Problem 1 (Do not submit): (ODE Review) Find the general solutions of the following equations:

a. \((1 + x^2) y' + 2xy = \cot x\)
b. \(xy' = y \ln y\)
c. \(y'' - 5y' + 4y = 0\)
d. \(4y'' + 4y' + 2y = 0, y(0) = 2, y'(0) = 3\)
e. \(y'' - 6y' + 9y = 2 \cos(\sqrt{3}x)\),

Problem 2 (Submit): (ODE Review) Find the general solutions of the following equations:

a. \(2x^2y'' - xy' + y = x\)
b. \(x^2y'' - xy' + y = 0\)
c. \(x^2y'' - xy' + 5y = 0\)

Problem 3 (Submit): (Power series solution): Consider the following linear ODEs:

\[
\begin{align*}
y' + xy &= 0 \quad (1) \\
y'' + y &= 0 \quad (2)
\end{align*}
\]

a. Use the methods you learned for solving ODEs to solve equations (1) and (2).

b. Expand the solutions to (1) and (2) as Taylor series about the point \(x_0 = 0\).

c. Now for each of equations (1) and (2) assume a power series solution of the form

\[
y(x) = \sum_{n=0}^{\infty} a_n x^n
\]

obtain a recursion for the coefficients \(a_n\). Use these recursions to determine the series representation of each solution. Compare these results to the series obtained in part b above.

Problem 4 (Do not submit): (Power series solution): Consider the following first order linear ODEs:
a. Solve the differential equations (4) and (5) using the appropriate integrating factors.

b. Expand the solution to (4) as Taylor series about the point \( x_0 = 0 \). Expand the exponential in the solution to (5) as a power series.

c. Now for (4) assume a power series solution of the form

\[
y(x) = \sum_{n=0}^{\infty} a_n x^n
\]  

obtain a recursion for the coefficients \( a_n \). Use these recursions to determine the series representation of the solution. Compare this result to the series obtained in part b above.

d. Try using the same power series expansion (6) to solve (5). What happens?

e. Consider the following recursive strategy to generate an approximate solution to (5). Rewrite (5) as

\[
xy' + 2y = xy
\]  

Now assuming \( x \to 0 \) and discarding the right hand side of (7), find a first order approximation \( y_0 \) as the solution to

\[
xy_0' + 2y_0 = 0
\]

Now substitute \( y_0 \) on the right side of (7) and solve for \( y_1 \)

\[
xy_1' + 2y_1 = xy_0
\]

Continue this process till you obtain \( y_2 \). How does \( y_2 \) compare with the series solution to (5) obtained in b? Can you use this series to motivate a modification to the series expansion (6) that would be appropriate to use to obtain a series solution to (5)?