Math 257/316, midterm 1, section 201

February 26, 2018
Duration: 50 min
Total marks: 100
Number of questions: 2

First Name: Last Name:

SID: Section:

Signature:

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Instructions:

- Notes, calculators, phones, computers and your cheat sheets are not allowed.
- The formula sheet is on the last page of the exam booklet.
- Show all your work. A correct answer without the intermediate steps will not receive credit.
1. (50 marks) Consider the second order differential equation
\[ Ly = (1 - x^2)y'' + xy' - y = 0, \]  
(1)
(a) Classify the points \(-\infty < x < \infty\) as ordinary points, regular singular points and irregular singular points.
(b) Use the appropriate series expansion about the point \(x = 0\) to determine two independent solutions to equation (1). You only need to determine the first three non-zero terms in each case.
(c) What can you say about the minimum radius of convergence of the series solution you found in (b)?

2. (50 marks) Apply the method of separation of variables to determine the solution to the one dimensional heat equation subject to the following boundary conditions and initial condition:
\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \text{ for } t > 0, \quad -1 \leq x \leq 1,
\]
\[
\text{BC: } u(-1, t) = u(1, t), \quad \text{and } \frac{\partial u(-1, t)}{\partial x} = \frac{\partial u(1, t)}{\partial x}
\]
\[
\text{IC: } u(x, 0) = f(x) = \begin{cases} 0 & \text{if } -1 \leq x < 0 \\ 1 & \text{if } 0 \leq x \leq 1 \end{cases}
\]
Please show all the cases when solving the appropriate eigenvalue problem.