Quiz 5 - Math221 - Sec 203 Feb 15, 2017

Name: _____ Student number: _____

No books. No notes. No calculator. No electronic device of any kind.

1. (*4 points*) Let A and its reduced row echelon form (RREF) be:

$$A = \begin{pmatrix} 1 & 3 & 3 & 2 & -9 \\ 0 & 0 & 2 & -8 & 2 \\ 0 & 0 & 0 & 7 & 1 \\ 0 & 0 & 0 & 7 & 10 \end{pmatrix} \sim \begin{pmatrix} \boxed{1} & 3 & 0 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 & 0 \\ 0 & 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 & \boxed{1} \end{pmatrix}$$

$Ax = 0$

$x_1 = -3x_2$

$x_3 = 0$

$x_4 = 0$

$x_5 = 0$

x_2 free

A. Find a basis for the column space of A

B. Find a basis for the nullspace of A

A. $C = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -8 \\ 7 \\ 7 \end{pmatrix}, \begin{pmatrix} -9 \\ 2 \\ 1 \\ 10 \end{pmatrix} \right\}$

B. $Ax = 0 \Rightarrow x = \begin{pmatrix} -3x_2 \\ x_2 \\ 0 \\ 0 \\ 0 \end{pmatrix} = x_2 \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow N = \left\{ \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

2. (*4 points*) Given that $B = \left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right)$ is a basis for $\mathbb{R}^2$, find the B-coordinate vector of the following vectors:

$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

$A[x]_B = x \Rightarrow [x]_B = A^{-1}x$

$[x]_B \xrightarrow{A} x$

$[x]_B \xleftarrow{A^{-1}} x$

x

- $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ $\quad \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

- $\begin{bmatrix} a \\ b \end{bmatrix}$ $\quad \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2a - b \\ -3a + 2b \end{bmatrix}$

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3. (*4 points*) Answer each question "True" or "False". No explanation is necessary.

1. If A is a 3×5 matrix, then the dimension of $Nul(A)$ must be at least 2. T
2. If a linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^3$ is onto, then the matrix T must have rank 5. F
3. If A is 3×3 matrix such that $A^3 = 0$ the Rank $A < 3$. T
4. If AB is not invertible, then A is not invertible. F

1) A: 3 rows, 5 cols $\Rightarrow$ maximum nb. of pivots = 3

$\Rightarrow$ at least 2 free variable $\Rightarrow \dim Null(A) \geq 2$ ✓

2) rank T = 3 (3 rows)

3) Rank A cannot be more than 3 (because it is 3×3)

Can we have Rank A = 3 ? No, if Rank A = 3 then

A is invertible $\Rightarrow A^3$ is invertible $\Rightarrow A^3 \neq 0$ ↳ not invertible

So Rank A must be less than 3 : Rank A < 3 ✓

4) counterexample: B = 0 A → it can be any invertable matrix

$\Rightarrow AB = 0 \Rightarrow AB$ not invertible

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