Show all your work. No calculators, no books/notes are allowed.

Name (please print): ________________________________
Student number: ________________________________

1. Questions (a)-(e) below all concern the function

\[ f(x) = \frac{1}{2x} \]

(a) [4 points] Use the definition of the derivative (and not any other method) to find \( f'(2) \).

\[
\begin{align*}
\frac{f(x+h) - f(x)}{h} & \rightarrow \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
\frac{f(2+h) - f(2)}{h} & \rightarrow \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}
\end{align*}
\]

\[
\begin{align*}
\frac{1}{2(2+h)} & - \frac{1}{2} \\
\lim_{h \to 0} & = \lim_{h \to 0} \frac{1}{2(2+h)} - \frac{1}{2} = \frac{0}{0}
\end{align*}
\]

*Do NOT forget to distribute (-) in all the term following it.

(b) [4 points] Use derivative rule(s) to find \( f'(2) \).

\begin{align*}
\text{Multiple methods:} & \quad \text{Power rule:} \\
& \quad \text{Quotient Rule:} \\
& \quad \text{\( y = \frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2} \cdot x^{-1} \) \Rightarrow \( y' = \frac{1}{2} \cdot (-1) \cdot x^{-1.1} \) \Rightarrow \( -\frac{1}{2} \cdot \frac{1}{x^2} \)} \\
& \quad \text{\( y' = \frac{(1) \cdot 2x - 1 \cdot (2x)}{(2x)^2} \)} \\
& \quad \text{\( = \frac{0 \cdot 2x - 2}{4x^2} \)} \\
& \quad \text{\( = -\frac{2}{4 \cdot x^2} = -\frac{1}{2x^2} \)}
\end{align*}
(c) [3 points] Find the equation of the tangent line to the graph of $f(x)$ at $x = 2$.

Slope: $m_{\text{tan}} = -\frac{1}{8}$

Point: $x = 2 \Rightarrow y = f(2) = \frac{1}{2 \cdot 2} = \frac{1}{4} \Rightarrow (2, \frac{1}{4})$

$y - y_0 = m(x - x_0)$

$y - \frac{1}{4} = -\frac{1}{8}(x - 2) \Rightarrow y = -\frac{1}{8}(x-2) + \frac{1}{4}$

(d) [2 points] Which option describes the graph of $f$ correctly? Give reasons for your choice.

i. Always increasing

ii. Always decreasing

iii. Increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$

iv. Decreasing on $(0, \infty)$ and increasing on $(-\infty, 0)$

\[ f'(x) = -\frac{1}{2x^2} \text{ for all values of } x; f'(x) \text{ is negative so slope of the tangent line always negative } \Rightarrow \text{ function itself is everywhere decreasing} \]

(e) [Bonus] [2 points] Sketch a rough graph of $f$ and its tangent line at $x = 2$.

$y = \frac{1}{2x}$ is similar to $y = \frac{1}{x}$

\[ y = \frac{1}{x} \]