Quiz 8 - Math221 - Sec 203 Mar 22, 2017

Name: ______________________ Student number: ____________________

No books. No notes. No calculator. No electronic device of any kind.

1. (*12 points*) Let $B = \{b_1, b_2\}$ be a basis for $\mathbb{R}^2$. If $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$,

(a) find $[b_2]_B$.

(b) if $a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, find $[a]_B$.

(c) if $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ is a linear map, and

$$T(b_1) = 2b_1 - b_2 \qquad T(b_2) = b_1 + b_2$$

Find $[T]_B$ and T.

Solution:

$$P = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \qquad P^{-1} = \begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix}$$

(a) $[b_2]_B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(b) $[a]_B = P^{-1}a = \begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

(c)

$$[T(b_1)]_B = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad [T(b_2)]_B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$[T]_B = [[T(b_1)]_B \qquad [T(b_2)]_B] = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$$

$$T = P[T]_B P^{-1} = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 6 & -7 \\ 3 & -3 \end{pmatrix}$$