the limit behaviour of a function.

(i) $\lim_{x \to a} f(x) = L$ and $f(a) = L$.

(ii) $\lim_{x \to a} f(x) = L$ but $f(a)$ is undefined.
(iii) \( \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = \lim_{x \to a} f(x) = L \), \( f(a) \) is defined but \( f(a) \neq L \).

(iv) \( \lim_{x \to a} f(x) = \text{DNE} \) but \( \lim_{x \to a^+} f(x) = f(a) \).

(v) \( \lim_{x \to a} f(x) = \text{DNE} \) and \( f(a) \) is undefined.

3. For each of the statements in Question 2, sketch a function graph that represents the situation described in the statement.
Reflection.

Have the problems in this lab’s worksheet helped you better understand the concept of limit of a function? State a summary of your understanding from these activities.