Warm-up.

Discuss in your group the meaning of the following statements, their translation in mathematical formulas and their graphical interpretation.

- A function has a vertical asymptote at some number $c$.
- A function has a horizontal asymptote at some number $b$.

Problems.

1. Compute the following limits.
   
   (a) $\lim_{x \to -1^+} \frac{x + 1}{1 - x^2}$
   
   (b) $\lim_{x \to 1^+} \frac{x + 1}{1 - x^2}$
   
   (c) $\lim_{x \to 3^+} \ln(x - 3)$
   
   (d) $\lim_{x \to -\infty} x^3 - 100x^2 + 1$

2. Find all vertical and horizontal asymptotes of the following functions. Ensure you compute (and show the computation of) all the relevant limits. Sketch a rough graph of the function around its asymptotes.

   (a) $\frac{x - 4}{x^2 - 2x - 8}$
   
   (b) $\frac{x^2 - 2x - 8}{x - 4}$
   
   (c) $\frac{e^x}{e^x - 2}$

3. Use the limit definition (and not any other method) to find the slope of the tangent line to the function $f(x) = \frac{1}{x}$ at $x = 2$ and use it to find the equation of the tangent line at this point.