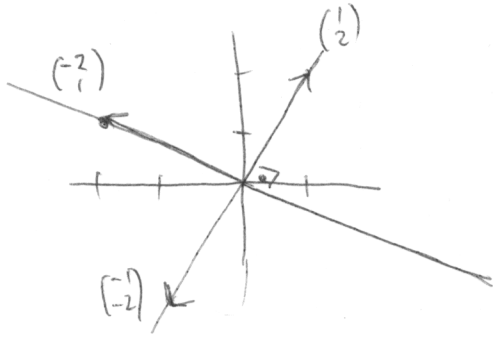
Problem 13 Reflection across $x+2y=0$.

The line is $\text{span}\begin{pmatrix}-2\\1\end{pmatrix}$ because $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-2\\1\end{pmatrix}$ solves $x+2y=0$.

the perpendicular line is $\text{span}\begin{pmatrix}1\\2\end{pmatrix}$ (the slope is the negative reciprocal.)



The points on the mirror $\text{span}\begin{pmatrix}-2\\1\end{pmatrix}$ are fixed, so $\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}-2\\1\end{pmatrix}=\begin{pmatrix}-2\\1\end{pmatrix}$.

The points on the line perpendicular to the mirror get mapped to their <u>negatives</u> (directly across the origin.)

so $\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}-1\\-2\end{pmatrix}$.

This gives the system of equations

$$\begin{aligned}-2a+b&=-2\\-2c+d&=1\\a+2b&=-1\\c+2d&=-2\end{aligned}$$

with augmented coefficient matrix

$$\left(\begin{array}{cccc|c}-2&1&0&0&-2\\0&0&-2&1&1\\1&2&0&0&-1\\0&0&1&2&-2\end{array}\right)\longrightarrow\left(\begin{array}{cccc|c}1&0&0&0&3/5\\0&1&0&0&-4/5\\0&0&1&0&-4/5\\0&0&0&1&-3/5\end{array}\right)$$

So the matrix is $\boxed{\begin{pmatrix}a&b\\c&d\end{pmatrix}=\frac{1}{5}\begin{pmatrix}3&-4\\-4&-3\end{pmatrix}.}$

<u>Check:</u> $\frac{1}{5}\begin{pmatrix}3&-4\\-4&-3\end{pmatrix}\begin{pmatrix}-2\\1\end{pmatrix}=\frac{1}{5}\begin{pmatrix}-10\\5\end{pmatrix}=\begin{pmatrix}-2\\1\end{pmatrix}$ ✓

$\frac{1}{5}\begin{pmatrix}3&-4\\-4&-3\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}=\frac{1}{5}\begin{pmatrix}-5\\-10\end{pmatrix}=\begin{pmatrix}-1\\-2\end{pmatrix}$ ✓